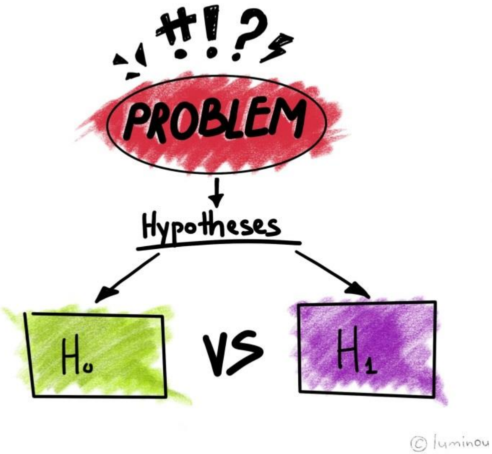
لجان الدفعات

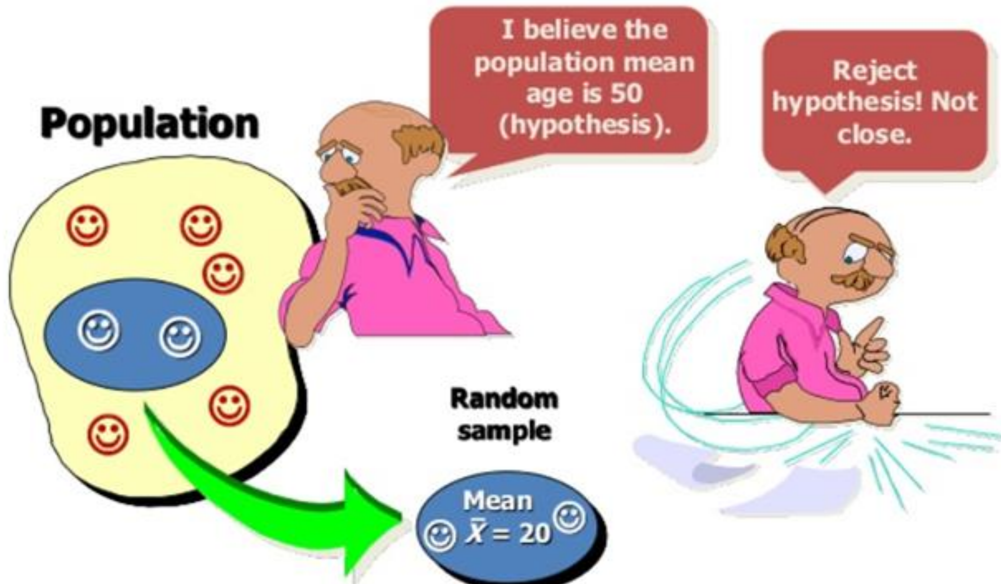
STATISTICS

MORPHINE ACADEMY

43

Hypothesis Testing





Hypothesis Testing

الهدف العام: استبعاد الصدفة كتفسير محتمل لنتائج الدراسة البحثية.

- The general goal of a hypothesis test is to rule out chance (sampling error) as a plausible explanation for the results from a research study.

التقنية

- Hypothesis testing is a technique to help determining whether a hypothesis is true (e.g. treatment, procedure has an effect in a population), or simply if a relationship exists between two or more variables.

التقنية: تساعد في تحديد ما إذا كان هناك تأثير لعلاج أو إجراء في السكان أو وجود علاقة بين متغيرات متعددة.

Null Hypothesis as an Assumption to be Challenged

نبدأ باعتقاد او افتراض انه صحة معامل المجتمع صحيحة.

- We might begin with a belief or assumption that a statement about the value of a population parameter is true.
- We then using a hypothesis test to challenge the assumption and determine if there is statistical evidence to conclude that the assumption is incorrect.

نستخدم اختبار الفرضيات لتحدي هذا الافتراض ونحدد ما اذا كان هناك دليل احصائي يثبت الخطأ.

- In these situations, it is helpful to develop the null hypothesis first.

Example

- A new drug is developed with the goal of lowering blood pressure more than the existing drug.
- **Null hypothesis**
- ➢ The new drug does not lower BP more than the existing drug.
- **Alternative hypothesis**
- ➢ The new drug lowers blood pressure more than the existing drug.

Null & Alternative Hypothesis

Gabapentin has no pharmacological effect

- The mean for population A is 20 (H_0: $\mu = 20$)
- The mean for population A is less than 20 (H_0: $\mu \le 20$)
- The mean for population A is larger than 20 (H_0: $\mu \ge 20$)

Gabapentin has a pharmacological effect

- The mean for population A is not 20 (H_1: $\mu \neq 20$)
- The mean for population A is larger than 20 (H_1: $\mu > 20$)
- The mean for population A is less than 20 (H_1: $\mu < 20$)

 # Accepting or rejecting a hypothesis is not a proof of the hypothesis!
Null hypothesis can be true or false, we only can reject it or not to reject it

قبول أو رفض الفرضية مش دليل قاطع، بل نرفض H_0 او لا نرفضها فقط

Null Hypothesis vs Alternative Hypothesis

Null hypothesis H_0	Alternative Hypothesis H_1
There is no relationship or difference	There is a relationship or difference
Refers to the population	Refers to the examined sample
Research aims to reject the null	Research aims to accept the alternative
Represent an original assumption	Prove statistically a systemic difference or relationship
Assumes a difference is due to chance	Assumes that difference is less likely to be due to chance.

