لجان الدُّفعات

STATISTICS

MORPHINE ACADEMY

The Sampling Distribution of a Sample Mean and the Central Limit Theorem

Properties of the Sampling Distribution of $\bar{x}$

- Let $X_1, X_2, \ldots, X_n$ be a random sample of size n drawn from any population with mean=μ and standard deviation = σ, then we have the following properties about the sampling distribution mean μ, that is:

الفكرة الرئيسية :

لما نوخذ عينة (sample) من (population) ونحسب المتوسط تبعها ($\bar{X}$) ، ∴ هاد المتوسط مش ثابت بيختلف من عينة لعينة .

∴ المتوسط نفسه $\bar{x}$ إله توزيع احتمالي اسمه [التوزيع العيني للمتوسط – sampling distribution of the mean]

ملاحظة :

كل ما كبر حجم العينة (n↑) ← قلّ الخطأ المعياري ← صار تقديرنا لل μ أدق.

مثال سريع التوضيح :

لو ال population عنده $\mu = 100$ و $\sigma = 20$

و اخذنا عينة حجمها $n = 25$

$$\sigma_{\bar{x}} = \frac{20}{\sqrt{25}} = \frac{20}{5} = 4 \ \#$$

: متوسط العينة بتوزّع حول 100 بانحراف معياري = 4 بس

(المفروض 20 ← يعني دقة اعلى)

Sample Mean Sampling Distribution: If the Population is **not** Normal

التوزيع العيني راح يصير يشبه التوزيع الطبيعي أكثر كلما زاد حجم العينة.

Sampling distribution properties:

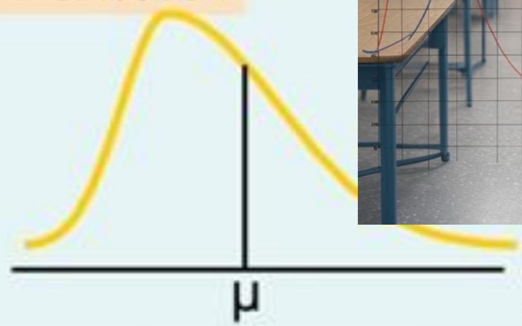
Central Tendency

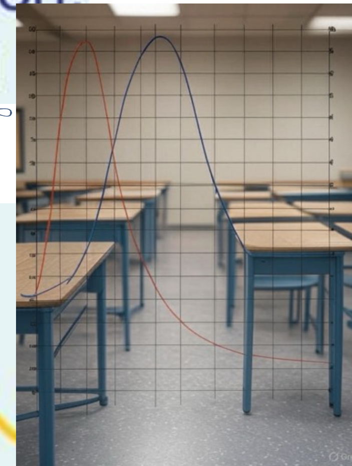
$$\mu_{\bar{x}} = \mu$$

Variation

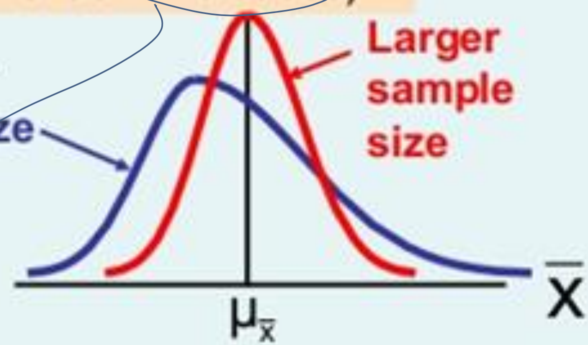
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Population Distribution





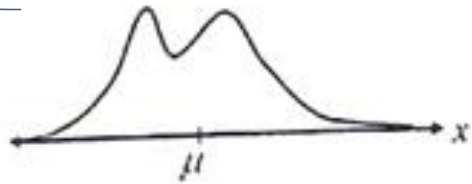
Sampling Distribution (becomes normal as n increases)



مع زيادة n ، بيصير التوزيع أكثر شبهاً (متشكل منحنى منتظم: شكله منحنى جرسي)

في حجم كافي التوزيع الطبيعي التقريبي بغض النظر عن شكل توزيع ال population

That is, if the sample size is large ($n \geq 30$), and the sample is drawn from any population with mean = μ and standard deviation = σ, as shown below:

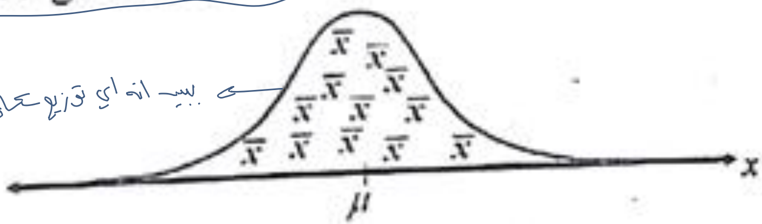


تحول من n معين الى n كبير

شكل غير منتظم

شكل جرسي

then the sampling distribution of the sample mean approximates a normal distribution. The greater the sample size, the better the approximation, as shown below:



يعني انه اي توزيع عاين مهما كان شكله يمكن يتحول توزيع متوسط العينات لتوزيع طبيعي عند $n \geq 30$.

Any Population Distribution

Distribution of Sample Mean, $n \geq 30$

Central Limit Theorem (CLT)

- Example: Interpreting the CLT

μ

Phone bill for residents of a city have a **mean of $64** and a **standard deviation of $9**. random samples of **36 phone bills** are drawn from this population and the mean of each sample is determined. Find the mean and standard error of the mean of the sampling distribution. Then sketch a graph of the sampling of sample mean?

σ n

$\mu_{\bar{x}} = \mu = 64$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{9}{\sqrt{36}} = \frac{9}{6} = 1.5$

Solution: Interpreting the Central Limit Theorem

- The mean of the sampling distribution is equal to the population mean:

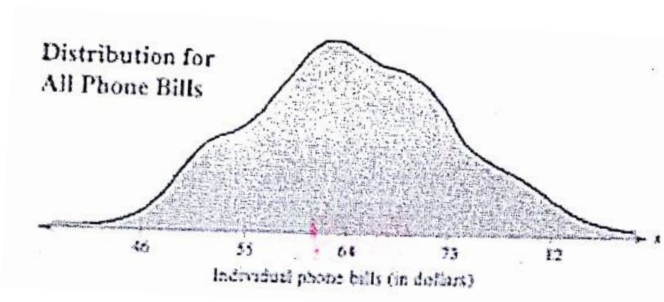
$$\mu_{\bar{x}} = \mu = 64$$

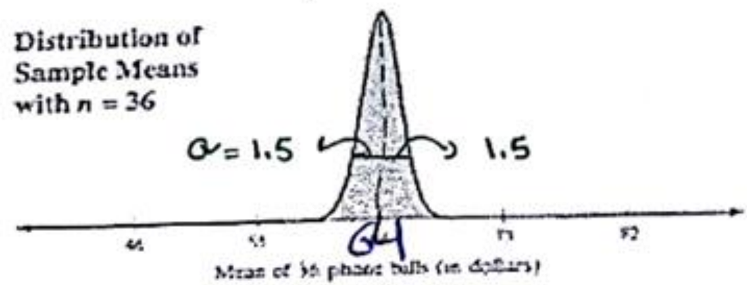
- The standard deviation (standard error of the mean) is equal to the population standard deviation divided by the square root of n:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{9}{\sqrt{36}} = 1.5$$

