

(A) Sampling Distribution of $\bar{x}$

الفكرة انه أنا بدى أوفد أكثر من Sample من ال Population
 ربيد من بدى أصب المتوسط لكل Sample صير رج يسير عندي أكثر من
 Mean لأنه عندي أكثر من Sample، صير ال Means رج أطلع ال Mean واه
 (Mean of the mean). وهى وى أنى قدر من أم كل ال Population، هذا

هذا المس يطلق على ال
 الـ (Mean of the mean)

Population (X)

$E(X) = \text{Mean} = \mu$
 expected value

$SD = \sigma$

Sample mean ($\bar{x}$)

$E(\bar{x}) = \text{mean of the mean} = \mu_{\bar{x}} = \mu$
 mean of $\bar{x}$ (sample mean)

standard deviation of sampling distribution:

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ → SD ال Population
 SD ال Sampling distribution
 Sample size

$$E(X) \\ = \text{mean} \\ = \mu$$

Standard deviation



نمونه گیری
Sample



میانگین و انحراف استاندارد نمونه



Mean of mean value
(میانگین از میانگینها)
(mean of sample mean)

$$\text{Mean of mean} = E(\bar{X}) = \mu_{\bar{X}} \\ = \mu$$

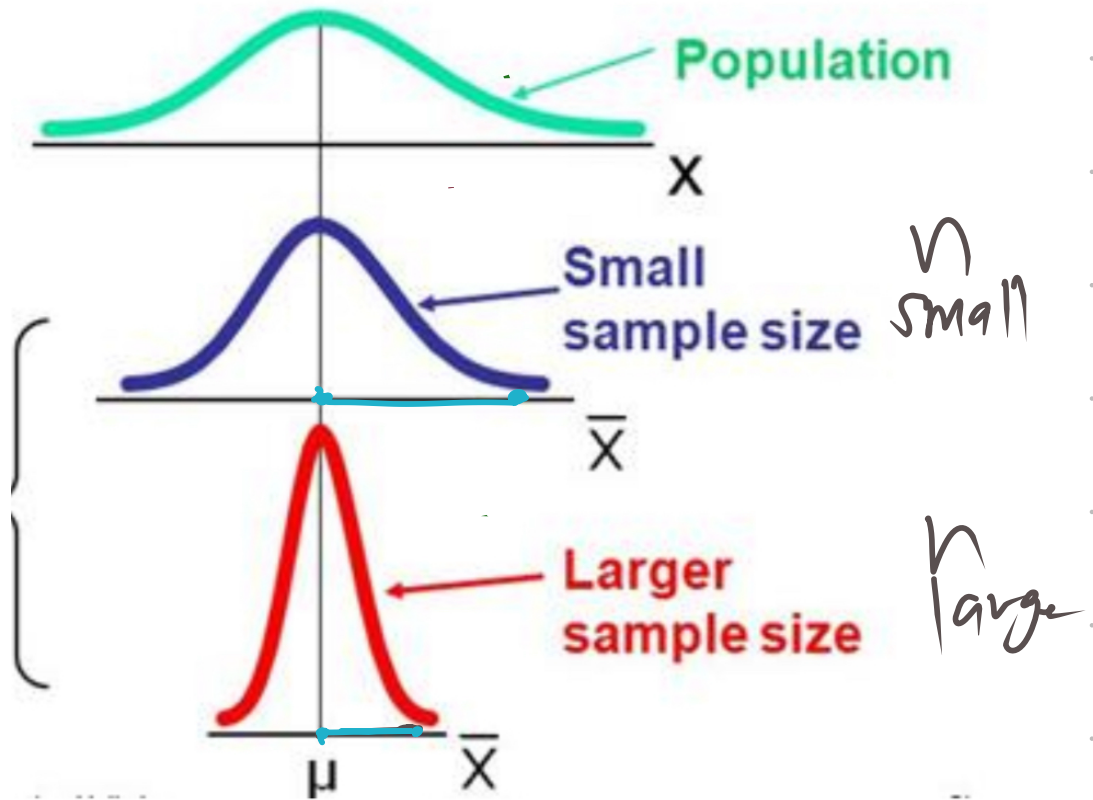


$$\text{Standard deviation of sampling mean} = \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