Errors in hypothesis testing

TRUTH

DECISION

	Ho is true (A=B)	Ho is false (A≠B)
Reject Ho (A≠B)	Type I error "giving a treatment that does not work"	Correct
Do not reject Ho (A=B)	Correct	Type II error "not giving a treatment that works"

Step 1

- Set your hypothesis

  Set both H_0 & H_1.

Example

It is believed that a candy machine makes chocolate bars that are on average 5g. A worker claims that the machine after maintenance no longer makes 5g bars. Write Ho and Ha.

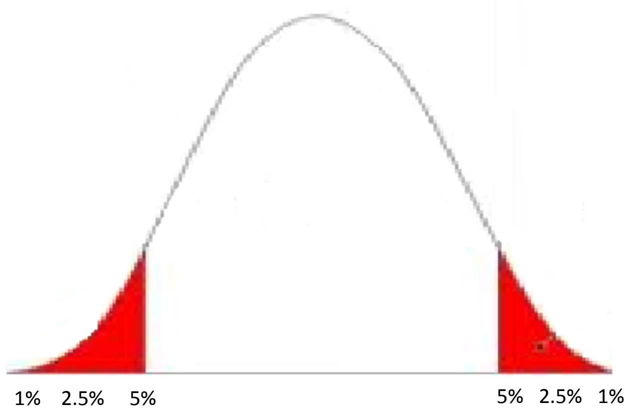
Answer:
$H_0: \mu = 5$
$H_1: \mu \neq 5$

A company has stated that their straw machine makes straws that are 4 mm diameter. A worker believes the machine no longer makes straws of this size and samples 100 straws to perform a hypothesis test with 99% confidence.

Answer:
$H_0: \mu = 4$
$H_1: \mu \neq 4$

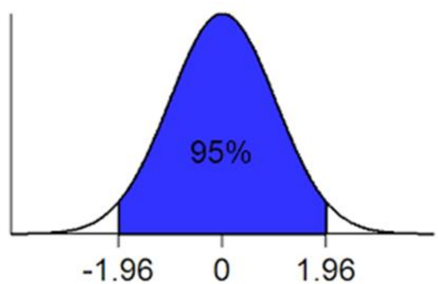
Step 2

- Set level of significance associated with the hypothesis.



Step 3

- Set the critical value needed to reject the null hypothesis (from Tables).
- You might need the table of Z-test, t-test, F-test.
- Based on the chosen table, look for the cut off value based on the level of significance you determined in step 2.



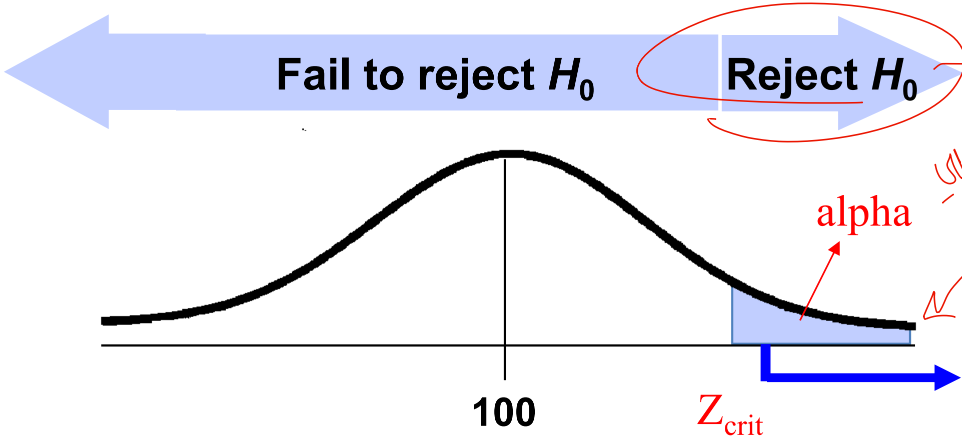
Right-tailed tests

H_0: μ = 100

H_1: μ > 100

Points Right

يتم رفض H_0 اذا كانت القيمة الاختبارية تقع في المنطقة الى اليمين من القيمة الحرجة (z-crit) ، وهي المنطقة المظللة التي تمثل مستوى الدلالة (α)



