



لجان الترقيات

STATISTICS



MORPHINE ACADEMY

MORPHINE
ACADEMY

Probability Distribution



Discrete



Binomial
Distribution



نوع من توزيعات الاحتمالات المنفصلة التي تُستخدم
لوصف النتائج العشوائية التي تكون من عدد محدد من التجارب
والاحتمالية نتيجة نجاح / فشل.



Continuous



Normal
Distribution

Example: Binomial Experiment

Testing the effectiveness of a drug

- Suppose that 10 patients with identical infirmities take a drug, for each patient, it is observed whether the drug is effective or not effective. Thus a success is a cure and failure is a non-cure. $n=10$
 $p=70\%$
 $q=30\%$ } independent (لا يتأثر نجاح العلاج لمرضى آخرين)
- The probability of success, p , is the effectiveness of the drug cures a patient, the probability of failure, $q=1-p$, is the probability that the drug does not cure a patient.
- Finally, we can assume that the results of administering the drug are independent from one patient to another. Hence the conditions of Binomial experiment are met.

لِمَ التوزيع الي ذكرناه فهو ثنائي؟

- * التجربة تتكون من ٢ خيارا متساوية.
- * كل نتيجة احدها نجاح او فشل فقط.
- * احتمالية النجاح ثابتة.
- * النتائج مستقلة.

اصدتوه - بالتوضيح

وكانه معنا قانونه كتاب في سورة التوحيد

$$P(X=k) = \binom{n}{k} p^k q^{n-k}$$

$\binom{n}{k}$: عدد الطرق لاختيار k نجاح من n تجارب.

في مثال الى اخذناه : $P(X=7) | p=0.7, n=10$

Binomial Distribution

Thinking Challenge

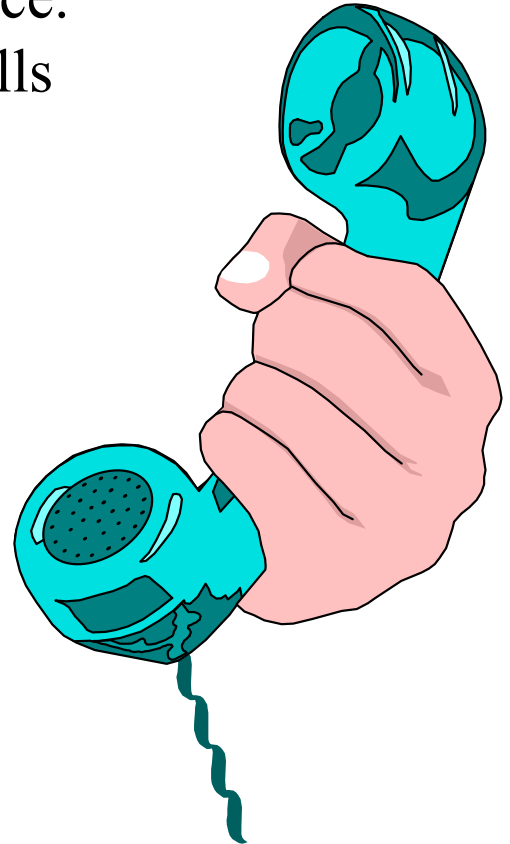
You're a telemarketer selling service.

You've sold 20 in your last 100 calls

($p = .20$). If you

call 12 people tonight, what's the probability of

- A. No sales?
- B. Exactly 2 sales?
- C. At most 2 sales?
- D. At least 2 sales?



Binomial Distribution Characteristics

Positive

$n = 5$ $p = 0.1$

Mean

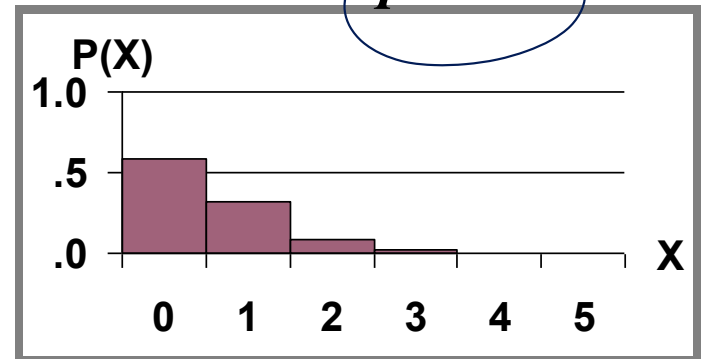
$$\mu = E(x) = np$$

Standard Deviation

$$\sigma = \sqrt{npq}$$

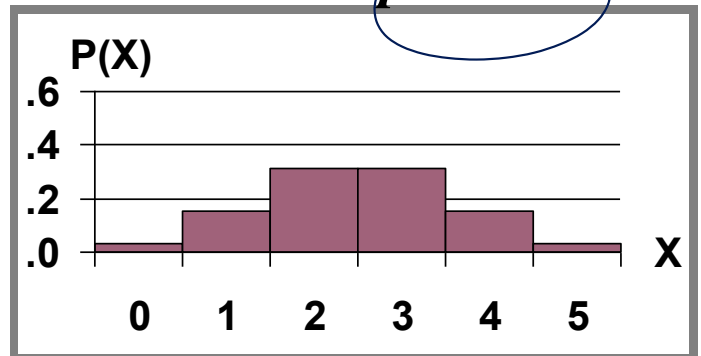
$$\sigma = \sqrt{5 * 0.1 * 0.9} = \sqrt{0.45} \approx 0.67$$

Recall that $q = 1 - p$



$n = 5$ $p = 0.5$

Symmetric



Example

- Suppose it is known a new drug is successful in curing a muscular pain in 22% of the cases. If it is tried on a random sample of 5 patients, then answer the following:

a) Find the probability that:

- 1) No one of patient will be cured?
- 2) Exactly 3 patients will be cured?
- 3) At least 2 patients will be cured?
- 4) At most 4 patients will be cured?
- 5) From 1 to 4 patients will be cured?

b) Find the mean and standard deviation for the distribution of the number of patients who are cured?

c) What is the approximate shape for the distribution of the number of patients who are cured?

: عالج 1
 $n = 5$
 $p = 0.22$
 $q = 1 - p = 0.78$
 $X \sim \text{Bin}(5, 0.22)$

Solution

- $n=5$, $p=0.22$, $X \sim \text{Bin}(5, 0.22)$

$$P(X = 0) = \binom{5}{0} (0.22)^0 (0.78)^{5-0} = \frac{5!}{0!5!} (1)(0.289) = (1)(1)(0.289) = 0.289$$

$$P(X = 1) = \binom{5}{1} (0.22)^1 (0.78)^{5-1} = \frac{5!}{1!4!} (0.22)(0.370) = (5)(0.22)(0.370) = 0.407$$

$$P(X = 2) = \binom{5}{2} (0.22)^2 (0.78)^{5-2} = \frac{5!}{2!3!} (0.0484)(0.475) = (10)(0.0484)(0.475) = 0.229$$

$$P(X = 3) = \binom{5}{3} (0.22)^3 (0.78)^{5-3} = \frac{5!}{3!2!} (0.010648)(0.6084) = (10)(0.010648)(0.6084) = 0.065$$

$$P(X = 4) = \binom{5}{4} (0.22)^4 (0.78)^{5-4} = \frac{5!}{4!1!} (0.00234256)(0.78) = (5)(0.00234256)(0.78) = 0.009$$

$$P(X = 5) = \binom{5}{5} (0.22)^5 (0.78)^{5-5} = \frac{5!}{5!0!} (0.0005153632)(1) = (1)(0.0005153632)(1) = 0.001$$

Continued

x	0	1	2	3	4	5	sum
P(X=x)	0.289	0.407	0.229	0.065	0.009	0.001	1

a) The probability:

- 1) Probability that no one of patients will be cured = $P(X=0)=0.289$.
- 2) Probability that exactly 3 patients will be cured = $P(X=3)=0.065$.
- 3) Probability that at least 2 patients will be cured = $P(X \geq 2)$
 $= P(X=2)+P(X=3)+P(X=4)+P(X=5)$
 $= 0.229+0.065+0.009+0.001$
 $= 0.304$ **OR**

The probability that at least 2 patients will be cured

$$\begin{aligned} &= P(X \geq 2) = 1 - P(X < 2) \\ &= 1 - (P(X=0) + P(X=1)) \\ &= 1 - (0.289 + 0.407) = 1 - 0.696 = 0.304 \end{aligned}$$

Continued

4) Probability that at most 4 patients will be cured

$$= P(X \leq 4)$$

$$= P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)$$

$$= 0.289+0.407+0.229+0.065+0.009 = 0.999$$

OR

Probability that at most 4 patients will be cured

$$= P(X \leq 4) = 1 - (P > 4) = 1 - (P(X=5)) = 1 - 0.001 = 0.999$$

5) Probability that from 1 to 4 patients will be cured

$$= P(1 \leq X \leq 4)$$

$$= P(X=1)+P(X=2)+P(X=3)+P(X=4)$$

$$= 0.407+0.229+0.065+0.009$$

$$= 0.71$$

Continued

b) The mean and the standard deviation for the number of patients who are cured can be calculated as follows:

➤ The mean $\mu = E(x) = np$
 $= (5)(0.22) = 1.1$ patients

➤ The standard deviation $\sigma = \sqrt{npq}$
 $\sigma = \sqrt{np(1 - q)} = \sqrt{(5)(0.22)(0.78)} = 0.926$ patient.