Standard deviation of population

sample size

$\uparrow n$ (sample size)
 $\downarrow \sigma_{\bar{x}}$
 $\uparrow \bar{x}$ becomes closer to μ

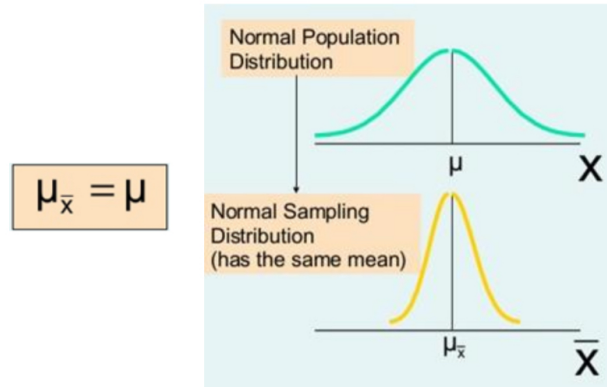


كلما زاد حجم ال sample size (2) نيل 1) standard deviation of $\bar{x}$
 ويزداد قرب $\bar{x}$ من μ

Theorem

If a random sample of n observations is selected from a population with a normal distribution, the sampling distribution of $\bar{x}$ will be a normal distribution.

That is, if a probability distribution is normal, then the sampling distribution of the sample mean is exactly normal distribution, regardless of the sample size (n) small or large, with:



یعنی اگر از یک توزیع normal
نمونه گیری کنیم و آن را
نمونه گیری کنیم و آن را
نمونه گیری کنیم و آن را

در هر یک از این موارد
mean

Mean of Population = Mean of sampling distribution.

Sample Size

Population

non-normal distribution

sampling distribution ($\bar{x}$)

لا يعتبر
normal
 $n < 30$

يمكن اعتباره
normal
لو كان

$n \geq 30$
sample size

$$\bar{X} \sim N(\mu_x, \frac{\sigma^2}{n})$$

CLT

Central limit theorem

Normal distribution

$\bar{X}$ (sampling distribution) is normal distribution.

regardless of sample size

$$\mu = \mu_{\bar{x}}$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$X \sim N(\mu, \sigma^2)$$

$$\bar{X} \sim N(\mu_{\bar{x}}, \frac{\sigma^2}{n})$$

السؤال بالامتحان $\hookrightarrow$ اذا كتبت انه normally distributed
 جداوله $\bar{x}$ انه normal

که لو ما حكي انه normally distributed

حريم اولي $\hookrightarrow$ sample size (n)
 $n < 30$ $n > 30$
 ما يقدر $n < 30$ $n > 30$
 normal $n > 30$
 من جدول

(Examples)

Phone bill for residents of a city have a **mean of \$64** and a **standard deviation of \$9**. random samples of **36 phone bills** are drawn from this population and the mean of each sample is determined. Find the mean and standard error of the mean of the sampling distribution. Then sketch a graph of the sampling of sample mean?

64 = mean

9 = SD

36 = n

يعمل بالسؤال $\hookrightarrow$

$\hookrightarrow$

$\hookrightarrow$

عدد من normal أو لا

ما حكي انه normal $\hookrightarrow$

sample size. $\hookrightarrow$ ان آبرو نكي

$\Rightarrow 36 > 30$

يعتبر $\rightarrow$ normal
 large sample size

$$\textcircled{1} \mu_{\bar{x}} = \mu_{\text{population}} = 64$$

$$\textcircled{2} \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{9}{\sqrt{36}} = 1.5$$

$$\bar{X} \sim N(\mu_{\bar{x}}, \sigma_{\bar{x}}^2)$$

$$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$$

$$\bar{X} \sim N(64, 2.25)$$

Example 2:

The height of fully white oak are normally distributed, with a mean of 90 feet and standard deviation of 3.5 feet. Random samples of size 4 are drawn from this population, and the mean of each sample is determined. Find the mean and standard error of the mean of the sampling distribution. Then sketch a graph of the sampling distribution of sample means?

①

$$\mu_{\bar{x}} = \mu = 90$$

$$\textcircled{2} \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3.5}{\sqrt{4}} = 1.75$$

$$\textcircled{3} \bar{X} \sim N(90, 3.0625)$$

$$\text{mean} = 90$$

$$SD = 3.5$$

$$n = 4$$

normally distributed

sample size

ما في داي اي فرق لي
I don't care if sample size is large or small.