- Since the sample size is greater than 30, the sampling distribution can be approximated by a normal distribution with:

$$\mu_{\bar{x}} = 64 \qquad \sigma_{\bar{x}} = 1.5$$

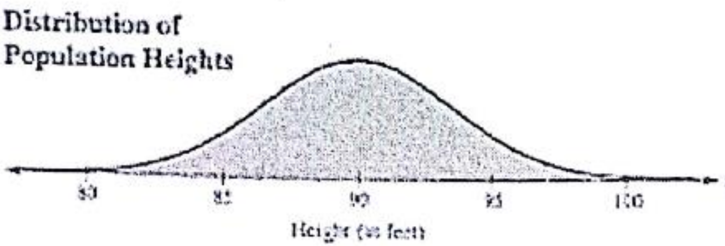




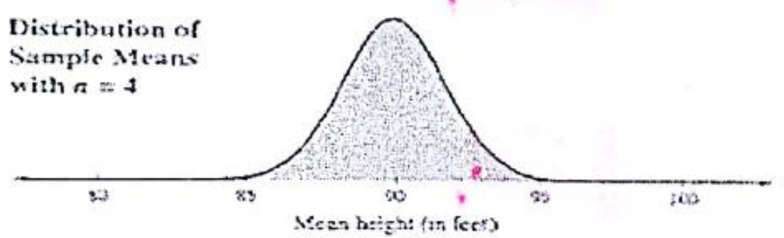
Solution

- **The mean of the sampling distribution is equal to the population mean:** $\mu_{\bar{x}} = \mu$ **= 90**
- **The standard error of the mean is equal to the population standard deviation divided by root of n:**

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \quad \mathbf{= 3.5/2 = 1.75}$$



$\mu_{\bar{x}} = 90 \qquad \sigma_{\bar{x}} = 1.75$



The Probability of a Sample Mean

- How to find the probability associated with a sample mean?
- To find the probability for the sample mean, we transform X to a Z-score as follows:

نحول $\bar{x}$ لصيغة
z-score

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \text{ is approximately } N(0, 1).$$

تتبع توزيع قياسي تقريبا لما
يكون حجم العينة كبير بما فيه الكفاية.

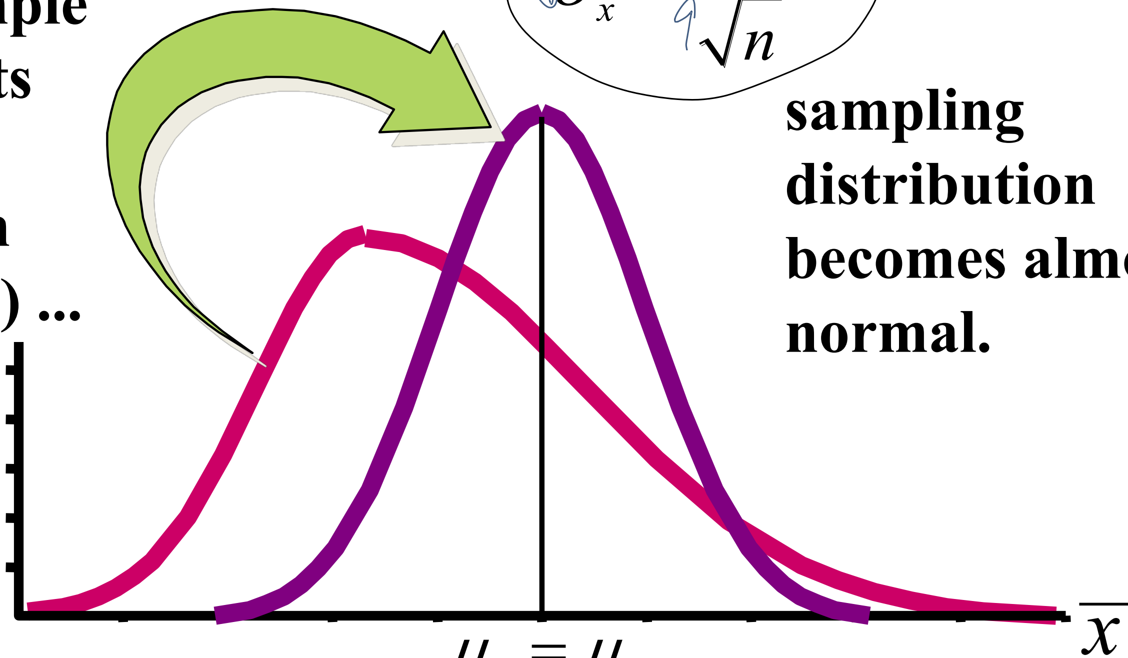
Central Limit Theorem

المبدأ: عندما يزداد حجم العينة (n) حتى يصبح كبيرًا بما فيه الكفاية (عادة $n \geq 30$)، يصبح توزيع المتوسطات العينية ($\bar{x}$) تقريبًا توزيعًا طبيعيًا.

As sample size gets large enough ($n \geq 30$) ...

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

sampling distribution becomes almost normal.



$$\mu_{\bar{x}} = \mu$$

Examples

The graph shows the length of time people spend driving each day. You randomly select $\overset{n}{50}$ drivers age 15 to 19. what is the probability that the mean time they spend driving each day is between 24.7 and 22.5 minutes?

بدنا الاحتمال انو يكون المتوسط وقت القيادة بين 24.7 و 22.5 دقيقة:

Assume that μ = 25 minutes and σ = 1.5 minutes?