Values that differ "significantly" from 100

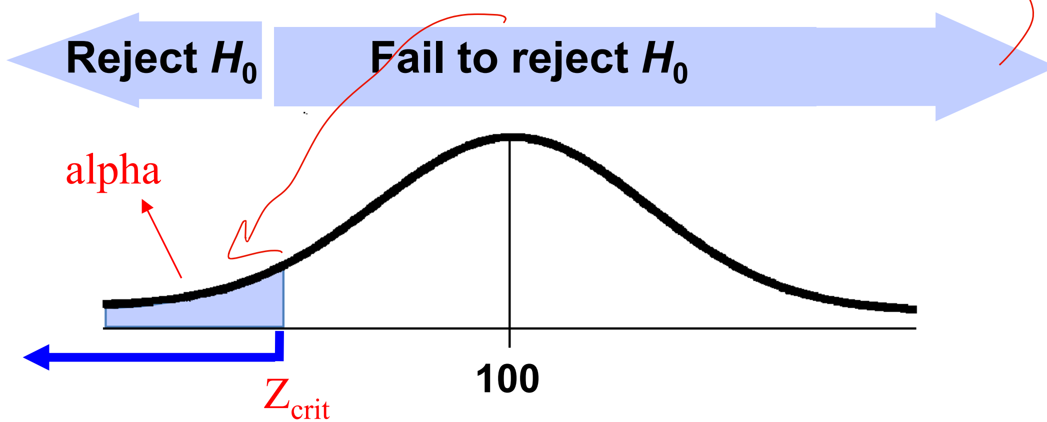
Left-tailed tests

H_0: $\mu = 100$

H_1: $\mu < 100$

Points Left

يتم رفضنا H_0 اذا كانت القيمة الاختبارية تقع في المنطقة الى اليسار
من القيمة الحرجة (z-crit) وهي المظللة.



Values that differ "significantly" from 100

Two-tailed hypothesis testing

- H_A is that μ is *either* greater or less than μ_{H0}

  H_A: $\mu \neq \mu_{H0}$

  2 او 7
  من 100

- α is divided equally between the two tails of the critical region

  α يقسم بالتساوي بين الذيلين.

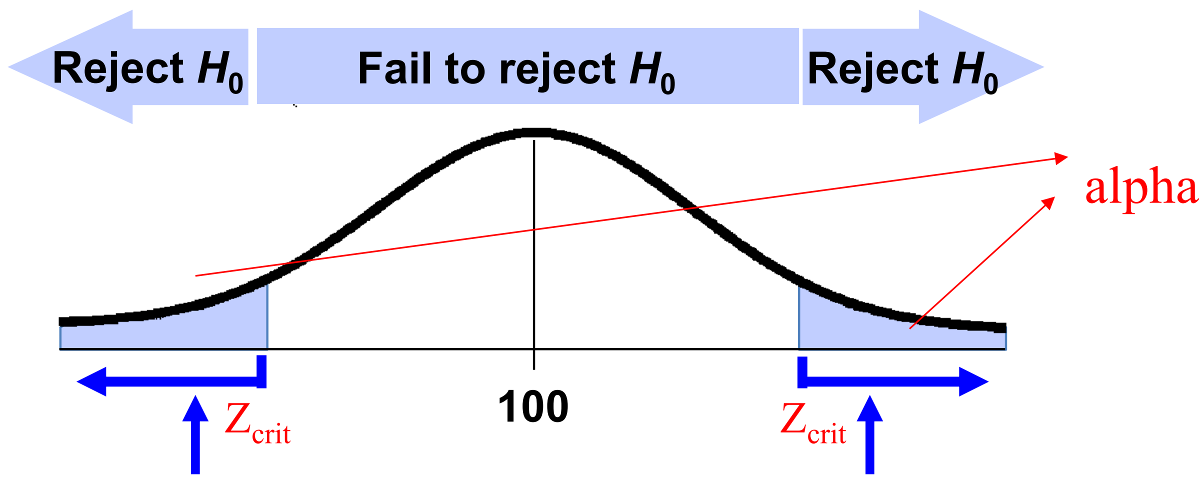
يتم رفض H_0 اذا كانت النتيجة الاختبارية تقع في احدى المنطقتين المظللتين (اقل من z-crit السالب or اكبر من z-crit العلوي)

Two-tailed hypothesis testing

H_0: μ = 100

H_1: μ ≠ 100

Means less than or greater than



Values that differ significantly from 100

القيم التي تختلف بشكل ملحوظ عن ١٠٠ تقع في كلا الجانبين .