Probability of sample mean ($\bar{x}$) طرحه به با اطلاع

$$Z = \frac{X - \mu}{\sigma} \Rightarrow Z = \frac{\bar{X} - \mu_{\bar{x}}}{\sigma / \sqrt{n}}$$

standard normal distribution
 $\mu = 0$
 $\sigma = 1$

طریقه حل ای سوال :-

① رسم Sketch $\bar{X}$

② جدول $\bar{X}$ را از Z به طرف الفابته

③ رسم Sketch Z $\Leftarrow Z \sim N(0,1)$

④ معروفه نتیجه Z اذا معرفة جدولی برای جدول
 Z المرجع و اذا سالبه برده برای جدول سالبه

⑤ **ملاحظه:** دائماً بطلعه فی المناسباته یا فی سائر الرسم
 کامله (وضوحه بده) تتوف المخلو و تلعب بالمناسباته

الكميات العددية - Example

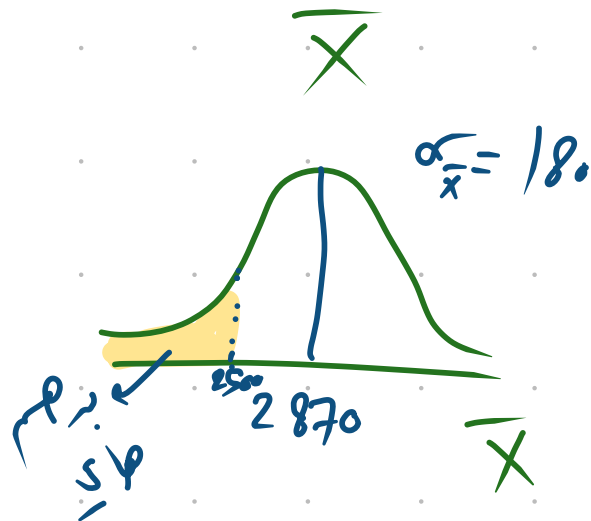
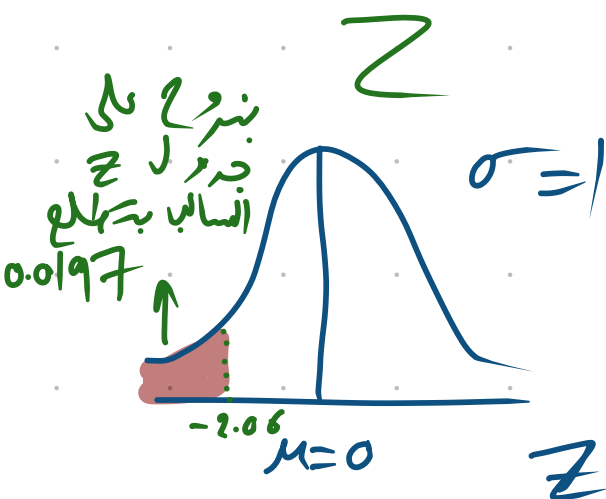
A bank auditor claims that credit card balances are normally distributed, with a mean of JD2870 and a standard deviation of JD900. you randomly select 25 credit card holders. What is the probability that their mean credit card balance is less than or equal to JD2500?