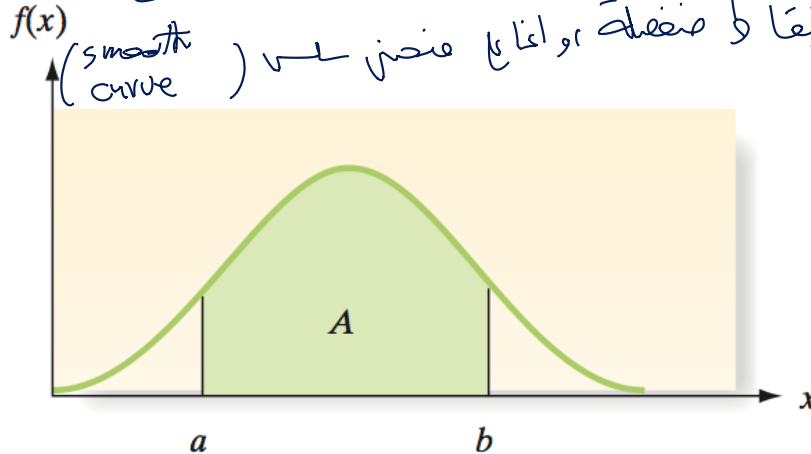
c) The approximate shape of the distribution of the number of patients who are cured is right-skewed (positive) because $p = 0.22$ which is less than 0.5.

(Right-skewed) $p = 0.22 < 0.5$
positive

Continuous Probability Density Function

The graphical form of the probability distribution for a continuous random variable x is a smooth curve

لما يكون المقياس العددي x (مستمر) فيزياء الفولت/الوزن/الوقت... فانه توزيع الاحتمال الخالص منه
لا يُعبر عنه بنقاط منفصلة او انما يعبر عنه بـ (smooth curve)



Probability Distributions for Continuous Random Variables

The Normal Distribution

Probability Density Function (pdf)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{1}{2}\right)\left(\frac{x-\mu}{\sigma}\right)^2}$$

where

μ = Mean of the normal random variable x

σ = Standard deviation

$\pi = 3.1415 \dots$

$e = 2.71828 \dots$

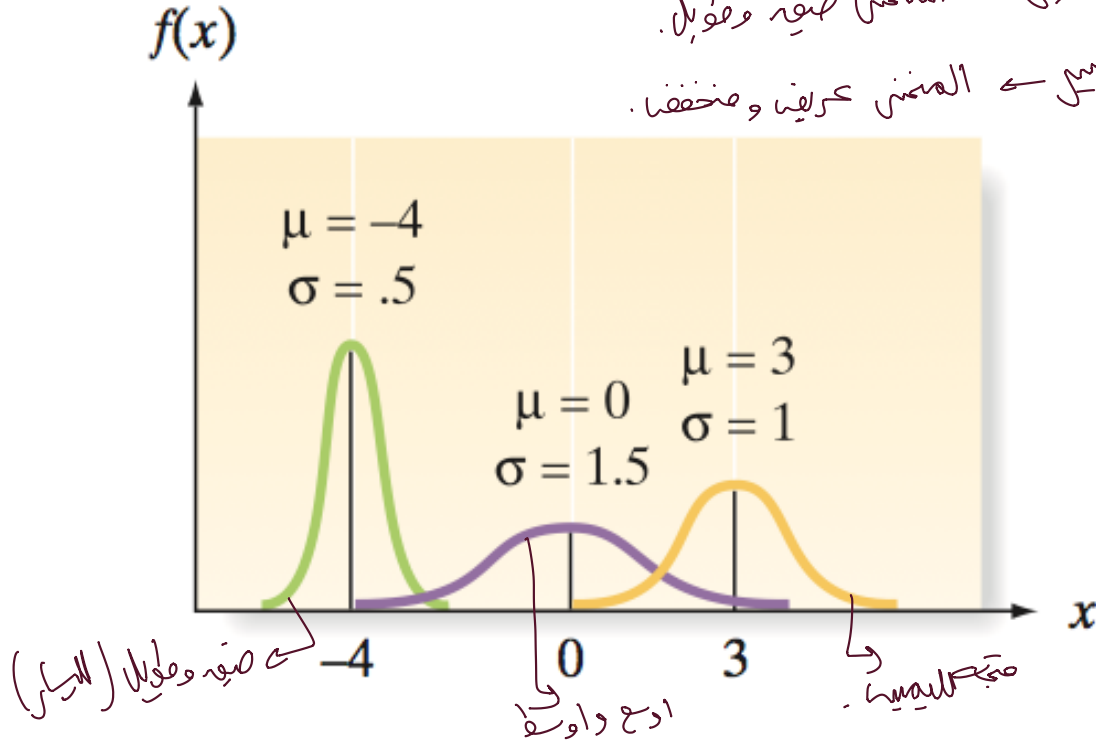
$P(x < a)$ is obtained from a table of normal probabilities

Effect of Varying Parameters (μ & σ)

مركز - مجرد موقع
المتوسط
المتوسط.

σ ← يحدد عرض
المتوسط.

إذا كان σ صغيراً ← المتوسط ضيقاً وطويلاً.
" " σ كبيراً ← المتوسط عريضاً ومختطفاً.



The Beauty of the Normal Curve

- Note that, when X is normally distributed with the mean μ and standard deviation σ , then we refer to that as follows:
 $X \sim N(\mu, \sigma^2)$.
- Example: 68-95-99 Rule

The Hashemite University students intelligence scores (X) are normally distributed with $\mu = 100$ and $\sigma = 15$; that is $X \sim N(100, 255)$.

- 68% of scores within $\mu \pm \sigma = 100 \pm 15 = \underline{85 \text{ to } 115}$
- 95% of scores within $\mu \pm 2\sigma = 100 \pm (2)(15) = \underline{70 \text{ to } 130}$
- 99.7% of scores within $\mu \pm 3\sigma = 100 \pm (3)(15) = \underline{55 \text{ to } 145}$

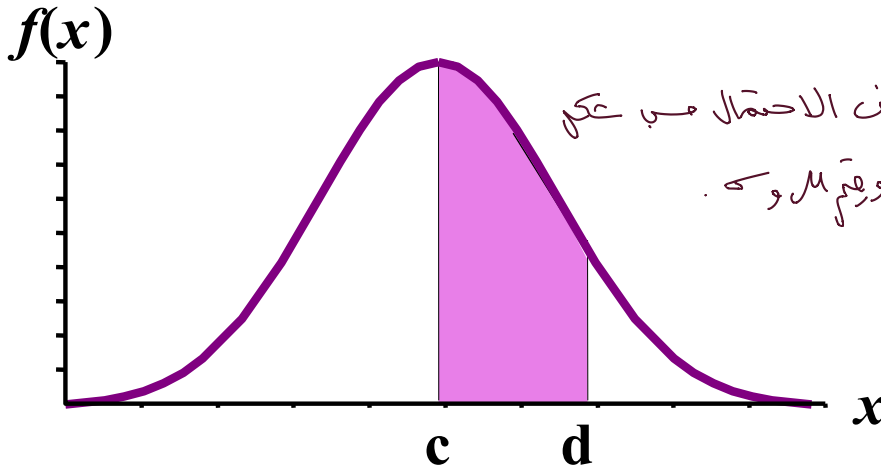
Normal Distribution

Probability

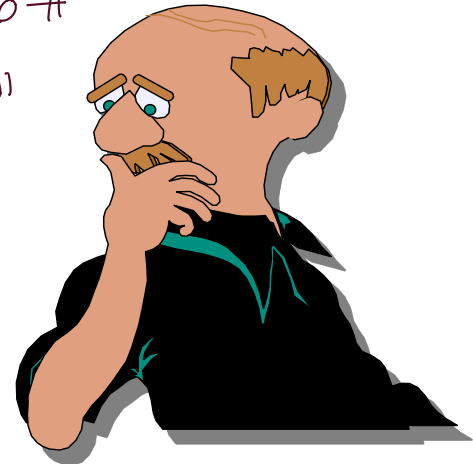
الاحتمال = المساحة تحت المنحنى
بـ القِيَمَة