Solution

From the central limit theorem (sample size is greater than 30), the sampling distribution of the sample mean is approximately normal with mean and standard deviation given as follows:

$$\mu_{\bar{x}} = \mu = 25 \qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = 1.5/\sqrt{50} \approx 0.21213$$

Then the probability that the mean time they spend driving each day is between 24.7 and 25.5 minutes can be calculated as follows:

Normal Distribution

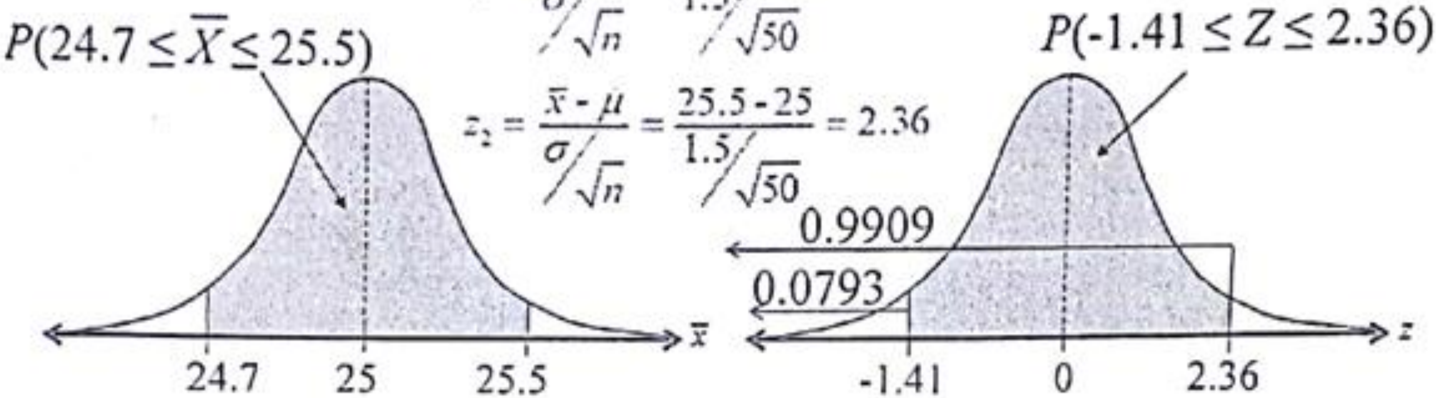
$\mu_{(xbar)} = 25 \quad \sigma_{(xbar)} = 0.21213$

Standard Normal Distribution

$\mu = 0 \quad \sigma = 1$

$$z_1 = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{24.7 - 25}{1.5/\sqrt{50}} = -1.41$$

$$z_2 = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{25.5 - 25}{1.5/\sqrt{50}} = 2.36$$



$$P(24.7 \leq \overline{X} \leq 25.5) = P(-1.41 \leq Z \leq 2.36)$$
$$= \mathrm{N}(2.36) - \mathrm{N}(-1.41) = 0.9909 - 0.0793 = 0.9116$$

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
+0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
+0.1	.53983	.54380	.54776	.55172	.55567	.55966	.56360	.56749	.57142	.57535
+0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
+0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
+0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
+0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
+0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
+0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
+0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
+0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
+1	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
+1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
+1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
+1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91308	.91466	.91621	.91774
+1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
+1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
+1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
+1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
+1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
+1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
+2	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
+2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
+2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
+2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
+2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
+2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
+2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
+2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
+2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
+2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
+3	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
+3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
+3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
+3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
+3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
+3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
+3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
+3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
+3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
+3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997
+4	.99997	.99997	.99997	.99997	.99997	.99997	.99998	.99998	.99998	.99998

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414
-0.1	.46017	.45620	.45224	.44828	.44433	.44034	.43640	.43251	.42858	.42465
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-1	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08692	.08534	.08379	.08226
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-2	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-3	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-4	.00003	.00003	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002

اوضح الحل على شكل الحقة الى فوق :

$\mu_{\bar{x}} = \mu = 25$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.5}{\sqrt{50}} = 0.21213$

لـ $\bar{x} = 24.7$: $Z = \frac{24.7-25}{0.21213} \approx -1.41$

لـ $\bar{x} = 25.5$: $Z = \frac{25.5-25}{0.21213} \approx 2.36$

الاحتمال :

$P(24.7 \leq \bar{x} \leq 25.5) = P(-1.41 \leq Z \leq 2.36)$

* من جداول التوزيع الطبيعي القياسي :

• $P(Z \leq 2.36) \approx 0.9909$

• $P(Z \leq -1.41) \approx 0.0793$

$0.9909 - 0.0793 = 0.9116 =$ الاحتمال

∴ النتيجة : الاحتمال ان يكون متوسط العينة بين 24.7 و 25.5 دقيقة هو حوالي 91.16%

#

Examples

A bank auditor claims that credit card balances are normally distributed, with a mean of JD2870 and a standard deviation of JD900. you randomly select 25 credit card holders. What is the probability that their mean credit card balance is less than or equal to JD2500?

<u>Solution</u>

Since the population is normally distributed, the sampling distribution of the sample mean is also normally distributed. Now, we are asked to find the probability associated with a sample mean and this can be calculated as follows:

$$\mu_{\bar{x}} = \mu = 2870$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{900}{\sqrt{25}} = 180$$

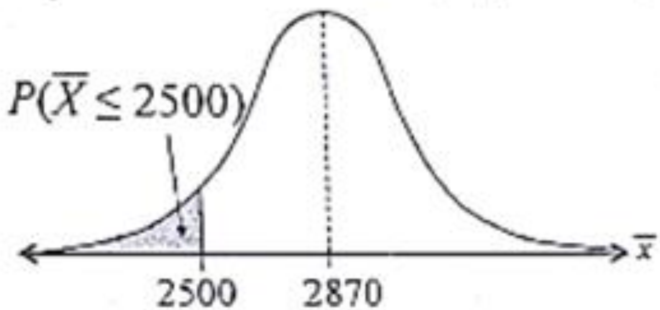
Normal Distribution

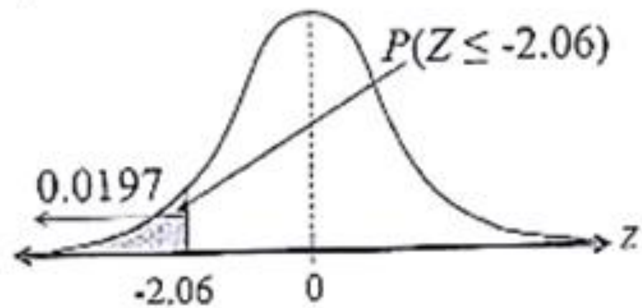
$\mu_{(xbar)} = 2870 \quad \sigma_{(xbar)} = 180$

Standard Normal Distribution

$\mu = 0 \quad \sigma = 1$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{2500 - 2870}{900 / \sqrt{25}} \approx -2.06$$

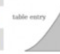




$$P(\bar{X} \leq 2500) = P(Z \leq -2.06) = \mathrm{N}(-2.06) = 0.0197$$

Conclusion: There is only a 2% chance that the mean of a sample of size 25 will have a balance less than or equal to JD2500 (*unusual event*). It is possible that the sample is unusual or it is possible that the auditor's claim that the mean is JD2870 is incorrect.