One tale critical values

alpha .05, Z_{crit}=1.64;

alpha .01, Z_{crit}=2.33

Two tale critical values

alpha .05, Z_{crit}=1.96;

alpha .01, Z_{crit}=2.58

Normal Distribution
(Population Standard Deviation σ is known or Standard Deviation σ is known but n is Large)

n ≥ 30

Normal Distribution

σ is known

[n < 30 (small) or n ≥ 30 (large)]

$$z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

Normal (σ is unknown) or unknown or non-normal distribution but n ≥ 30 (large)

$$z = \frac{\bar{X} - \mu_0}{S / \sqrt{n}}$$

Normal Distribution
(Population Standard Deviation σ is Unknown and n is small)

Normal Distribution

σ is unknown

N < 30 (small)

$$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$$

Summary of Forms for Null and Alternative Hypotheses about a Population Mean

$\mu = \mu_0 \rightarrow H_0$

The equality part of the hypotheses always appears in the null hypothesis H_0. In general, a hypothesis test about the value of a population mean μ must take one of the following three forms (where μ_0 is the hypothesized value of the population mean).

a) One – tailed test

i. $H_0: \mu = \mu_0$ vs $H_1: \mu < \mu_0$ (less than or smaller than)

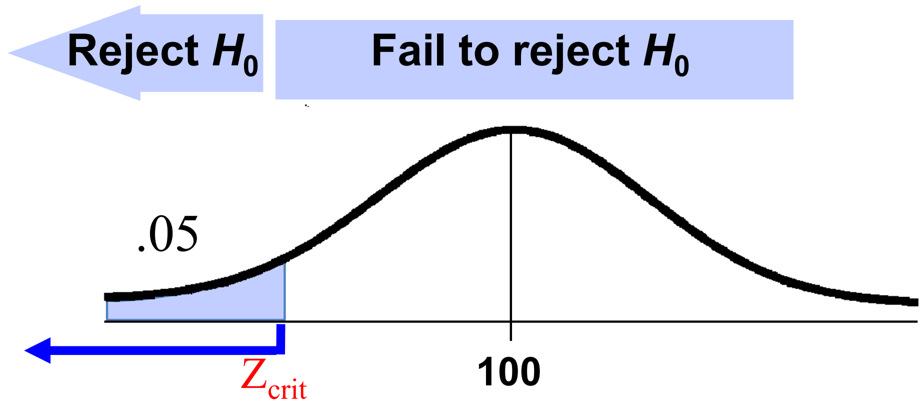
ii. Upper (right)-tailed test

$H_0: \mu = \mu_0$ vs $H_1: \mu > \mu_0$ (more than or greater than)

b) Two – tailed test

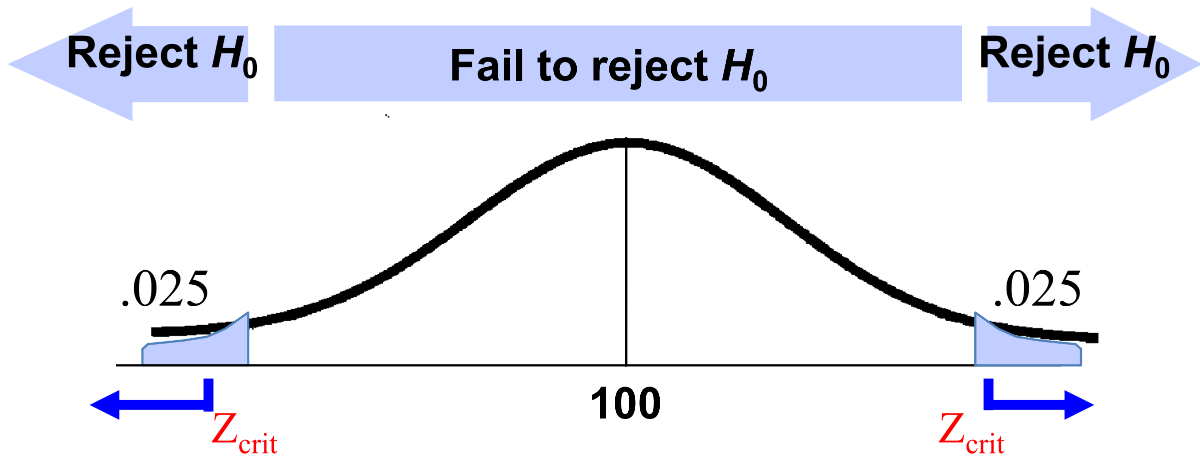
$H_0: \mu = \mu_0$ vs $H_1: \mu \neq \mu_0$ (does not equal or different from)

One tail



Values that differ “significantly” from 100

Two tail



Values that differ significantly from 100

Step 5

t-value z-value

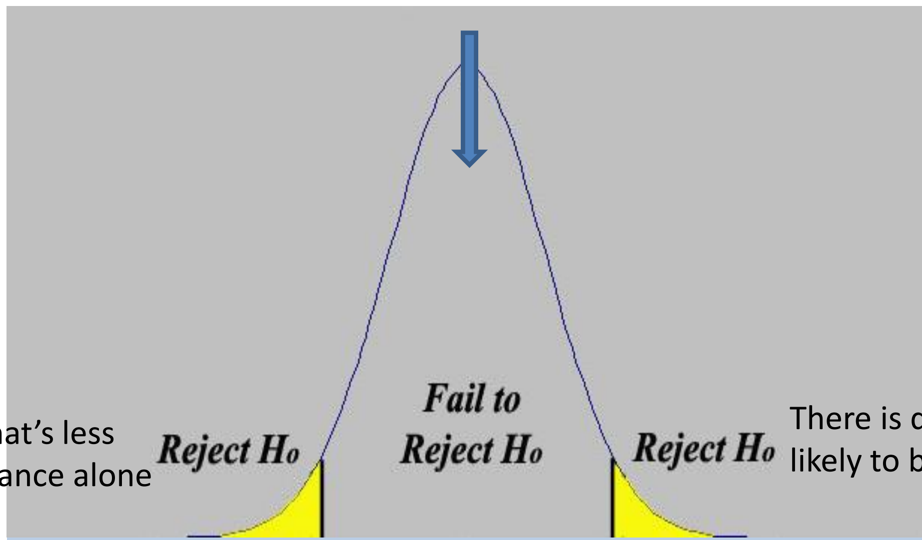
- Compare the test statistics value with the critical value of rejection.