Solution

$$P(\bar{X} \leq 2500)$$

$$\text{mean } \bar{x} = 2870$$

$$\sigma_{\bar{x}} = \frac{900}{\sqrt{25}} = 180$$



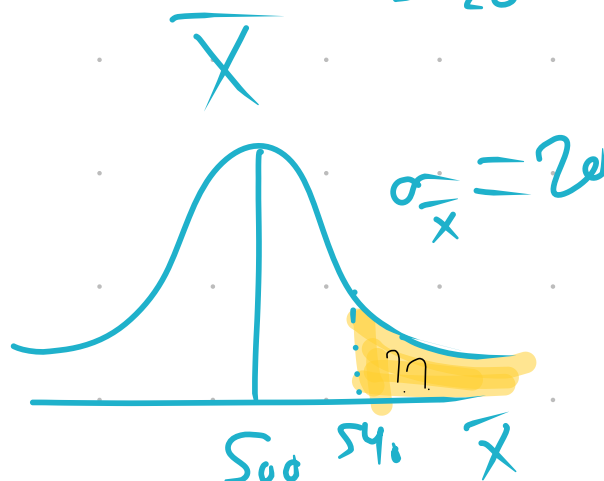
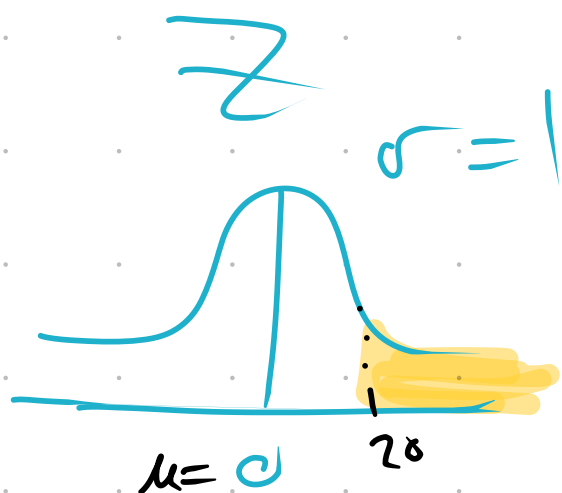
$$\begin{aligned} Z &= \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \\ &= \frac{2500 - 2870}{180} = -2.06 \\ P(Z \leq -2.06) &= 0.0197 \end{aligned}$$

Suppose that the TOFEL exam scores for pharmacy students graduated from the Hashemite University are normally distributed with $\mu = 500$ and $\sigma = 100$.

In a random sample of size $n = 25$ students, what is the probability that the sample mean would be greater than 540?

$$P(\bar{X} > 540)$$

$$\mu_{\bar{X}} = \mu = 500$$
$$\sigma_{\bar{X}} = 100 / \sqrt{25} = 20$$



$$Z = \frac{540 - 500}{20}$$

$$Z = 2$$

$$P(Z > 2.0)$$

$$= 1 - P(Z < 2)$$

$$= 1 - 0.9772$$

$$= 0.0228$$

الجزء من
الكل

You're an operations analyst for AT&T. Long-distance telephone calls are normally distributed with $\mu = 8$ min. and $\sigma = 2$ min. If you select random samples of $n = 25$ calls, what percentage of the **sample means** would be between **7.8** & **8.2** minutes?