Probability is
area under
curve!

$$P(c \leq x \leq d) = \int_c^d f(x) dx?$$



صِبْغًا فَبَلَّتِ الاحتمال من عَمَلِ
المنزل وقيَمته وكـ

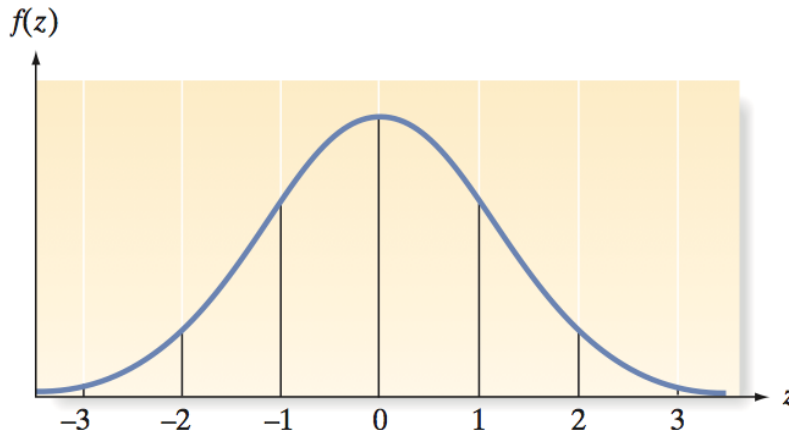


Standard Normal Distribution

The standard normal distribution is a normal distribution with $\mu = 0$ and $\sigma = 1$. A random variable with a standard normal distribution, denoted by the symbol z , is called a standard normal random variable.

$$Z \sim N(0, 1).$$

كل التوزيعات الطبيعية
عكس منحنى التوزيع
معايير الانحراف.



وحاد بجمع
لا يتزامن صيغ
z بل صلات
معددة.

Property of Normal Distribution

If x is a normal random variable with mean μ and standard deviation σ , then the random variable z , defined by the formula

$$z = \frac{x - \mu}{\sigma}$$

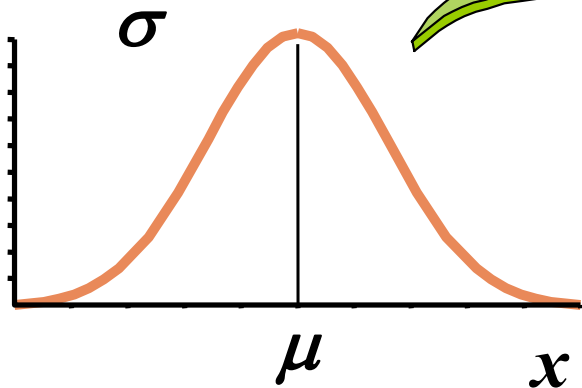
has a standard normal distribution. The value z describes the number of standard deviations between x and μ .

این از آنجا که z معیار استاندارد است و μ میانگین است، این فرمول z را می‌دهد.
بنابراین، z معیار استاندارد است.

انه لو تم تحويل الى توزيع طبيعي الى توزيع طبيعي معيار باستخدام الصيغة :
$$z = \frac{x - \mu}{\sigma}$$

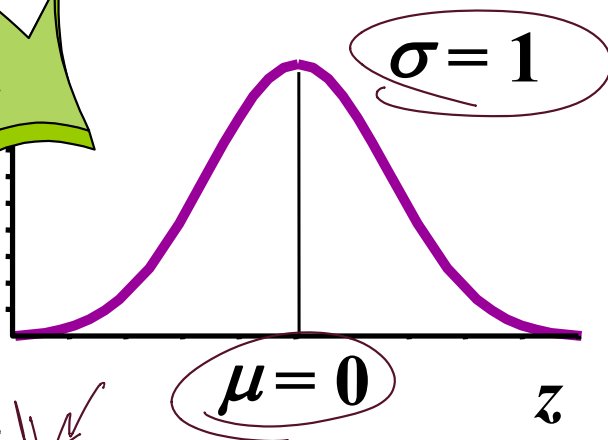
Standardize the Normal Distribution

Normal Distribution



$$z = \frac{x - \mu}{\sigma}$$

Standardized Normal Distribution

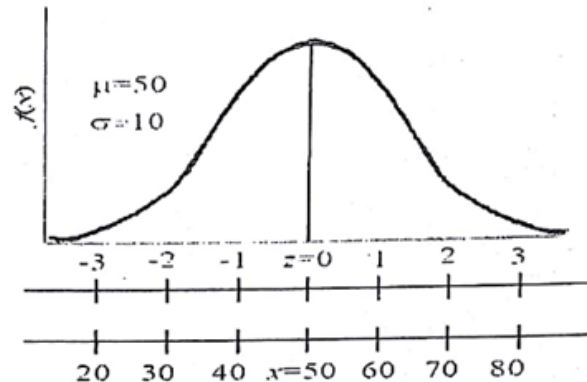
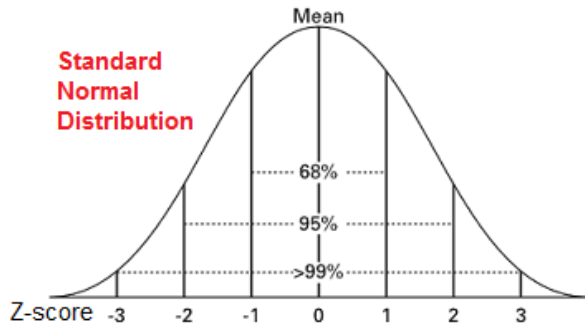
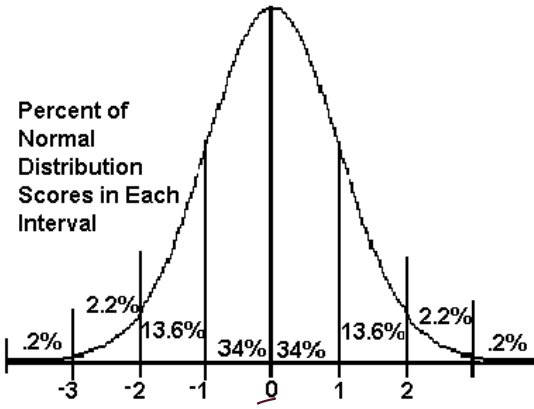
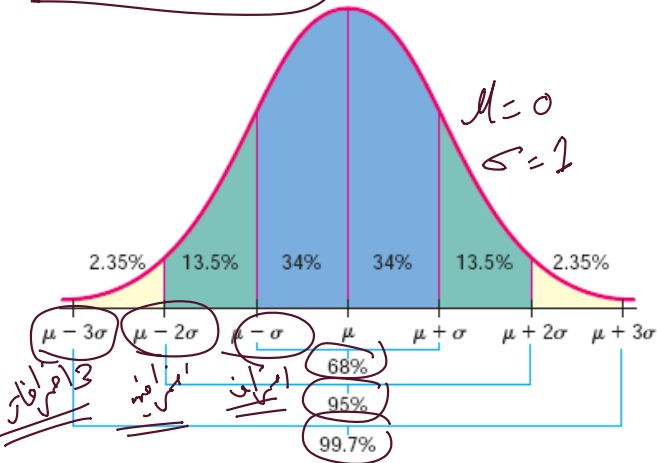


النتيجة ←

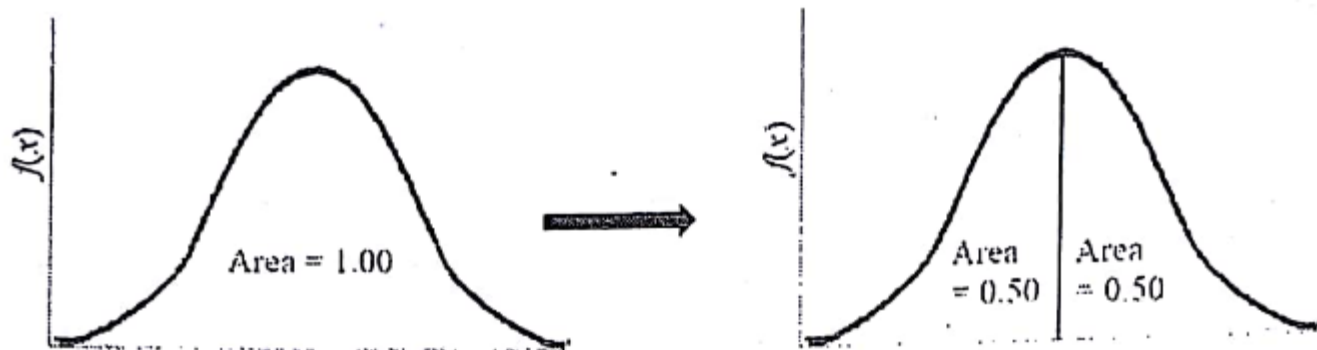
One table!