Does Z-value = or ≠ value from the table

Step 6

- Decide whether to reject the null hypothesis and confidence statement.

There is difference that's less likely to be due to chance alone



There is difference that's less likely to be due to chance alone

critical

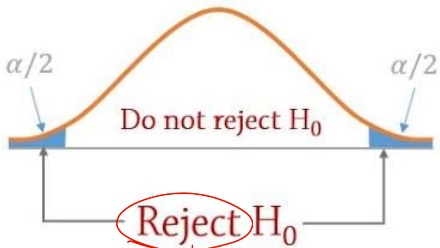
اذا كانت القيمة الاختبارية تقع بالمنطقة المظللة (الحرجة) ← يتم رفض Ho، مما يشير الى ان الفرق قد لا يكون بسبب الصدفة
وحدها. اذا لم تقع في تلك المنطقة، يتم الاحتفاظ بـ Ho.

Hypothesis Testing

One-tailed

Two-tailed

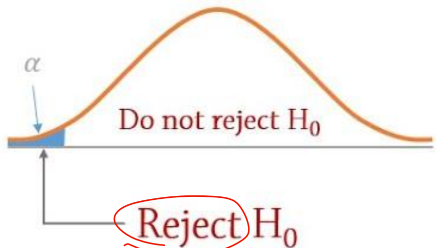
$$H_0:\ \mu = 23$$
$$H_1:\ \mu \neq 23$$



يكون لما تكون القيمة أقل او أكبر
من الحدود النقدية الموزعة ع الذيلين.

Left-tailed

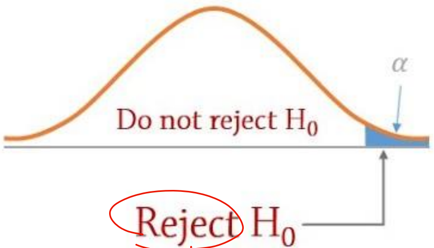
$$H_0:\ \mu \geq 23$$
$$H_1:\ \mu < 23$$



يكون لما تكون القيمة اقل
من الحد النقدي.

Right-tailed

$$H_0:\ \mu \leq 23$$
$$H_1:\ \mu > 23$$



يكون لما تكون القيمة أكبر
من الحد النقدي.

Rejection Rule

الفرضيات

Lower (left)-tailed test

Hypotheses

H_0: $\mu = \mu_0$

vs H_1: $\mu < \mu_0$ (less than or smaller than)

Rejection Rule

Reject H_0 at a level of significance α if:

i. $Z < -Z_{1-\alpha}$ (In case of using Z-distribution)

ii. $t < -t_{(\alpha,\, n-1)}$ (In case of using t-distribution)

Rejection Rule

Upper (right)-tailed test

Hypotheses

H_0: $\mu = \mu_0$

vs H_1: $\mu > \mu_0$ (more than or greater than)

<u>Rejection Rule</u>

Reject H_0 at a level of significance α if:

i. $Z > Z_{1-\alpha}$ (In case of using Z-distribution)

ii. $t > t_{(\alpha, n-1)}$ (In case of using t-distribution)