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414
-0.1	.46017	.45620	.45224	.44828	.44433	.44034	.43640	.43251	.42858	.42465
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-1	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08692	.08534	.08379	.08226
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-2	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-3	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-4	.00003	.00003	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002



$\mu = \mu_{\bar{x}} = 2870$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{900}{\sqrt{25}} = 180$

$\bar{x} = 2500$ J : $Z = \frac{2500 - 2870}{180} \approx -2.06$

الاحتمال :

$P(\bar{x} \leq 2500) = P(Z \leq -2.06)$

من الجدول :

$P(Z \leq -2.06) \approx 0.0197$

الاحتمال أن يكون متوسط حسب العينة أقل من أو يساوي 2500 د.أ هو 1.97% ، وبما أن هنا الاحتمال أقل من 2%

فإن العينة غير معتادة مما يشير إلى أن ادعاء المحاسب قد يكون غير صحيح

Examples

Suppose that the TOFEL exam scores for pharmacy students graduated from the Hashemite University are normally distributed with μ = 500 and σ = 100.

In a random sample of size n = 25 students, what is the probability that the sample mean would be greater than 540?

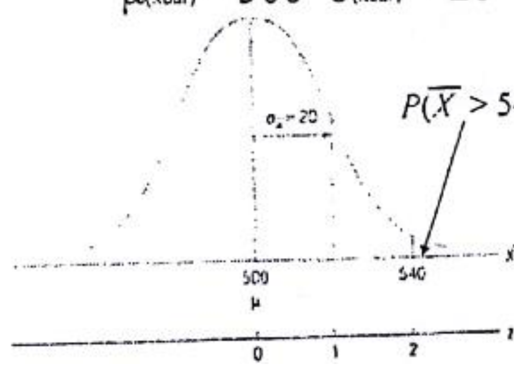
Solution

Since the population is normally distributed, the sampling distribution of the sample mean is also normally distributed. Now, we are asked to find the probability associated with a sample mean and this can be calculated as follows:

$$P(\overline{X} > 540) = 1 - P(\overline{X} \leq 540)$$
$$= 1 - P(Z \leq (540 - 500)/20)$$
$$= 1 - P(Z \leq 2)$$
$$= 1 - N(2)$$
$$= 1 - 0.9772$$
$$= 0.0228$$

Normal Distribution

$\mu_{(xbar)} = 500 \quad \sigma_{(xbar)} = 20$



Thinking Challenge

You're an operations analyst for AT&T. Long-distance telephone calls are normally distributed with μ = **8** min. and σ= **2** min. If you select random samples of **25** calls, what percentage of the **sample means** would be between **7.8** & **8.2** minutes?

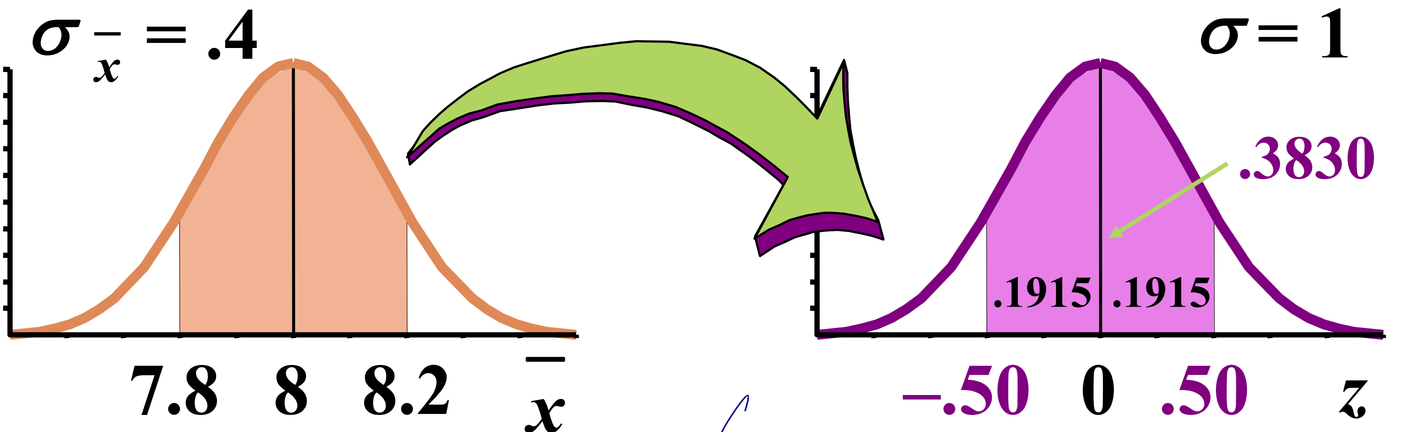
Sampling Distribution Solution*

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{7.8 - 8}{2 / \sqrt{25}} = -.50$$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{8.2 - 8}{2 / \sqrt{25}} = .50$$

Sampling Distribution

Standardized Normal Distribution



z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
+0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
+0.1	.53983	.54380	.54776	.55172	.55567	.55966	.56360	.56749	.57142	.57535
+0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
+0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
+0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
+0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
+0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
+0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
+0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
+0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
+1	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
+1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
+1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
+1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91308	.91466	.91621	.91774
+1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
+1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
+1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
+1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
+1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
+1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
+2	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
+2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
+2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
+2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
+2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
+2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
+2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
+2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
+2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
+2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
+3	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
+3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
+3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
+3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
+3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
+3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
+3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
+3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
+3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
+3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997
+4	.99997	.99997	.99997	.99997	.99997	.99997	.99998	.99998	.99998	.99998

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414
-0.1	.46017	.45620	.45224	.44828	.44433	.44034	.43640	.43251	.42858	.42465
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-1	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08692	.08534	.08379	.08226
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-2	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-3	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-4	.00003	.00003	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002

(8.2 ، 7.8 بين) $n = 25$ / $\sigma = 2$ / $\mu = 8$

soln:

$\mu_{\bar{x}} = \mu = 8$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{25}} = 0.4$

$\bar{x} = 7.8$ ل: $z = \frac{7.8 - 8}{0.4} = \frac{-0.2}{0.4} = -0.5$

$\bar{x} = 8.2$ ل: $z = \frac{8.2 - 8}{0.4} = \frac{0.2}{0.4} = 0.5$

* احصائياً:

$P(7.8 \le \bar{x} \le 8.2) = P(-0.5 \le z \le 0.5)$

* من الجدول:

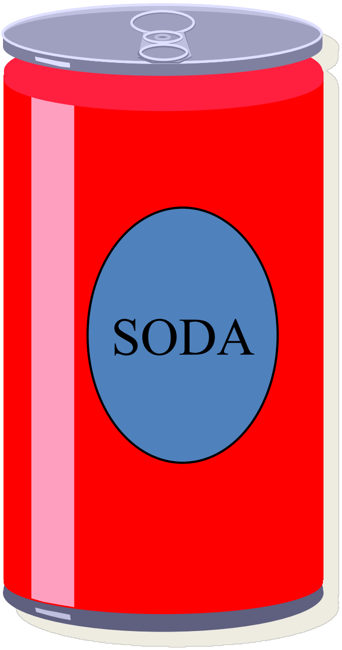
$P(z \le 0.5) \approx 0.6915$

$P(z \le -0.5) \approx 0.3085$

∴ حوالي 38.3% من متوسطات العينة بتكون بين 7.8 و 8.2 دقيقة.

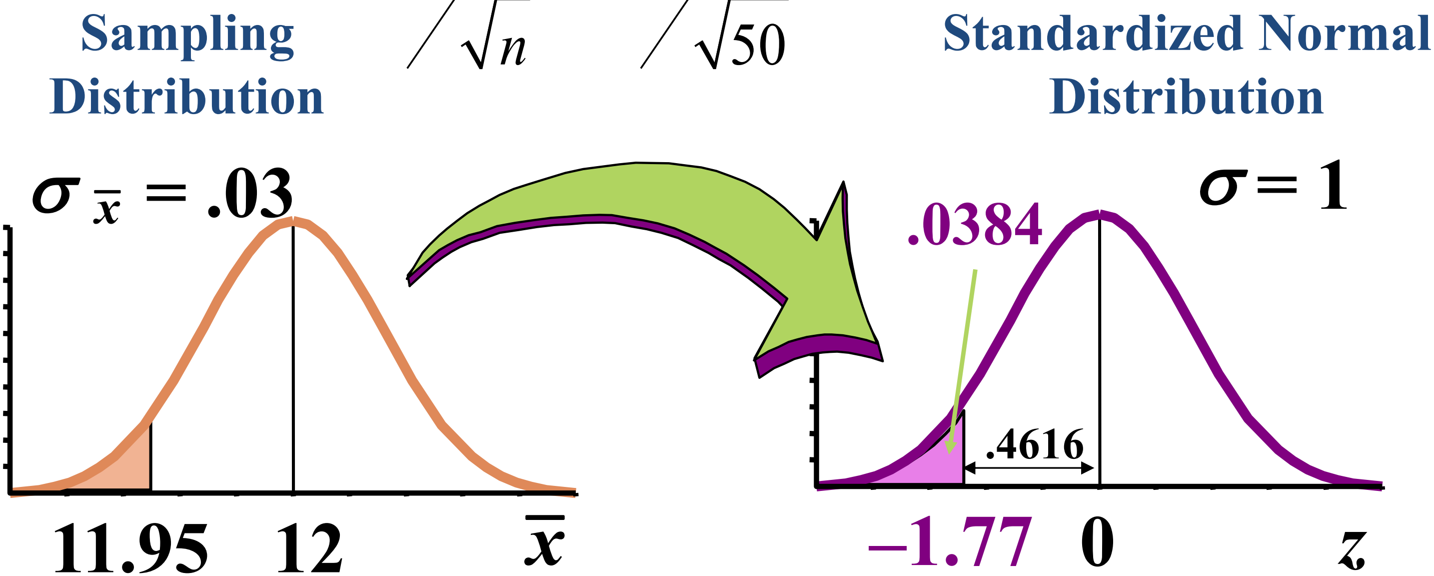
Central Limit Theorem Example

The amount of soda in cans of a particular brand has a mean of **12** oz and a standard deviation of **.2** oz. If you select random samples of **50** cans, what percentage of the **sample means** would be less than **11.95** oz?



Central Limit Theorem Solution*

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{11.95 - 12}{.2 / \sqrt{50}} = -1.77$$



Shaded area exaggerated

Sampling Distributions

Common Statistics & Parameters

	Sample Statistic	Population Parameter
Mean	$\bar{x}$	μ
Standard Deviation	s	σ
Variance	s^2	σ^2
Binomial Proportion	$\hat{p}$	p

Sampling Distribution

The **sampling distribution** of a sample statistic calculated from a sample of *n* measurements is the probability distribution of the statistic in all possible samples of size n taken from the same population.

The **sampling distribution** are used to calculate the probability that sample statistics could have occurred by chance and thus to decide whether something that is true of sample statistics is also likely to be true of a population parameter.

له بساعد باتخاذ قرار حول ما اذا كان الاستنتاج العيني ينطبق على السكان.

Developing
Sampling Distributions

Suppose There's a Population ...

- Population size, $N = 4$
- Random variable, x
- Values of x: 1, 2, 3, 4
- Uniform distribution

لكل قيمة الها احتمال متساوي

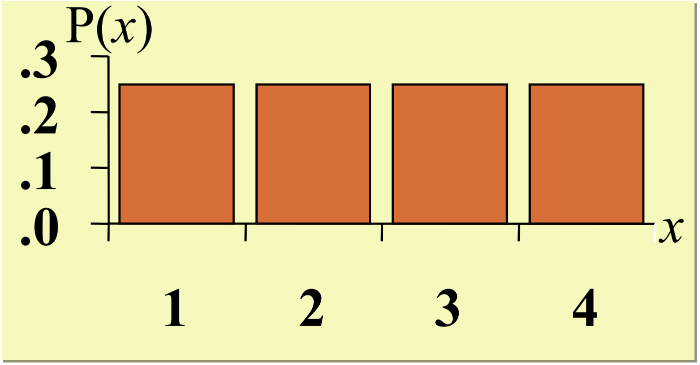
$P(x) = 0.25$

Population Characteristics

Summary Measure

$$\mu = \frac{\sum_{i=1}^{N} x_i}{N} = 2.5 = \frac{1+2+3+4}{4}$$

Population Distribution



All Possible Samples of Size $n = 2$

مع الاستبدال

16 Samples

1st Obs	2nd Observation			
	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

16 Sample Means

1st Obs	2nd Observation			
	1	2	3	4
1	1.0	1.5	2.0	2.5
2	1.5	2.0	2.5	3.0
3	2.0	2.5	3.0	3.5
4	2.5	3.0	3.5	4.0