Standard Normal Distribution

Area Under a Normal Curve

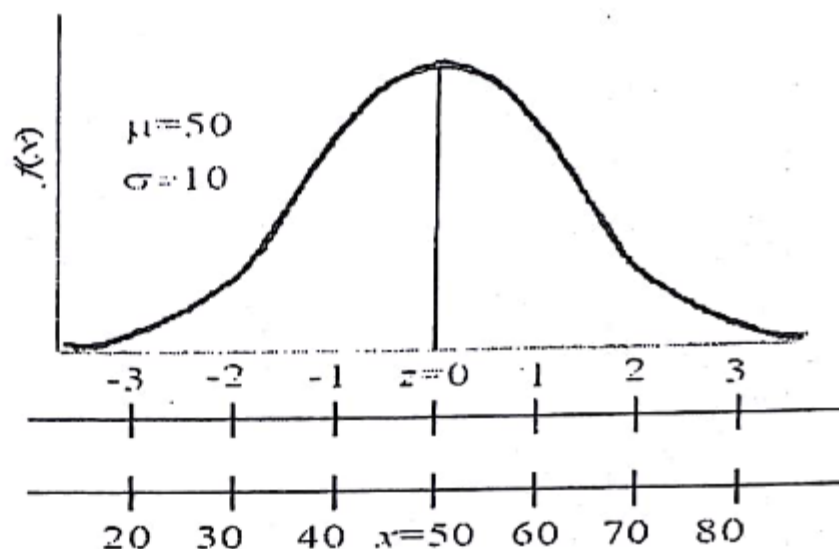


STANDARD NORMAL DISTRIBUTION



$$z = \frac{X - \mu}{\sigma}$$

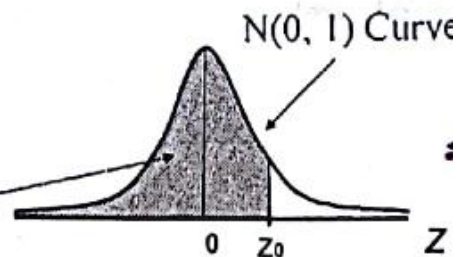
$$\text{or, } X = \mu + z\sigma$$



Finding Areas Under the Standard Normal Curve

- (a) To find the area to the *left* of z_0 , find the area that corresponds to z_0 in the Standard Normal Table by using the following rule:

$$P(Z \leq z_0) = N(z_0)$$

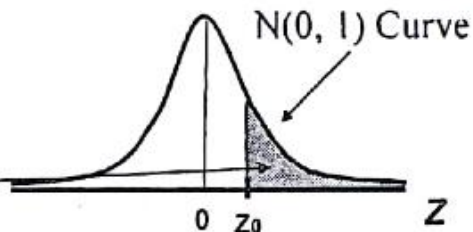


- (b) To find the area to the *right* of z_0 , use the Standard Normal Table to find the area that corresponds to *left* of z_0 . Then subtract the area from 1 by using the following formula:

$$P(Z > z_0) = 1 - P(Z \leq z_0) = 1 - N(z_0)$$

Or

$$P(Z > z_0) = P(Z \leq -z_0) = N(-z_0)$$



Non-standard Normal $\mu = 5$, $\sigma = 10$:

$$P(5 < x < 6.2)$$

$$0 = 0.5 \quad 0.12 = 0.5478$$

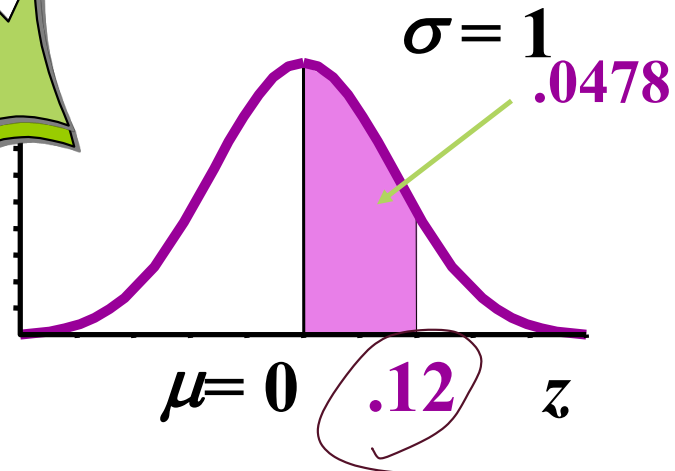
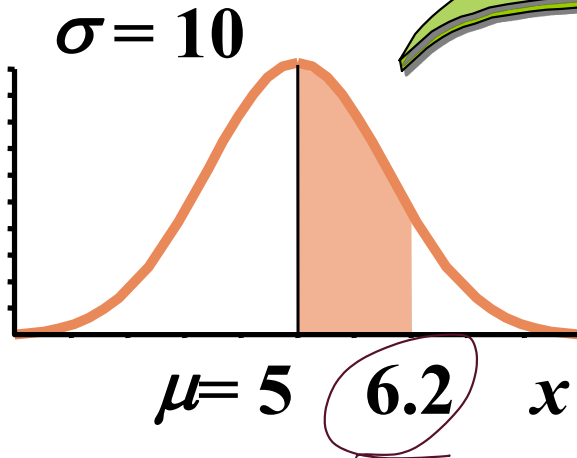
$$P(0 < z < 0.12) = 0.5478 - 0.5 \\ = 0.0478$$

$$0 < z < 0.12$$

$$z = \frac{x - \mu}{\sigma} = \frac{6.2 - 5}{10} = .12 \quad Z = (5 - 5)/10 = 0$$

Normal
Distribution

Standardized Normal
Distribution



Shaded area exaggerated

The Standard Normal Table:

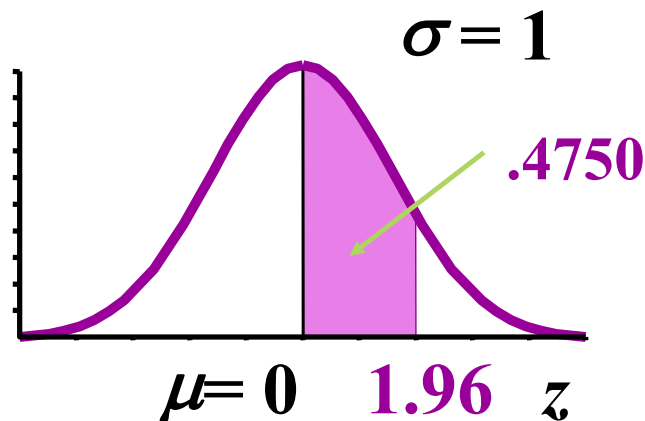
$$P(0 < z < 1.96)$$

Standardized Normal
Probability Table

Z	.04	.05	.06
1.8	.96512	.96784	.96856
1.9	.97381	.97441	.97500
2.0	.97932	.9798	.98030
2.1	.98382	.98422	.984616

$$0 = 0.5 \quad 1.96 = 0.9750$$

$$P(0 < z < 1.96) = 0.9750 - 0.5 = 0.4750$$



Probabilities

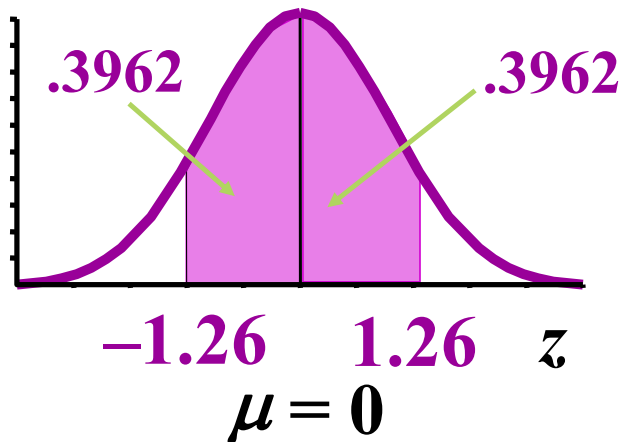
Shaded area
exaggerated

The Standard Normal Table:

$$P(-1.26 \leq z \leq 1.26)$$

Standardized Normal Distribution

$$\sigma = 1$$



Shaded area exaggerated

$$-1.26 = 0.1038 \quad 1.26 = 0.8962$$

$$P(-1.26 \leq z \leq 1.26) = 0.8962 - 0.1038 = 0.7924$$

$$P(-1.26 \leq z \leq 1.26)$$

$$= 0.8962 - 0.1038$$

$$= 0.7924$$

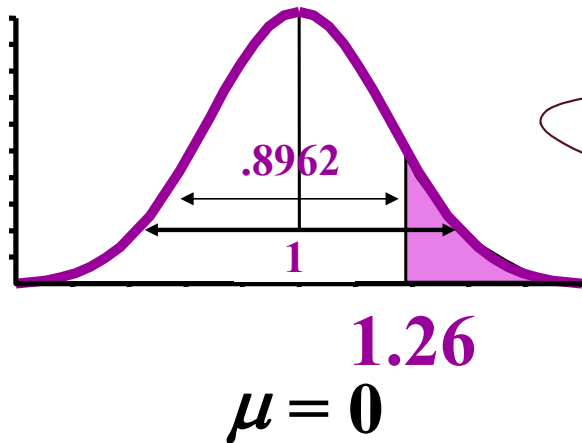
لنفس الاحتمال لـ $z = 1.26$ و $z = -1.26$
صالحه
 $P(z \leq 1.26) \approx 0.8962$