Rejection Rule

Two-tailed test

Hypotheses

$H_0: \mu = \mu_0$

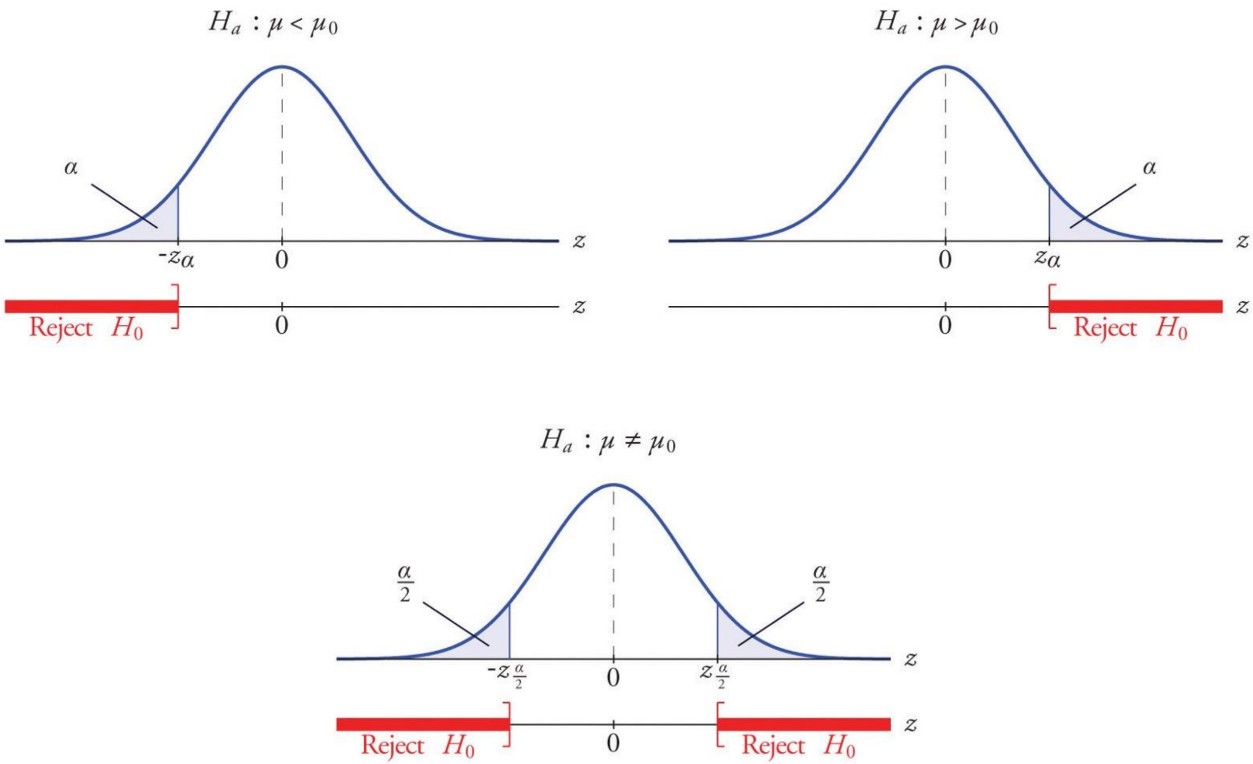
vs $H_1: \mu \neq \mu_0$ (does not equal or different from)

Rejection Rule

Reject H_0 at a level of significance α if:

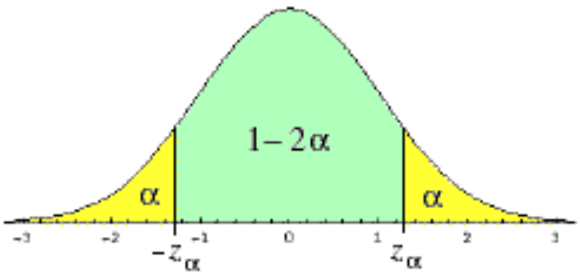
i. $|Z| > Z_{1-\frac{\alpha}{2}}$ (In case of using Z-distribution)

ii. $|t| > t_{(\frac{\alpha}{2},\, n-1)}$ (In case of using t-distribution)



$H_a : \mu < \mu_0$
α
$-z_\alpha$
0
z
Reject H_0
$H_a : \mu > \mu_0$
α
0
z_α
z
Reject H_0
$H_a : \mu \neq \mu_0$
$\frac{\alpha}{2}$
$\frac{\alpha}{2}$
$-z_{\frac{\alpha}{2}}$
0
$z_{\frac{\alpha}{2}}$
z
Reject H_0
Reject H_0

Common Critical Values



α = tail area	central area = 1 − 2α	z_α
0.10	0.80	$z_{.10}$ = **1.28**
0.05	0.90	$z_{.05}$ = **1.645**
0.025	0.95	$z_{.025}$ = **1.96**
0.01	0.98	$z_{.01}$ = **2.33**
0.005	0.99	$z_{.005}$ = **2.58**

α	1 − α	$z_{1-\alpha}$
0.10	0.90	$z_{0.90}$ = 1.28
0.05	0.95	$z_{0.95}$ = 1.645
0.01	0.99	$z_{0.99}$ = 2.33

Example Weight

Salem believes that his “true weight” is 72kg with a standard deviation of 3kg.

Salem weighs himself once a week for four weeks. The average of these four measurements is 75.4kg.

Are the data consistent with Salem’s belief?

① تحديد الفرضيات :

$H_0 : \mu = 72$

$H_1 : \mu > 72$ ، (ذيل واحد لليمين)

$\alpha = 0.05$

نوع الاختبار : ذو ذيل واحد لليمين ، لأننا نتحقق ما اذا كان الوزن أكبر من 72 كجم.