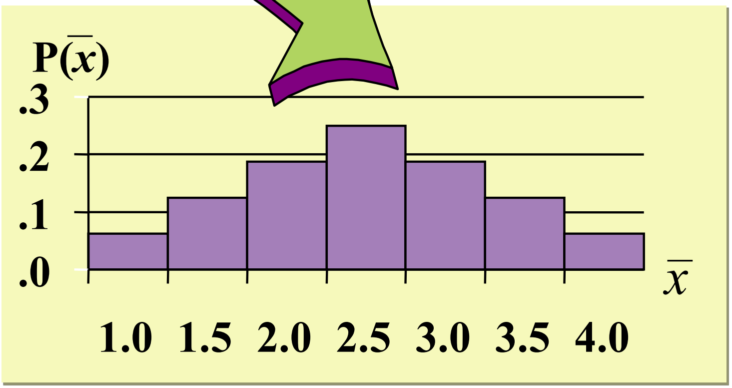
Sample with replacement

Sampling Distribution of All Sample Means

16 Sample Means

1st Obs	2nd Observation 1	2	3	4
1	1.0	1.5	2.0	2.5
2	1.5	2.0	2.5	3.0
3	2.0	2.5	3.0	3.5
4	2.5	3.0	3.5	4.0

Sampling Distribution of the Sample Mean



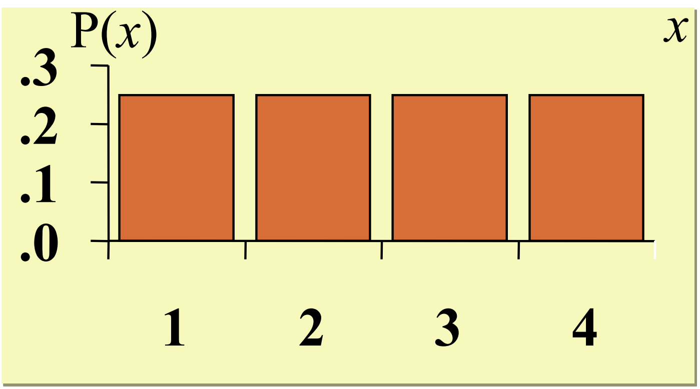
Summary Measure of All Sample Means

$$\mu_{\bar{X}} = \frac{\sum_{i=1}^{N} \bar{x}_i}{N} = \frac{1.0 + 1.5 + \ldots + 4.0}{16} = 2.5$$

= متوسط ال Population.

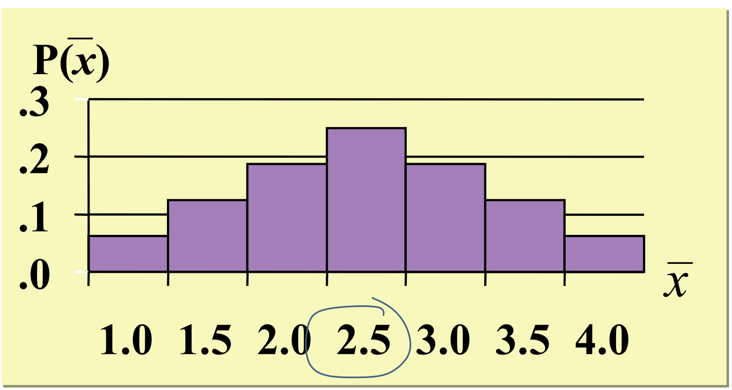
Comparison

Population



$$\mu = 2.5$$

Sampling Distribution

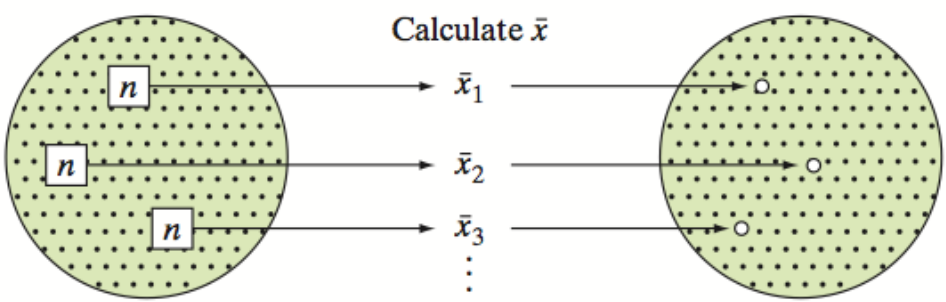


$$\mu_{\bar{x}} = 2.5$$

Key Ideas

Generating the Sampling Distribution of $\overline{x}$

Select sample size n (large) from target population



Population:
Mean = μ
Std. Dev. = σ
Unknown shape

Repeat this process an infinite number of times

Sampling distribution of $\bar{x}$
(i.e., theoretical population of $\bar{x}$'s)
Mean = $\mu_{\bar{x}} = \mu$
Std. Dev. = $\sigma_{\bar{x}} = \sigma/\sqrt{n}$
Normal distribution (Central Limit Theorem)

The Sampling Distribution of the Sample Proportion $\hat{p}$

The Sampling Distribution of the Sample Proportion $p^{\wedge}$

- We hope that the sample proportion is close to the population proportion.
- How close can we expect it to be?
- Would it be worth it to collect a larger sample?
  - If the sample were larger, would we expect the sample proportion to be closer to the population proportion?
  - How much closer? كلما زاد حجم العينة، كلما اقترب $\hat{p}$ من p ،

Example

The Jordanian Ministry of Health did a survey of in a random sample of 10904 Jordanian peoples and smoking habits. The researchers defined :

Frequent smoking as having 5 or more cigarettes in a row three or more times in the past two hours. According to this definition, 2486 persons were classified as frequent smoking. Based on these data, estimate the proportion p of all Jordanian peoples who admits to frequent smoking?

<u>Solution</u>

The true proportion (p) of Jordanian peoples who are frequent smoking can be estimated by using the sample proportion ($\hat{p}$) which can be calculated as follows:

$$\hat{p} = \frac{\textit{Number of Jordanian persons who are classified as frequent smokin}}{\textit{Number of Jordanian persons in the sample}}$$

$$\Longrightarrow \quad \hat{p} = \frac{X}{n} = \frac{2486}{10904} = 0.228$$

#

The Sampling Distribution of the Sample Proportion $p^\wedge$

- Choose an SRS of size n from a large population that contains population proportion p of successes. Let $p^\wedge$ be the sample proportion of successes,

ع/ح أ يصير التوزيع طبيعي

$$p^\wedge = \frac{\textit{count of successes in the sample}}{n}$$

- Then :

➢ As the sample size increases, the sampling distribution of p becomes approximately normal.