The Standard Normal Table:

$$P(z > 1.26)$$

Standardized Normal Distribution

$$\sigma = 1$$



$$P(z > 1.26) = 1 - P(z \leq 1.26)$$

$$= 1 - .8962$$

$$= .1038$$

Or

$$P(z > 1.26) = P(z \leq -1.26)$$

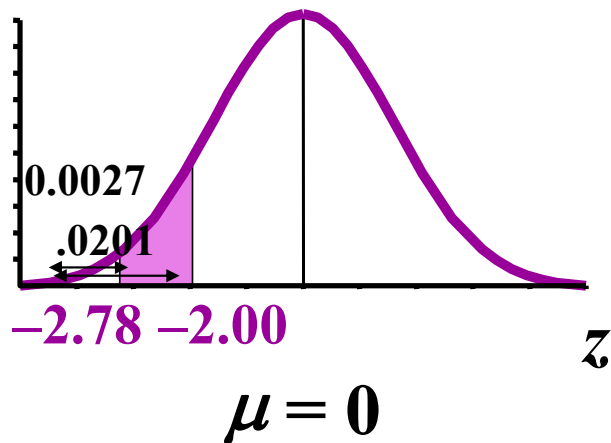
$$= .1038$$

The Standard Normal Table:

$$P(-2.78 \leq z \leq -2.00)$$

Standardized Normal Distribution

$$\sigma = 1$$



$$P(-2.78 \leq z \leq -2.00)$$

$$= .0228 - .0027$$

$$= .0201$$

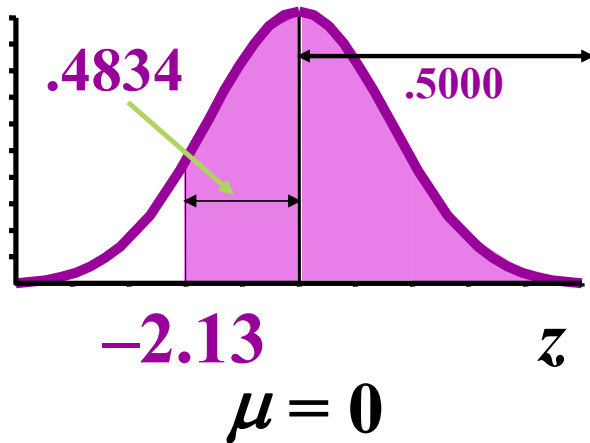
Shaded area exaggerated

The Standard Normal Table:

$$P(z > -2.13)$$

Standardized Normal Distribution

$$\sigma = 1$$



Shaded area exaggerated

$$\begin{aligned} P(z > -2.13) &= P(z \leq 2.13) \\ &= 0.9834 \end{aligned}$$

OR

$$\begin{aligned} P(z > -2.13) &= 1 - P(z \leq -2.13) \\ &= 1 - 0.0166 \\ &= .9834 \end{aligned}$$

لِصِفِ اَوَّلِ اسْتِثْنَاً : ① تحديده الاحتمالية المطلوبة :

• مصدر النفاذ الذي يبين ما هي الاحتمالية

مثل $P(a < Z < b)$ أو $P(Z > c)$.

② تحويل الى صيغة Z :

• اذا كان التوزيع غير معيار، استندم الصيغة : $Z = \frac{x - \mu}{\sigma}$

• احسب قيمة Z لكل صفة بالنفاذ (مثل Z_1 و Z_2).

③ استندلم جدول Z :

• ارجع لجدول التوزيع الصفي القياسي لإيجاد الاحتمالية المشتركة

$P(Z \leq z)$ لكل قيمة Z .

• الجدول يعطيك المنطقة من اليسار، $(-\infty)$ الى Z المحدد

④ صاب الاحتمالية :

• لنفرض ببيتي قمتين $P(Z_1 < Z < Z_2)$:

$$P(Z_1 < Z < Z_2) = P(Z \leq Z_2) - P(Z \leq Z_1)$$

• للقيم الأكبر من Z $P(Z > Z_2)$:

$$P(Z > Z_2) = 1 - P(Z \leq Z_2)$$

• استخدم التناظر (لأنه المنحنى متماثل) لهذا نلزم الأمر :

$$P(Z > Z_2) = P(Z < -Z_2)$$

وهي خلاصة طرفية الى

الاحتمال

Non-standard Normal $\mu = 5, \sigma = 10$:

$$P(3.8 \leq x \leq 5)$$

$$z = \frac{x - \mu}{\sigma} = \frac{3.8 - 5}{10} = -.12$$

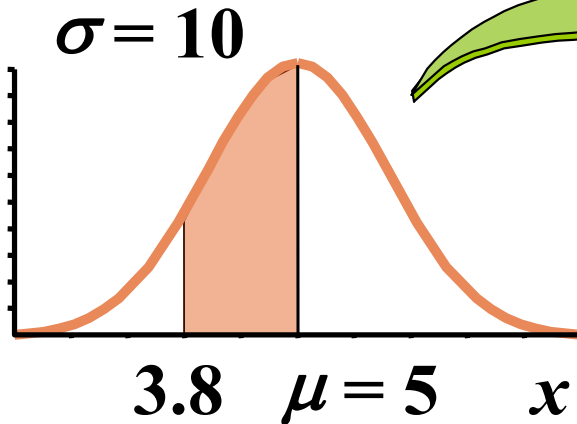
Z score for $-.12 = 0.4522$

$P(-.12 \leq z \leq 0)$

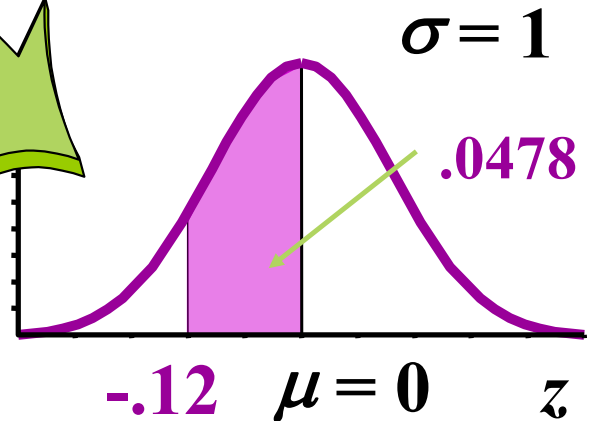
$= 0.5 - 0.4522$

$= 0.0478$

Normal
Distribution



Standardized Normal
Distribution



Shaded area exaggerated

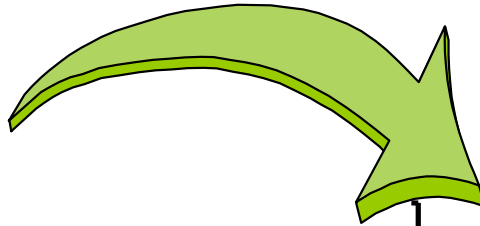
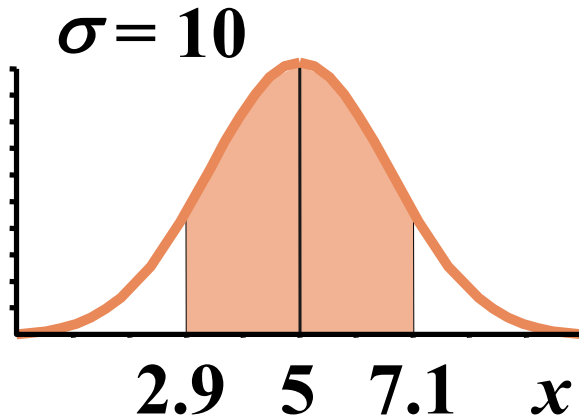
Non-standard Normal $\mu = 5, \sigma = 10$:

$$P(2.9 \leq x \leq 7.1)$$

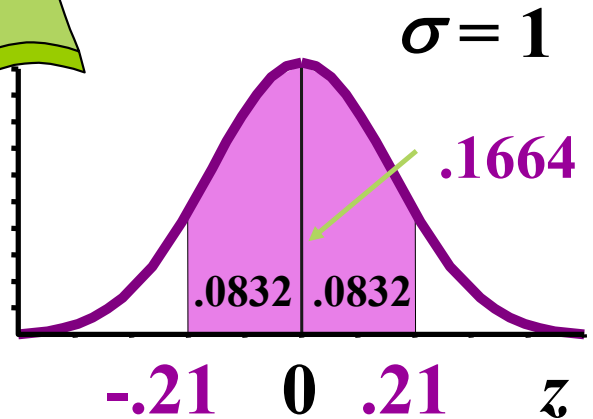
$$z = \frac{x - \mu}{\sigma} = \frac{2.9 - 5}{10} = -.21$$

$$z = \frac{x - \mu}{\sigma} = \frac{7.1 - 5}{10} = .21$$

Normal
Distribution



Standardized Normal
Distribution



Shaded area exaggerated

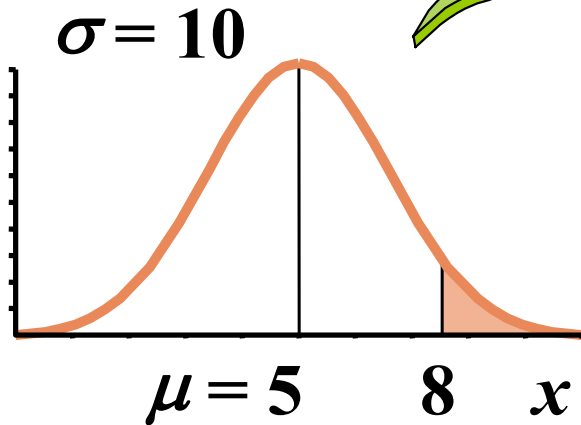
$$\begin{aligned} P(-.21 \leq z \leq .21) \\ &= 0.5832 - 0.4522 \\ &= 0.1664 \end{aligned}$$

Non-standard Normal $\mu = 5, \sigma = 10$:

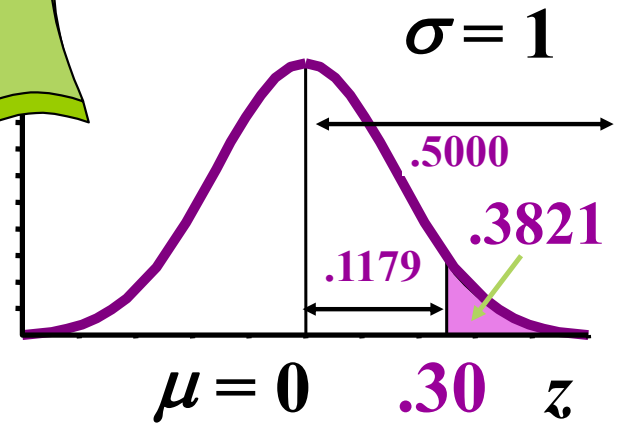
$$P(x \geq 8)$$

$$z = \frac{x - \mu}{\sigma} = \frac{8 - 5}{10} = .30$$

Normal
Distribution



Standardized Normal
Distribution



Shaded area exaggerated

$$\begin{aligned} P(z \geq 0.3) &= 1 - P(z \leq 0.3) \\ &= 1 - 0.6179 \\ &= 0.3821 \end{aligned}$$

Non-standard Normal $\mu = 5, \sigma = 10$:

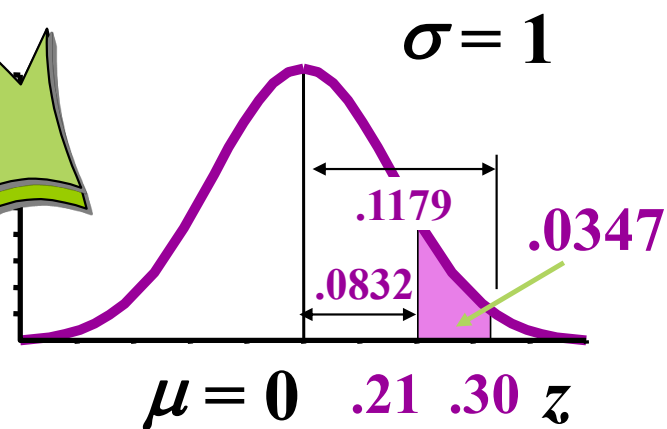
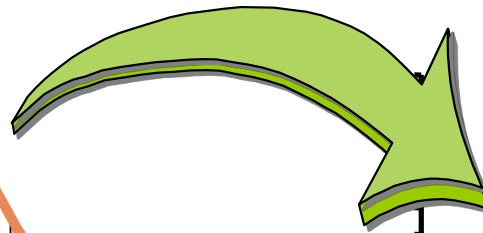
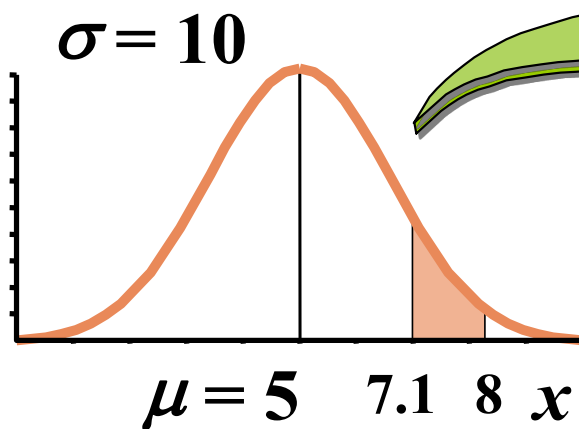
$P(7.1 \leq X \leq 8)$

$$z = \frac{x - \mu}{\sigma} = \frac{7.1 - 5}{10} = .21$$

Normal
Distribution

$$z = \frac{x - \mu}{\sigma} = \frac{8 - 5}{10} = .30$$

Standardized Normal
Distribution



Shaded area exaggerated

$$\begin{aligned} P(0.21 \leq z \leq 0.3) \\ &= 0.6179 - 0.5832 \\ &= 0.0347 \end{aligned}$$

أما طرف صيغة الأس هو:

① الخطوة الأولى هي تحويل القيم X إلى Z باستخدام

$$Z = \frac{X - \mu}{\sigma}$$

② استخدام جدول Z لإيجاد الاحتمالات المشتركة، ثم حساب

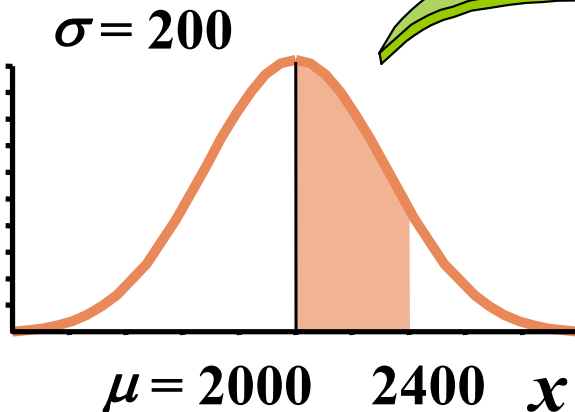
الفرق أو الفرق حسب النسبة.

المعيار ◇	التوزيع الطبيعي القياسي (Standard) ◇	التوزيع الطبيعي غير القياسي (Non-standard) ◇
تعريف	توزيع طبيعي بمتوسط $\mu = 0$ وانحراف معياري $\sigma = 1$.	توزيع طبيعي بمتوسط μ وانحراف معياري σ يختلفان عن 0 و1.
رمز	$Z \sim N(0, 1).$	$X \sim N(\mu, \sigma).$
حاجة للتحويل	لا حاجة، القيم جاهزة كـ Z مباشرة.	يجب تحويل القيم X إلى Z باستخدام $Z = \frac{X - \mu}{\sigma}$.
استخدام جدول Z	يُستخدم مباشرة لإيجاد $P(Z \leq z)$.	يُستخدم بعد التحويل إلى Z لإيجاد الاحتمالية.
أمثلة على الحساب	لحساب $P(0 < Z < 1.96)$: استخدم الجدول مباشرة (0.4750).	لحساب $P(3.8 \leq X \leq 5)$ مع $\mu = 5, \sigma = 10$: احسب $Z = -0.12$ و $Z = 0.38$ ، ثم $0.5 - 0.4522 = 0.0478$.
تعقيد الحساب	أقل تعقيدًا لأن القيم جاهزة.	أكثر تعقيدًا بسبب الحاجة للتحويل أولاً.
الرسم البياني	منحنى مركز حول 0 مع انحراف معياري 1.	منحنى قد يكون مركّزًا حول أي قيمة μ مع انحراف σ مختلف.