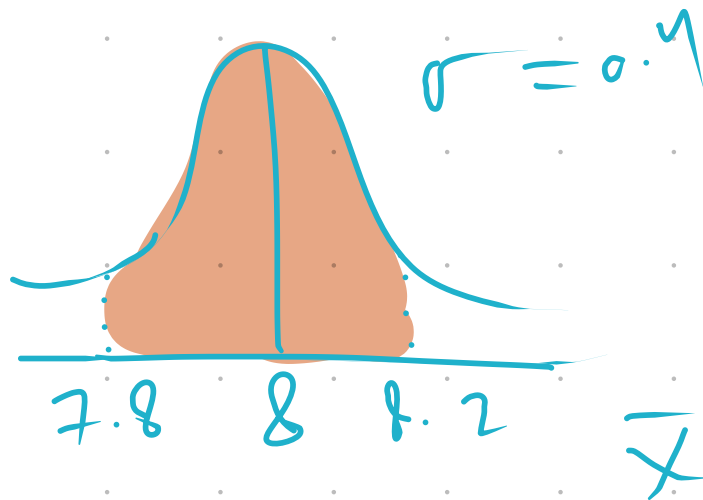


بہتر فنر
مس کا ریف
لیٹر ۱

$$P(7.8 < \bar{x} < 8.2)$$

$$\mu_{\bar{x}} = \mu = 8$$

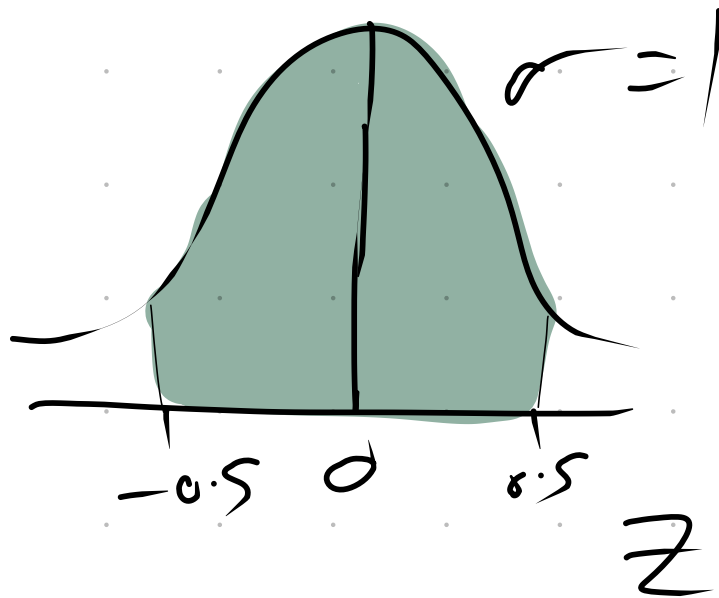
$$\sigma_{\bar{x}} = \frac{2}{\sqrt{25}} = \frac{2}{5} = 0.4$$



$$z = (8.2 - 8) / 0.4 = 0.5$$

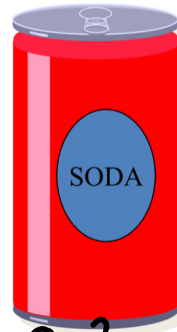
$$z = (7.8 - 8) / 0.4 = -0.5$$

$$P(-0.5 < z < 0.5)$$



$$P(Z < 0.5) - P(Z < -0.5) = 0.3830$$

The amount of soda in cans of a particular brand has a mean of **12** oz and a standard deviation of **.2** oz. If you select random samples of **50** cans, what percentage of the **sample means** would be less than **11.95** oz?



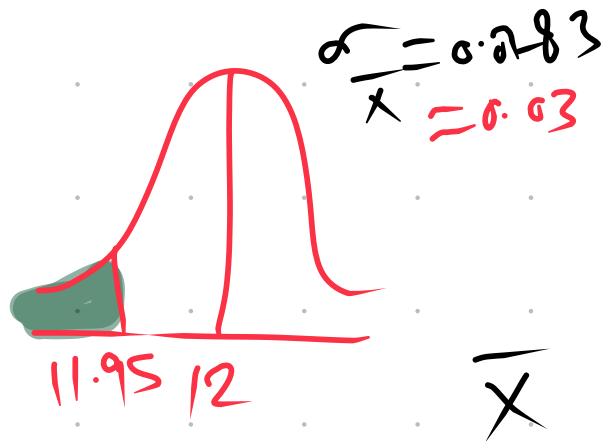
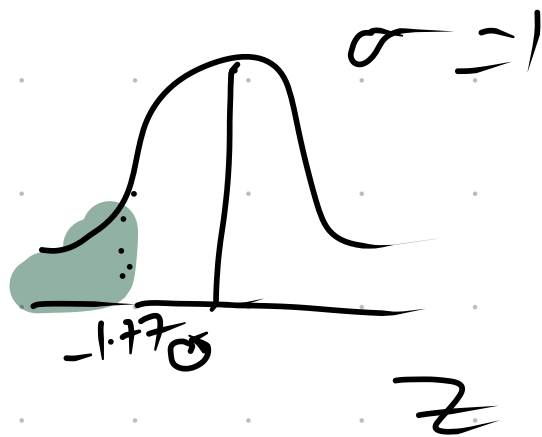
$$P(\bar{x} < 11.95) = ??$$

أول الشيء هو
نمط N n n n

$50 > 30$
بنية تفرع
normal

$$\mu_{\bar{x}} = \mu = 12$$

$$\sigma_{\bar{x}} = \frac{0.2}{\sqrt{50}} = 0.0283$$



$$z = \frac{11.95 - 12}{0.0283} = -1.77$$

$$P(z < -1.77) = 0.0384$$

Suppose There's a Population ...