➢ The **mean** of the sampling distribution is **p**.

➢ The standard deviation of the sampling distribution is

#

$$\boxed{\text{Standard deviation of } \hat{p} = \sqrt{\frac{p(1-p)}{n}}}$$

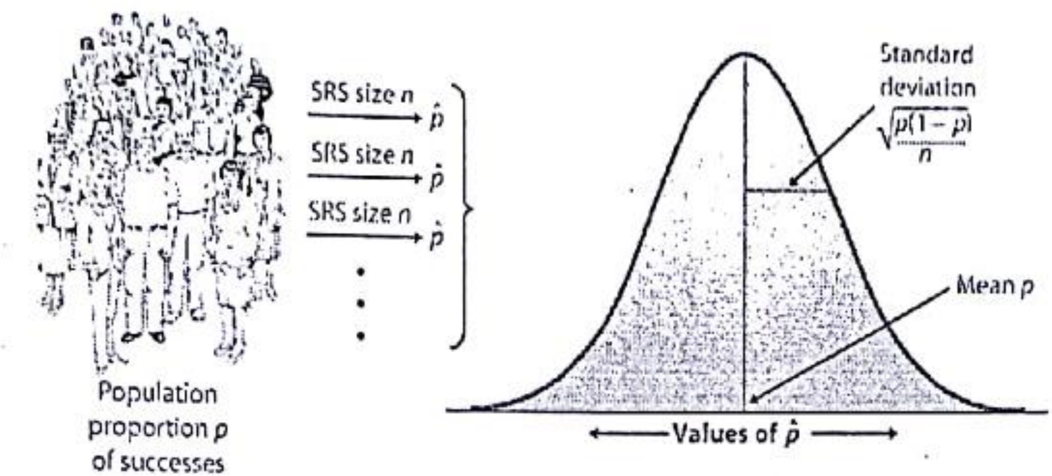
The Sampling Distribution of *p*^

- It turns out that the sampling distribution of *p*^ is approximately normal with the following parameters.

$$\text{Mean of } \hat{p} = p$$

$$\text{Variance of } \hat{p} = \frac{p(1-p)}{n}$$

$$\text{Standard deviation of } \hat{p} = \sqrt{\frac{p(1-p)}{n}}$$



The Sample Proportion

- Let p be the population proportion.
- Then p is a fixed value (for a given population).
- Let $p^{\wedge}$ ("p-hat") be the sample proportion.
- Then $p^{\wedge}$ is a random variable; it takes on a new value every time a sample is collected.
- The sampling distribution of $p^{\wedge}$ is the probability distribution of all the possible values of $p^{\wedge}$.

The Central Limit Theorem for the Sample Proportion ($\hat{p}$)

For any population proportion (p), the sampling distribution of the sample proportion ($\hat{p}$) is *approximately normal if the sample size (n) is sufficiently large*. As a general guideline, the normal distribution approximation is justified when $np \geq 5$ and $n(1 - p) \geq 5$, that is:

Sampling Distribution of the Sample Proportion

$\hat{p}$ $\widetilde{Approximately}$ $N\left(p, \frac{p(1-p)}{n}\right)$ if n is sufficiently large.

- The approximation to the normal distribution is excellent if:

$$\boxed{np \geq 5 \text{ and } n(1-p) \geq 5.}$$

Example

Suppose that the population of interest is the Hashemite University pharmacy students. Assume that the proportion of students in the population who wear eyeglasses is **p=0.25**. if a random sample (SRS) of **50** students is to be selected, then define the characteristics of the sampling distribution of the sample proportion ***p*^**, where ***p*^** is the proportion of HU pharmacy students in the random sample who wear eyeglasses?

Example

Solution

We know that the mean of the sampling distribution of $\hat{p}$ is:

$$\mu_{\hat{p}} = p = 0.25$$

and we know that the variance and standard deviation of the sampling distribution of $\hat{p}$ are:

$$\sigma^2_{\hat{p}} = \frac{p(1-p)}{n} = \frac{(0.25)(0.75)}{50} = 0.00375 \quad ; \quad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{0.00375} = 0.0612$$

Finally, for sample size of $n = 50$, we have $np = (50)(0.25) = 12.5$ and $n(1-p) = (50)(0.75) = 37.5$ thus the large sample conditions are satisfied and the sampling distribution of $\hat{p}$ can be approximated by a normal distribution, that is: $\hat{p}$ *Approximately* $N\left(p, \frac{p(1-p)}{n}\right) = N(0.25, 0.00375)$

How to Find the Probability associated with a Sample Proportion ($\hat{p}$)?

Since the sampling distribution of $\hat{p}$ can be approximated by a normal distribution, we can use the areas under the curve of the standard normal distribution to answer the probability question. With the normal distribution, we need to compute a Z-score value, where

$$Z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{\hat{p} - P}{\sqrt{\frac{p(1-p)}{n}}} \sim N(0,1)$$

Notation: The above Z-score is identical to the Z value we have always used, with the exception of the notation $\hat{p}$ and $\sigma_{\hat{p}}$, which is used to remind us we are dealing with the sampling distribution of the sample proportion ($\hat{p}$).

$$\underset{nq}{\underline{n(1-p)}} \geq 5 \,,\; np \geq 5$$

✓ • $np = 50 * 0.25 = 12.5 \geq 5$

✓ • $n(1-p) = 50(1-0.25) = 37.5 \geq 5$

$$\mu_{\hat{p}} = p = 0.25$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.25 * (1-0.25)}{50}} = \sqrt{\frac{0.1875}{50}} = \sqrt{0.00375} \approx 0.06124$$

#

Example

Thiazide diuretics are often the first, but not the only choice in high blood pressure medications. Suppose that 20% of all doctors in Jordan favour Thiazide diuretics. If the manger of a pharmaceutical industries company in Jordan take a random sample (SRS) of size n = 600 doctors, then what is the probability that the sample proportion ($\hat{p}$) of doctors who favour Thiazide diuretics will be:

1. Between 0.18 and 0.22? $P(0.18 \le \hat{p} \le 0.22)$
2. Less than or equal to 0.18? $P(\hat{p} \le 0.18)$
3. More than 0.22? $P(\hat{p} \ge 0.22)$

(الحل الأول صح)

✓ $nP = 600 * 0.2 = 120 \geq 5$

✓ $nq = 600 * 0.8 = 480 \geq 5$

$\mu_{\hat{p}} = P = 0.2$

$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.2 * 0.8}{600}} = \sqrt{\frac{0.16}{600}} = \sqrt{0.0002667} \approx 0.01633$, $N(0.2, 0.01633)$