Example Weight

1. H_0: μ =72 H_1: μ >72 This is a one tail test

2. a=0.05

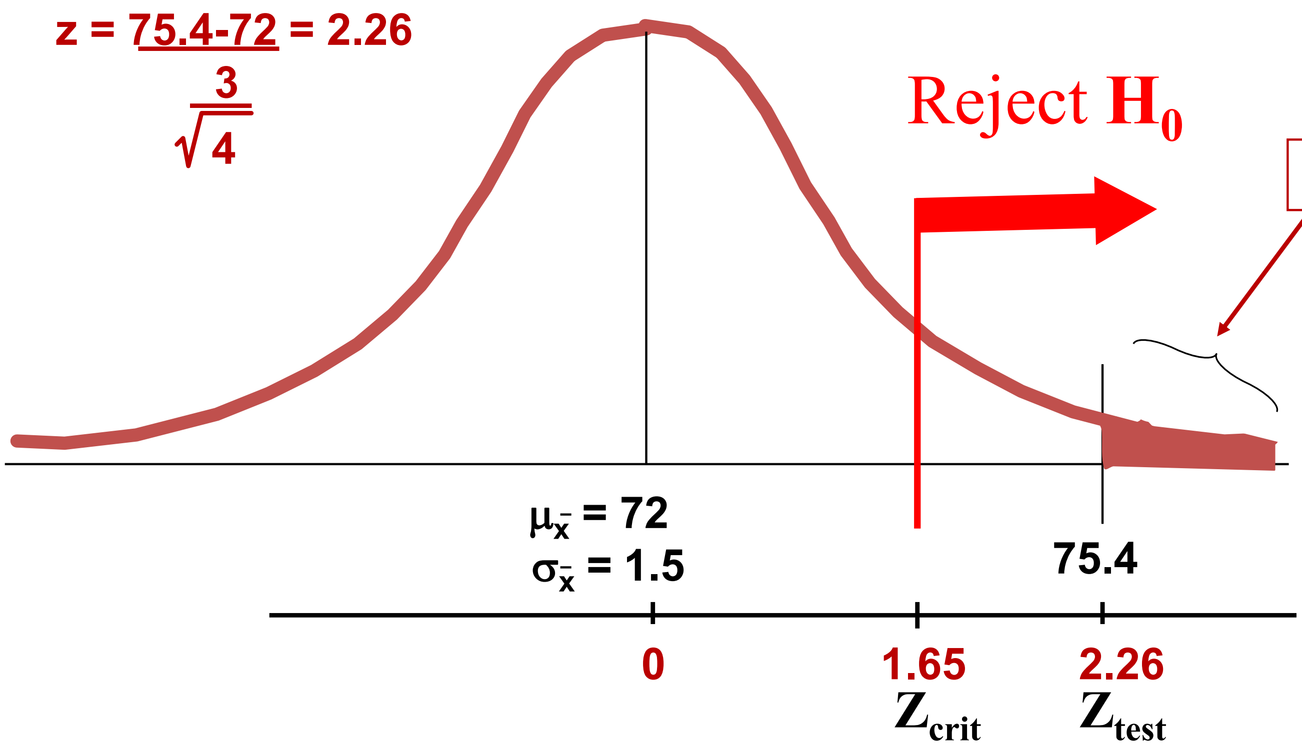
3. μ > 72 (one tail test)

4. Z_{crit} = ZReject H_0 if z $\underline{>}$ 1.645

5. $Z = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} = \frac{75.4 - 72}{3/\sqrt{4}} = 2.26$ $P(Z > 2.26) = .012$

6. Since 2.26 >1.645, we <u>Reject</u> H_0. There is a statistically significant evidence at a=0.05 to show that the mean weight measured is higher than his original belief about his weight. The chance that the measured weight and initial (belief) weight means are different due to chance only is less than 5%.

Example Weight illustrated



Example

Researcher are interested in the mean age of a certain population. They are wondering if the **mean age** is more than **25** years. Assuming that the **population** is **normally** distributed with **variance** equal to **20**. A **random sample** of **10** individuals drawn from the population of interest. From this sample, a **mean** of **27** is calculated. Construct the proper hypothesis, test your hypothesis, and then state the proper conclusion? Use **α = 0.05** to test the hypothesis?

Solution

We have:

1- Normal distribution.

2- The standard deviation σ is known.

$\Rightarrow$ Then we will use the standard normal distribution (Z).

$$n = 10 \;,\; \mu_0 = 25 \;,\; \bar{X} = 27 \;,\; \sigma = \sqrt{\sigma^2} = \sqrt{20} = 4.472 \;,\; \alpha = 0.05$$

Example

Hypotheses

$H_0: \mu = \mu_0 \Rightarrow H_0: \mu = 25$

vs $H_1: \mu > \mu_0 \Rightarrow H_1: \mu > 25$ (more than)

Rejection Rule

Reject H_0 at a level of significance $\alpha = 0.05$ if:

$Z > Z_{1-\alpha}$, $Z_{1-\alpha} = Z_{1-0.05} = Z_{0.95} = 1.645$

Test statistics (calculated value)

$$z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} = \frac{27 - 25}{4.472 / \sqrt{10}} = 1.414$$

$H_0 : \mu = 25$ (المتوسط الحقيقي = 25)

$H_1 : \mu > 25$ (المتوسط الحقيقي > 25)

لاحظوا أنه الاختبار اتجاه واحد (one-tailed test) لأنو بس مهتمين إذا أكبر

* بما أنّ σ معلومة ← نستخدم توزيع Z .