- Population size, $N = 4$
- Random variable, x
- Values of x : 1, 2, 3, 4
- Uniform distribution



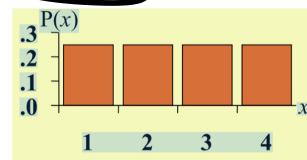
این
کل
ار
Population

Summary Measure

$$\mu = \frac{\sum_{i=1}^N x_i}{N} = 2.5$$

میانگین
Mean
Population = 2.5

Population Distribution



آنها 16 sample
من این population
یک sample
نشان

16 Samples

1st Obs	2nd Observation			
1	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

Sample with replacement

16 Sample Means

1st Obs	2nd Observation			
	1	2	3	4
1	1.0	1.5	2.0	2.5
2	1.5	2.0	2.5	3.0
3	2.0	2.5	3.0	3.5
4	2.5	3.0	3.5	4.0

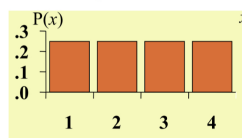
Population

اینها 16 sample
من این population
یک sample
نشان

$$\mu_{\bar{x}} = \frac{\sum_{i=1}^N \bar{x}_i}{N} = \frac{1.0 + 1.5 + \dots + 4.0}{16} = 2.5$$

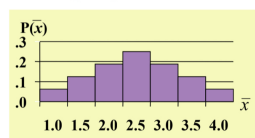
Comparison

Population



$$\mu = 2.5$$

Sampling Distribution



$$\mu_{\bar{x}} = 2.5$$

الهدف من هذا كله:

آنها 16 sample
من این population
یک sample
نشان

اینها 16 sample
من این population
یک sample
نشان

③

Sampling Distribution of $\hat{p}$

Proportion
(percentage)

P : proportion of population

$\hat{p}$: proportion of sample

$$\hat{p} = \frac{\text{count of successes in sample}}{n} = \frac{x}{n}$$

↙ (sample size)

Example:-

The Jordanian Ministry of Health did a survey of in a random sample of 10904 Jordanian peoples and smoking habits. The researchers defined:

Frequent smoking as having 5 or more cigarettes in a row three or more times in the past two hours. According to this definition, 2486 persons were classified as frequent smoking. Based on these data, estimate the proportion p of all Jordanian peoples who admits to frequent smoking?

$$\hat{p} = \frac{2486}{10904} = 0.228$$

بشكر تكرار
بدراسة متكررة
المعدل

السؤال حول تعريف متي هيا 5 او اكثر في
الوقت الاخير الي عدد 2486

$\hat{p}$: proportion
of the sample

P : Probability : Proportion
of population.

Mean of $\hat{p} = P$
 $\mu_{\hat{p}} = P$

large sample
size.



$$np \geq 5$$

$$n(1-p) \geq 5$$

approximation
that is normal
distribution

$$\Rightarrow \hat{p} \sim N\left(P, \frac{P(1-P)}{n}\right)$$

Standard
deviation of $\hat{p} = \sqrt{\frac{pq}{n}}$

$q = (1-p)$

کتاب ویدیو آفلاین از Probability

$\hat{p}$ و μ

① رسم Sketch $\hat{p}$

$$Z = \frac{\hat{p} - \mu_{\hat{p}}}{\sqrt{p(1-p)/n}}$$

② قول $\hat{p}$ و Z $\leftarrow$

$$Z \sim N(0, 1)$$

③ رسم Sketch Z

④ رسم Z

Example

Example

Suppose that the population of interest is the Hashemite University pharmacy students. Assume that the proportion of students in the population who wear eyeglasses is $p=0.25$. if a random sample (SRS) of 50 students is to be selected, then define the characteristics of the sampling distribution of the sample proportion $\hat{p}$, where $\hat{p}$ is the proportion of HU pharmacy students in the random sample who wear eyeglasses?

$$\mu_{\hat{p}} = p = 0.25$$

$$\sigma_{\hat{p}} = \sqrt{\frac{(0.25)(0.75)}{50}} = 0.0612$$

حساب مقدار صحت أو خطأ
normal approximation distribution

الزمن في صحة الشرح:

$$\textcircled{1} np > 5 \rightarrow 50 \times \frac{1}{4} = 12.5 > 5$$

$$\textcircled{2} nq > 5 \rightarrow 50 \times (1 - \frac{1}{4}) = 37.5 > 5$$

لحجم العينة - sample size

معظمهم بنعم ✓

$$\hat{p} \sim N(p, \frac{p(1-p)}{n})$$

$$\hat{p} \sim N(0.25, 0.00375)$$

Example:-

Thiazide diuretics are often the first, but not the only choice in high blood pressure medications. Suppose that 20% of all doctors in Jordan favour Thiazide diuretics. If the manager of a pharmaceutical industries company in Jordan take a random sample (SRS) of size $n = 600$ doctors, then what is the probability that the sample proportion ($\hat{p}$) of doctors who favour Thiazide diuretics will be:

1. Between 0.18 and 0.22? $P(0.18 \leq \hat{p} \leq 0.22)$
2. Less than or equal to 0.18? $P(\hat{p} \leq 0.18)$
3. More than 0.22? $P(\hat{p} \geq 0.22)$

$$\mu_{\hat{p}} = p = \frac{20}{100} = 0.2$$

$$\sigma_{\hat{p}} = \sqrt{\frac{0.2(0.8)}{600}} = 0.0169$$

$$np \geq 5$$

$$(600)(0.2) \geq 5$$

$$120 \geq 5$$

$$\checkmark$$

$$nq \geq 5$$

$$(600)(0.8) \geq 5$$

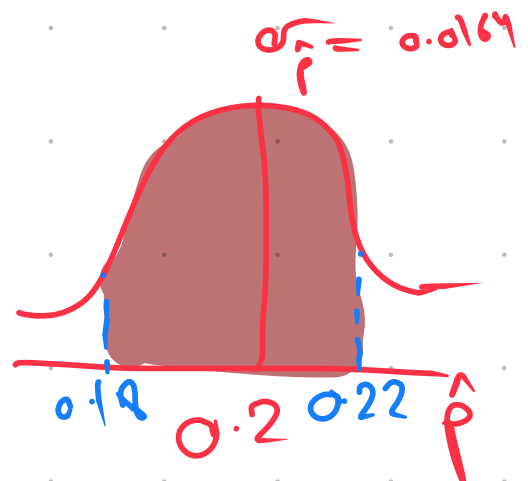
$$480 \geq 5$$

$$\checkmark$$

large sample

$$\hat{p} \sim N(0.2, 0.00027)$$

A $P(0.18 \leq \hat{p} \leq 0.22)$

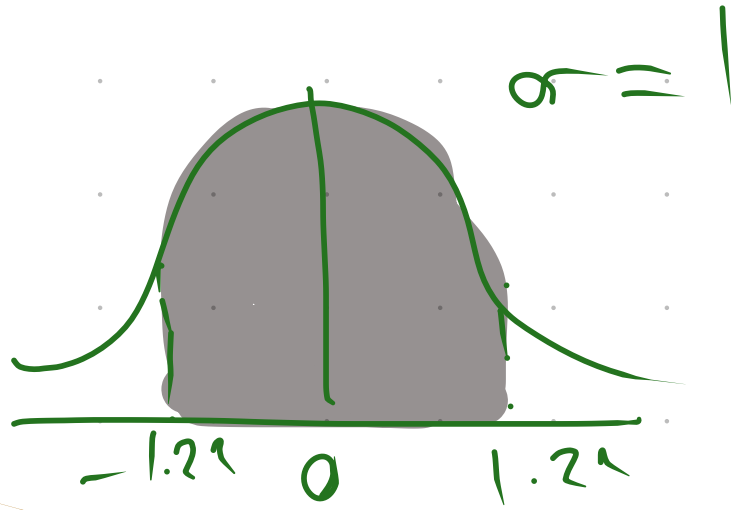


نوع کا قیاس $\hat{p}$ سے z کے ذریعے

$$z = \frac{0.18 - 0.2}{0.0164} = -1.22$$

$$z = \frac{0.22 - 0.2}{0.0164} = 1.22$$

$$P(-1.22 < Z < 1.22) \quad \#$$



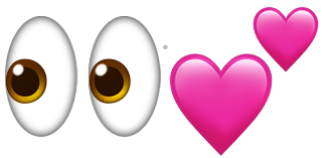
منطقة
المنطقة

منطقة Z

$$= P(Z < 1.22) - P(Z < -1.22)$$

$$= 0.8888 - 0.1112$$

$$= 0.7776$$



دعواتكم