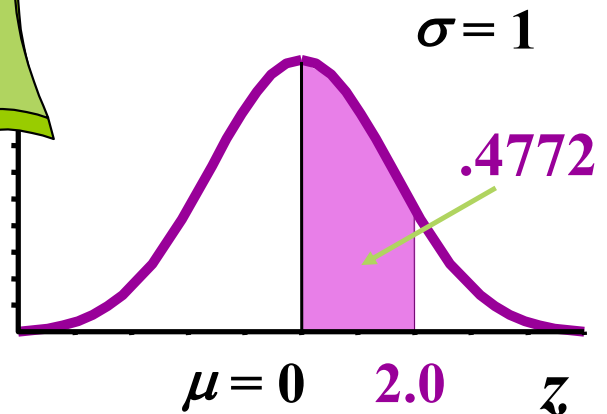
Solution* $P(2000 \leq x \leq 2400)$

$$z = \frac{x - \mu}{\sigma} = \frac{2400 - 2000}{200} = 2.0$$

**Normal
Distribution**



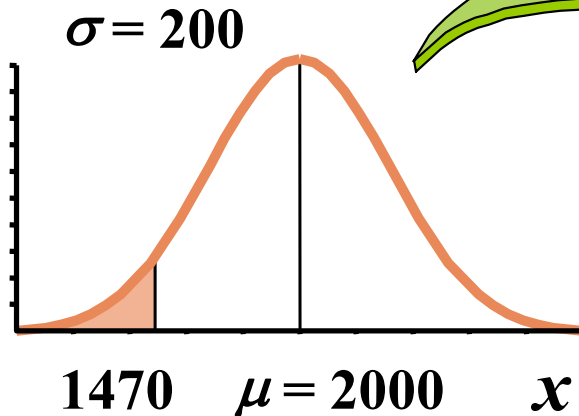
**Standardized Normal
Distribution**



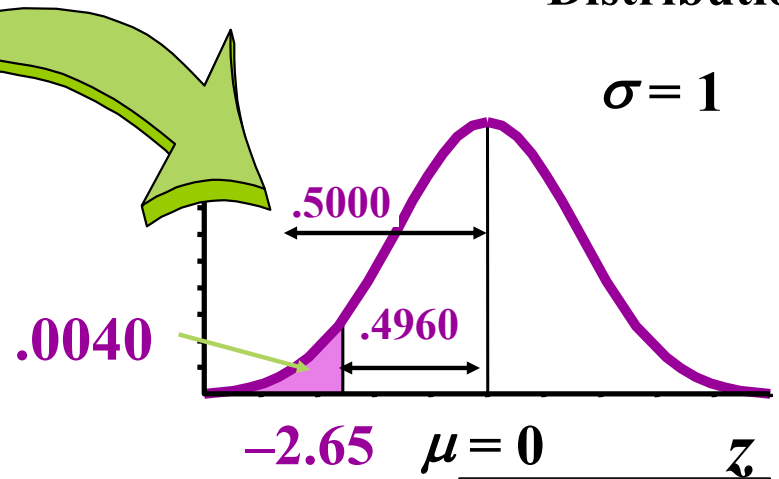
Solution* $P(x \leq 1470)$

$$z = \frac{x - \mu}{\sigma} = \frac{1470 - 2000}{200} = -2.65$$

**Normal
Distribution**



**Standardized Normal
Distribution**



$P(z \leq -2.65)$ $= 0.0040$

Example

A survey in Jordan indicates that pharmacies use their computers in an average of 2.4 years before upgrading to a new machine. The standard deviation is 0.5 year. A pharmacy is selected at random. Find the probability that the pharmacy will use it for less than or equal 2 years before upgrading. Assume that the variable X is normally distributed?

Continued

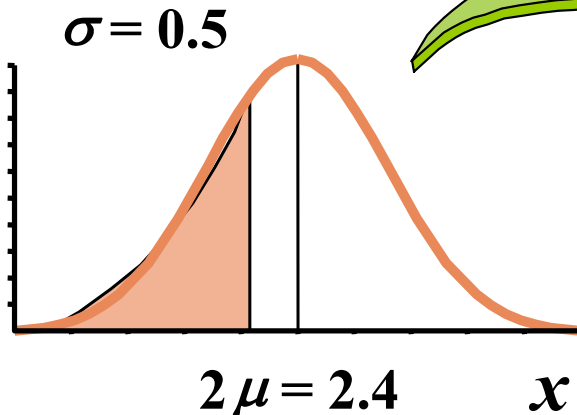
Solution

We need to find $P(X \leq 2)$ as follows: $(x - \mu) / \sigma = (2 - 2.4) / 0.5 = -0.8$

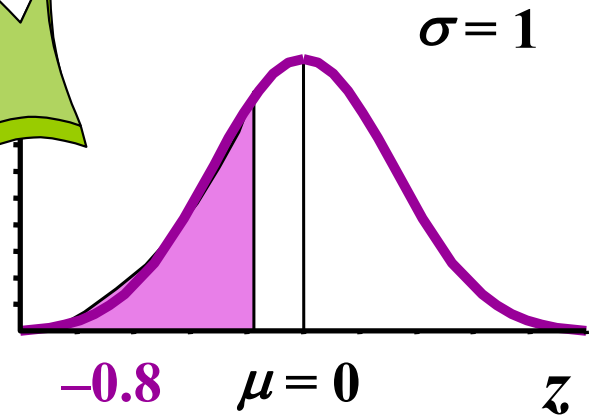
So $P(z \leq -0.8) = 0.2119$

$$z = \frac{X - \mu}{\sigma} = \frac{2 - 2.4}{0.5} = -0.8$$

**Normal
Distribution**

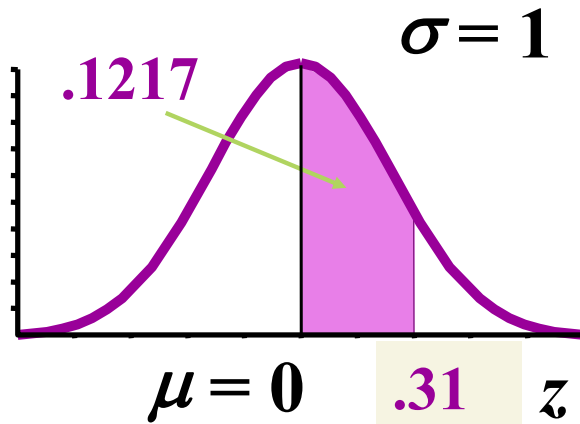


**Standardized Normal
Distribution**



Finding z -Values for Known Probabilities

What is Z , given
 $P(z) = .1217$?



z	0	0.01	0.02
+0	.50000	.50399	.50798
+0.1	.53983	.54380	.54776
+0.2	.57926	.58317	.58706
+0.3	.61791	.62172	.62552

The probability between the z and the mean = 0.1217

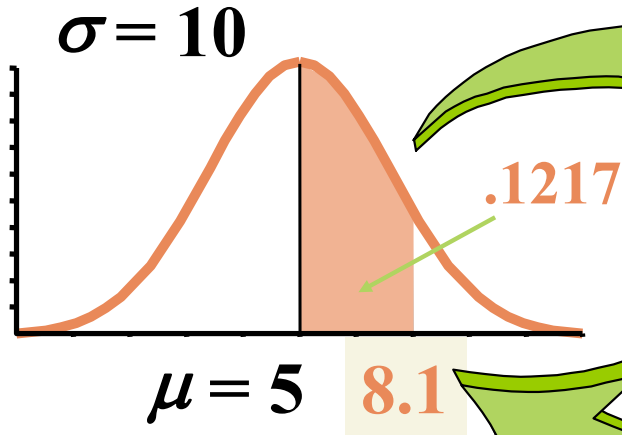
$$0.1217 + 0.500 = 0.6217$$

So it is corresponding to 0.31

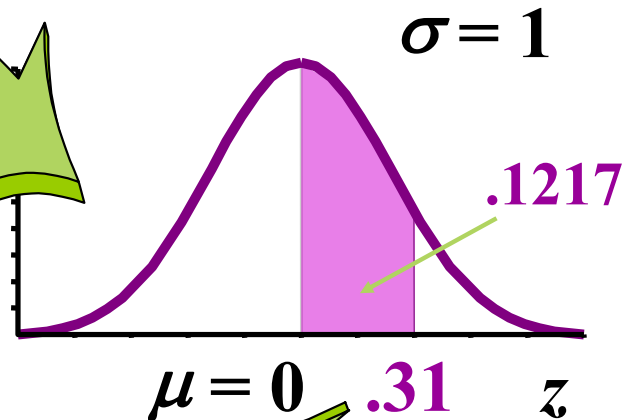
Shaded area
exaggerated

Finding x Values for Known Probabilities

Normal Distribution



Standardized Normal Distribution



$$x = \mu + z \cdot \sigma = 5 + (.31)(10) =$$

Shaded areas exaggerated

$$\mu = 2.4 \quad / \quad \sigma = 0.5$$

$$Z = \frac{X - \mu}{\sigma} = \frac{2 - 2.4}{0.5} = -0.8$$

← استخدام جدول Z :

$$P(Z \leq -0.8) \approx 0.2119$$

∴ غالباً محتملة : $P(X \leq 2) = P(Z \leq -0.8) \approx 0.2119$

معني ← في حوالي 21.19% احتمال ان النتيجة تقدم الجواز لدرجة

2 سنة أو أقل.

