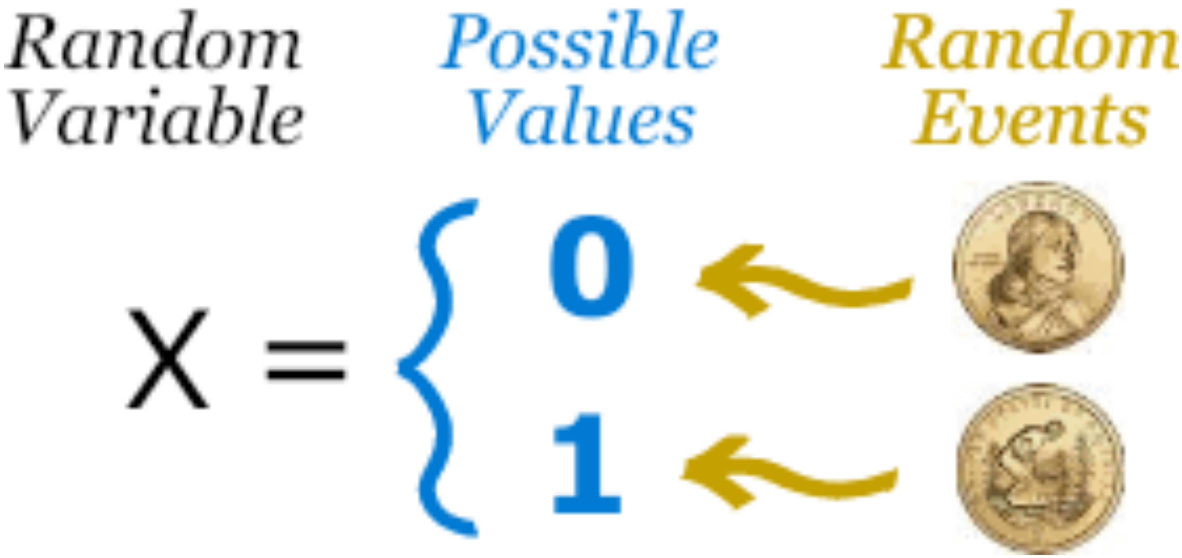
لجان الدُّفعات

STATISTICS

MORPHINE ACADEMY

Discrete Random Variable



Discrete Random Variable Examples

احفظوا هاي الأمثلة

Experiment	Random Variable	Possible Values
Make 100 Sales Calls	# Sales	0, 1, 2, ..., 100
Inspect 70 Radios	# Defective	0, 1, 2, ..., 70
Answer 33 Questions	# Correct	0, 1, 2, ..., 33
Count Cars at Toll Between 11:00 & 1:00	# Cars Arriving	0, 1, 2, ..., ∞

Continuous Random Variable Examples

برضو احفظوا هاي الأمثلة.

Experiment	Random Variable	Possible Values
Weigh 100 People	Weight	45.1, 78, ...
Measure Part Life	Hours	900, 875.9, ...
Amount spent on food	$ amount	54.12, 42, ...
Measure Time Between Arrivals	Inter-Arrival Time	0, 1.3, 2.78, ...

Probability Distributions for Discrete Random Variables

توزيع الاحتمالات للمتغير المنفصل : طريقة لوصف كيفية توزيع الاحتمالات على القيم الممكنة لمتغير عشوائي منفصل

- The **probability distribution for a discrete random variable *x*** resembles the relative frequency distributions we constructed previously.

طبقاً نستطيع عرضه على شكل جدول او رسم بياني (هستوغرام) بحيث انه مربوط كل قيمة محتملة بالاحتمالية المرتبطة فيها.

It is a graph, table or formula that gives the possible values of *X* and the probability *p(X=x)* associated with each value.

- The probability distribution for a discrete variable X must satisfy the following two conditions:

1) $0 \leq p(X = x) \leq 1, and$ → احتمالية كل قيمة لازم تكون بين 0 و 1
2) $\sum p(X=x)=1$ → مجموع جميع الاحتمالات الممكنة لازم تساوي 1

Example

- Toss a fair coin three times and define ***X*** **= number of heads.**

		x
HHH	1/8	3
HHT	1/8	2
HTH	1/8	2
THH	1/8	2
HTT	1/8	1
THT	1/8	1
TTH	1/8	1
TTT	1/8	0

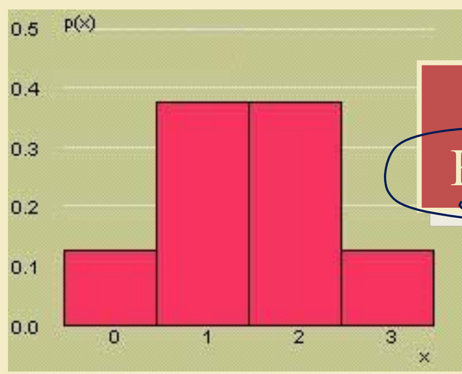
$P(X = 0) = 1/8$

$P(X = 1) = 3/8$

$P(X = 2) = 3/8$

$P(X = 3) = 1/8$

x	$p(x)$
0	1/8
1	3/8
2	3/8
3	1/8



Probability Histogram for x

Discrete Probability Distribution Example

Experiment: Toss 2 coins. Count number of tails.