$z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}}$

① $P(0.18 \leq \hat{p} \leq 0.22)$: $z_1 = \frac{0.18 - 0.2}{0.01633} \approx \frac{-0.02}{0.01633} \approx -1.22$; $\hat{p} = 0.18$

$z_2 = \frac{0.22 - 0.2}{0.01633} \approx \frac{0.02}{0.01633} \approx 1.22$; $\hat{p} = 0.22$

$P(-1.22 \leq z \leq 1.22) = P(z \leq 1.22) - P(z \leq -1.22)$

من الجدول: $P(z \leq 1.22) \approx 0.8888$

لأن التوزيع متماثل ← $P(z \leq 1.22) \approx 0.8888$

الأحتمال → $0.8888 - 0.1112 = 0.7776$ #

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
+0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
+0.1	.53983	.54380	.54776	.55172	.55567	.55966	.56360	.56749	.57142	.57535
+0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
+0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
+0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
+0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
+0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
+0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
+0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
+0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
+1	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
+1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
+1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
+1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91308	.91466	.91621	.91774
+1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
+1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
+1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
+1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
+1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
+1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
+2	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
+2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
+2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
+2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
+2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
+2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
+2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
+2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
+2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
+2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
+3	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
+3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
+3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
+3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
+3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
+3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
+3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
+3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
+3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
+3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997
+4	.99997	.99997	.99997	.99997	.99997	.99997	.99998	.99998	.99998	.99998

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414
-0.1	.46017	.45620	.45224	.44828	.44433	.44034	.43640	.43251	.42858	.42465
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-1	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08692	.08534	.08379	.08226
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-2	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-3	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-4	.00003	.00003	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002

Solution

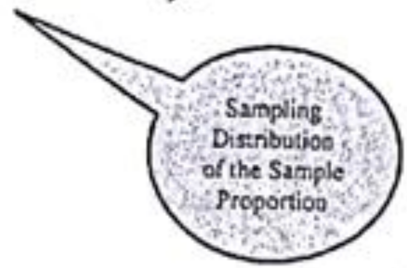
We know that the mean of the sampling distribution of $\hat{p}$ is:

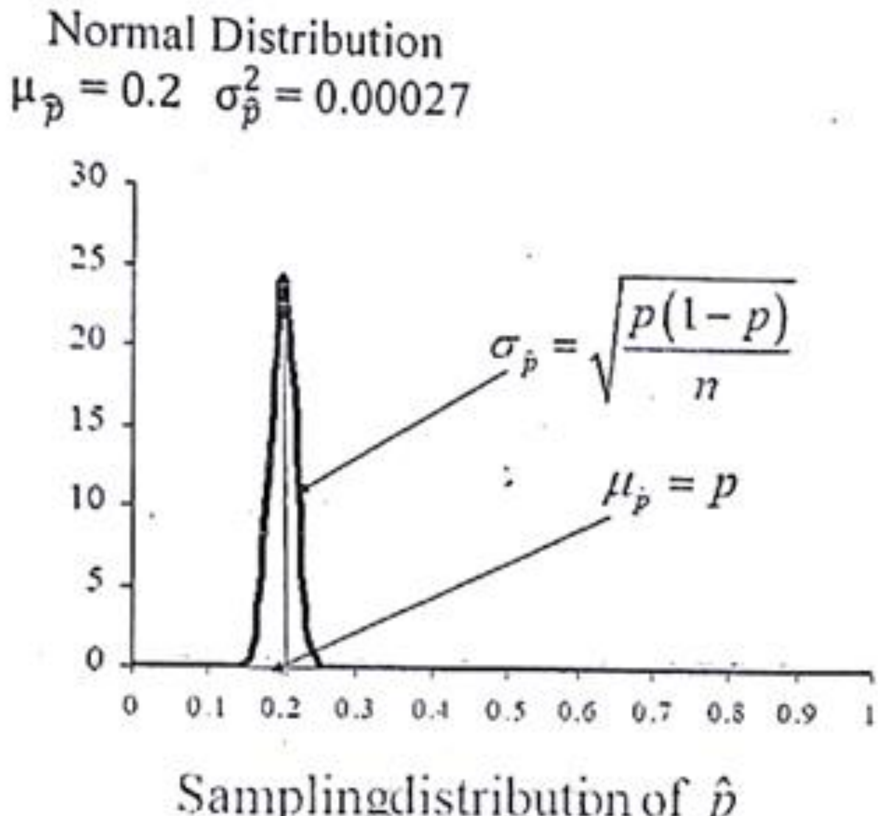
$$\mu_{\hat{p}} = p = 20\% = 0.2$$

and we know that the variance and standard deviation of the sampling distribution of $\hat{p}$ are:

$$\sigma^2_{\hat{p}} = \frac{p(1-p)}{n} = \frac{(0.2(0.8)}{600} = 0.00027 \quad ; \quad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{0.00027} = 0.0164$$

Finally, for sample size of $n = 600$, we have $np = (600)(0.2) = 120$ and $n(1-p) = (600)(0.8) = 480$; thus the large sample conditions are satisfied and the sampling distribution of $\hat{p}$ can be approximated by a normal distribution, that is: $\hat{p} \ \widetilde{Approximately} \ N\left(p, \frac{p(1-p)}{n}\right) = N(0.2, 0.00027)$.





Now, the required probabilities can be found as follows:

(i) Probability that the sample proportion ($\hat{p}$) of doctors who favor Thiazide diuretics will be between 0.18 and 0.22 can be calculated as follows:

$$P(0.18 \leq \hat{p} \leq 0.22) = P\left(\frac{0.18-0.2}{0.0164} \leq \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} \leq \frac{0.22-0.2}{0.0164}\right)$$

$$= P(-1.22 \leq Z \leq 1.22)$$

$$= N(1.22) - N(-1.22) = 0.8888 - 0.1112$$

$$= 0.7776$$

(ii) Probability that the sample proportion ($\hat{p}$) of doctors who favor Thiazide diuretics will be less than or eq 18 can be calculated as follows:

$$P(\hat{p} \leq 0.18) = P\left(\frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} \leq \frac{0.18-}{0.0164}\right.$$

$$= P(Z \leq -1.22) = N(-1.22) = 0.1112$$

(iii) Probability that the sample proportion ($\hat{p}$) of doctors who favor Thiazide diuretics will be more than 0.22 can be calculated as follows:

$$P(\hat{p} > 0.22) = 1 - P(\hat{p} \leq 0.22) = 1 - P\left(\frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} \leq \frac{0.22-0.2}{0.0164}\right)$$

$$= 1 - P(Z \leq 1.22) = 1 - N(1.22)$$

$$= 1 - 0.8888 = 0.1112$$