قيمة Z الحرجة عند $\alpha = 0.05$ (لجهة واحدة) $= 1.645$ « القاعدة: نرفض H_0 إذا Z المحسوبة > 1.645

احسب قيمة Z :

$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \rightarrow Z = \frac{27 - 25}{4.47\ 2/\sqrt{10}} = \frac{2}{1.414} = 1.414$$

Decision

$-Z_{critical} = 1.645$ ، $-Z = 1.414$ « وبما أنه $Z = 1.414 < 1.645$ ← ما بنرفض H_0.

الاستنتاج: ما في أيّ دليل احصائي كافي عند مستوى دلالة 0.05 انه متوسط العمر أكبر من 25 ← ∴ نستنتج انه المتوسط لساته 25 سنة.

الخلاصة:

- فرضية بديلة $(\mu > 25)$ غير مدعومة.
- القرار: قبول الفرضية الصفرية $(\mu = 25)$

#

Example

The **mean** maximum oxygen uptake for a sample of **242** women was **32.3** with a **standard deviation** of **12.14**, we wish to know if, on the basis of the data, can we conclude that the **mean** score for a population of such women is **smaller** than **33**? Use **α = 0.01** to test the hypothesis?

Solution

We have:

1- Unknown distribution (population).
2- The standard deviation σ is unknown ($S = 12.14$).
3- The sample size (n) is large ($n = 242 > 30$).

Then we will use the standard normal distribution (Z).

$$n = 242 \ , \ \mu_0 = 33 \ , \ \bar{X} = 32.30 \ , \ S = 12.14 \ , \ \alpha = 0.01$$

Continued

<u>Lower (Left) -Tailed Test</u>

<u>Hypotheses</u>

$$H_0 : \mu = 33$$
$$\text{vs } H_1 : \mu < 33 \quad \text{(smaller than).}$$

Critical Value(*Tabulated Value*)

$$Z_{1-\alpha} = Z_{1-0.01} = Z_{0.99} = 2.33$$
$$\Rightarrow \quad -Z_{1-\alpha} = -Z_{1-0.01} = -Z_{0.99} = -2.33$$

الانحراف المعياري مش معروف لكن n كبير جداً (30<242) ← نستخدم توزيع Z.

<u>Rejection Rule</u>

Reject H_0 at level of significance $\alpha = 0.01$ if :

$$Z < -Z_{1-\alpha}$$

Test Statistic (*Calculated Value*)

$$Z = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$$

$$\Rightarrow \quad Z = \frac{32.30 - 33}{12.14/\sqrt{242}} = -0.897$$

Decision

We get $Z = -0.897 > -Z_{1-\alpha} = -2.33$

Then the rejection rule is not satisfied and therefore the decision will be <u>do not reject (accept)</u> H_0 at $\alpha = 0.01$ and therefore we conclude that the mean score for a population of such women is 33, that is, $\mu = 33$.

Example

The body mass index (BMI) of a **group** of **14** healthy adult males has a **mean of 30.5** and a **standard deviation of 10.6392**, can we conclude that the **mean** BMI of the **population** is equal to **36** assuming that the population is normally distributed? Use **α = 0.1** to test the hypothesis?

<u>Solution</u>

We have:

1- Normal distribution (Normal population).
2- The standard deviation σ is unknown ($S = 10.6392$).
3- The sample size (n) is small ($n = 14 < 30$).

➡ Then we will use the t-distribution.

$n = 14$, $\mu_0 = 36$, $\bar{X} = 30.5$, $S = 10.6392$, $\alpha = 0.10$

Two -Tailed Test

Hypotheses

$$H_0 : \mu = 36$$

vs $H_1 : \mu \neq 36$ (does not equal).

Rejection Rule

Reject H_0 at level of significance $\alpha = 0.10$ if :

$$|t| > t_{(\frac{\alpha}{2}, n-1)}$$

Test Statistic (*Calculated Value*)	Critical Value(*Tabulated Value*)
$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$	$t_{(\frac{\alpha}{2}, n-1)}$
$\Rightarrow t = \frac{30.5-36}{10.6392/\sqrt{14}} = -1.934$	$= t_{(\frac{0.10}{2}, 14-1)}$
	$= t_{(0.05, 13)}$
	$= 1.771$

Decision

We get $|t| = |-1.934| = 1.934 > t_{(\frac{\alpha}{2}, n-1)} = t_{(0.05, 13)} = 1.771$

Then the rejection rule is satisfied and therefore the decision will be reject H_0 at $\alpha = 0.10$ and therefore we conclude that the mean BMI of the population is not equal to 36, that is, $\mu \neq 36$. In other words H_1 is accepted at $\alpha = 0.10$.