Example

The daily sales volume in JD for a given pharmacy in Jordan is normally distributed with a mean of 67 JD per day and a standard deviation of 4 JD per day. Find the daily sales volume x corresponding to z-score 1.96, -2.33, and 0?

- $z = 1.96$, $x = 67 + 1.96(4) = 74.84$ JD per day
- $z = -2.33$, $x = 67 + (-2.33)(4) = 57.68$ JD per day.
- $z = 0$, $x = 67 + 0(4) = 67$ JD per day.
- Notice that 74.84 JD is above the mean, 57.68 JD is below the mean, and 67 JD is equal to the mean.

$$\sigma = 4 / \mu = 67$$

$$74.84 \leq x : 1.96 = \frac{7.84}{4} = \frac{74.84 - 67}{4} = z = \frac{x - \mu}{\sigma}$$

$$57.68 \leq x : -2.33 = \frac{-9.32}{4} = \frac{57.68 - 67}{4} = z = \frac{x - \mu}{\sigma}$$

$$67 \leq x : 0 = \frac{67 - 67}{4} = z = \frac{x - \mu}{\sigma}$$

يعني: * 74.84 دينار فوفه المتوسطا بحدار 1.96 انحراف معياري.

* 57.68 دينار تحت المتوسطا بحدار 2.33 انحراف معياري.

* 67 دينار يساوي المتوسطا (فتر انحراف)

Other Discrete Distributions:

Poisson

Characteristics of a Poisson Random Variable

يتكوّن من عدد المرات التي تحدث فيها صفة في وحدة زمنية أو مساحة.

1. Consists of counting number of times an event occurs during a given unit of time or in a given area or volume (any unit of measurement).
2. The probability that an event occurs in a given unit of time, area, or volume is the same for all units.
3. The number of events that occur in one unit of time, area, or volume is independent of the number that occur in any other mutually exclusive unit.
4. The mean number of events in each unit is denoted by λ .

المتوسط أو المتوسط الحسابي

Poisson Random Variable

متغير عشوائي

① يتكرر عدد المرات التي تحدث فيها حدث معين في وحدة زمنية أو مكانية.

② احتمال حدوث حدث مرة واحدة / عدة مرات / لا يحدث هو نفس الشيء.

③ عدد الأحداث في وحدة زمنية واحدة مستقلة عن عدد الأحداث في وحدة زمنية أخرى.

④ المتوسط الحسابي للأحداث $\leq \lambda$

Poisson Probability Distribution Function

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad (x = 0, 1, 2, 3, \dots)$$

$$\mu = \lambda$$

$$\sigma^2 = \lambda$$

$p(x)$ = Probability of x given λ

λ = Mean (expected) number of events in unit

e = 2.71828 ... (base of natural logarithm)

x = Number of events **per unit**

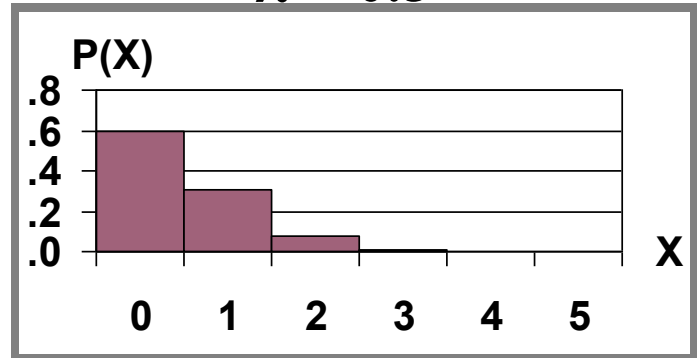
Poisson Probability Distribution Function

يمكن جعل صيغة الامتحان في صورة زخرف او صيغة
وهو ينه $E(x)$

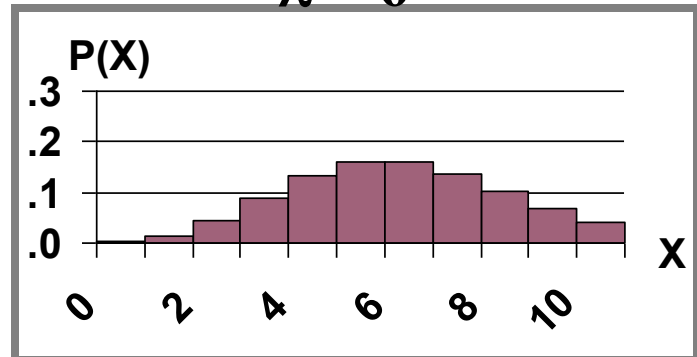
Mean

$$\mu = E(x) = \lambda$$

$\lambda = 0.5$



$\lambda = 6$



Standard Deviation

$$\sigma = \sqrt{\lambda}$$

Poisson Distribution Example

Customers arrive at a
rate of 72 per hour.

What is the probability
of 4 customers arriving
in 3 minutes?



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Poisson Distribution Solution

72 Per Hr. = 1.2 Per Min. = 3.6 Per 3 Min. Interval

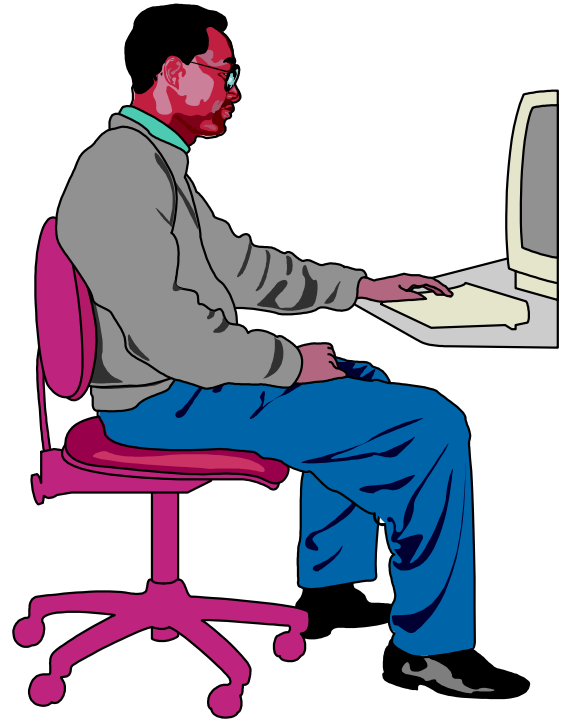
$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$p(4) = \frac{(3.6)^4 e^{-3.6}}{4!} = .1912$$

احتمال وقوع 4 حوادث في 3 دقائق
هو حوالي 19.12%

Thinking Challenge

You work in Quality Assurance for an investment firm. A clerk enters **75** words per minute with **6** errors per hour. What is the probability of **0 errors** in a **255-word** bond transaction?



Poisson Distribution Solution: Finding λ^*

- $75 \text{ words/min} = (75 \text{ words/min})(60 \text{ min/hr})$
 $= 4500 \text{ words/hr}$
- $6 \text{ errors/hr} = 6 \text{ errors}/4500 \text{ words}$
 $= .00133 \text{ errors/word}$
- In a **255**-word transaction (interval):
 $\lambda = (.00133 \text{ errors/word})(255 \text{ words})$
 $= .34 \text{ errors}/255\text{-word transaction}$

Poisson Distribution Solution: Finding $p(0)^*$

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$p(0) = \frac{(.34)^0 e^{-.34}}{0!} = .7118$$

لحساب الاحتمالية ان لا يتكبد

الاشغال في معالجة 255

من حوالي 71.18%