Probability Distribution

Values, x	Probabilities, $p(x)$
0	1/4 = .25
1	2/4 = .50
2	1/4 = .25

Visualizing Discrete Probability Distributions

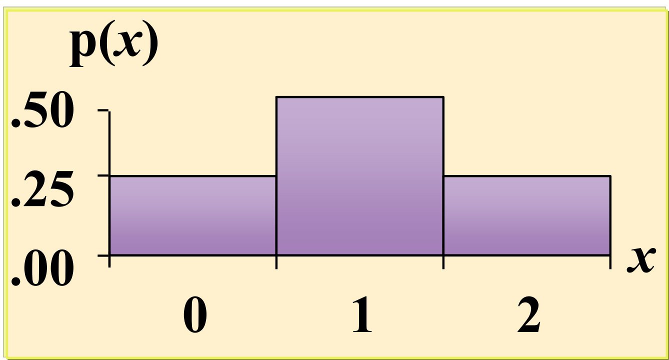
Listing

{ (0, .25), (1, .50), (2, .25) }

Table

# Tails	f(x) Count	p(x)
0	1	.25
1	2	.50
2	1	.25

Graph



Formula

$$p(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

Example

In the random experiment of rolling two dice once, then the number of the outcomes = n(S) = 6*6=36, and if we define X as follows:

X=the sum of the two numbers comes up

Then we will have the probability distribution for the discrete random variable X (X takes on value from 2 to 12) as follows:

عدد النتائج الممكنة : لما نرمي نردين (وكل نرد اله 6 وجوه)، فإن العدد الإجمالي للنتائج = $\boxed{6 \times 6 = 36}$
وهاد بيمثل مجموع جميع الاحتمالات الممكنة (n(S) = 36)

المتغير العشوائي X : يمثل مجموع العددين الظاهرين على الوجهين العلويين للنرد

القيم الممكنة لـ X :- تتراوح من 2 (عندما تكون النتيجة 1+1) الى 12 (عندما تكون النتيجة 6+6)

X (المجموع)	عدد الطرق الممكنة	$P(X)$ (الاحتمال)
2 الأقل احتمالية، لأنه عنده طريقة وحدة.	1 (1+1)	$1/36$
3	2 (1+2, 2+1)	$2/36$
4	3 (1+3, 2+2, 3+1)	$3/36$
5	4 (1+4, 2+3, 3+2, 4+1)	$4/36$
6	5 (1+5, 2+4, 3+3, 4+2, 5+1)	$5/36$
7 أعلى احتمالية لأنه عنده أكبر عدد من الطرق (6 طرق)	6 (1+6, 2+5, 3+4, 4+3, 5+2, 6+1)	6/36
8	5 (2+6, 3+5, 4+4, 5+3, 6+2)	$5/36$
9	4 (3+6, 4+5, 5+4, 6+3)	$4/36$
10	3 (4+6, 5+5, 6+4)	$3/36$
11	2 (5+6, 6+5)	$2/36$
12 الأقل احتمالية، لأنه عنده طريقة وحدة.	1 (6+6)	$1/36$

$\boxed{36/36} = 1$

Example

- Below is the probability distribution table for the random variable X whose values are the possible numbers of defective computers purchased by pharmacy in Jordan

x	0	1	2	3	4
P(X = x)	0.16	0.53	0.20	0.08	0.03

= 1

- Use this table to answer the questions that follow:

a) What is the probability a randomly selected pharmacy has exactly 2 defective computers? 0.2

b) What is the probability a randomly selected pharmacy has less than 2 defective computer?

The Mean and the Standard Deviation of a Discrete Random Variable

مثال

لنفرض انك رميت عملة مرتين، و x هو عدد الرؤوس:

القيم الممكنة لـ x: $0, 1, 2$

الاحتمالات:

- $P(x=0) = 1/4$ (فقط TT)
- $P(x=1) = 1/2$ (فقط HT او TH)
- $P(x=2) = 1/4$ (فقط HH)

حساب المتوسط المتوقع:

$$\mu = \left(0 * \frac{1}{4}\right) + \left(1 * \frac{1}{2}\right) + \left(2 * \frac{1}{4}\right)$$

$$= 0 + 0.5 + 0.5 = 1$$

Summary Measures Calculation Table

x	$p(x)$	$x\,p(x)$	$x - \mu$	$(x - \mu)^2$	$(x - \mu)^2 p(x)$
القيم الممكنة للمتغير x.	الاحتمالية لكل قيمة x.	الوزن المرجح لكل قيمة (لتحديد μ)	الفرق بين كل قيمة والمتوسط.	مربع الفرق.	الوزن المرجح للمربع (لتحديد σ^2)
Total		$\Sigma x\,p(x) = \mu$			$\Sigma(x - \mu)^2 p(x) = \sigma^2$

التباين (σ^2): يوضح مدى انتشار القيم حول المتوسط.

الانحراف المعياري (σ): يعطي مقياس أسهل لفهم هذا الانتشار.

عملياً الجدول بيساعدنا بتنظيم الحسابات أول بأول.

Thinking Challenge

You toss 2 coins. You're interested in the number of tails. What are the **expected value**, **variance**, and **standard deviation** of this random variable, number of tails?

Expected Value & Variance Solution*

x	$p(x)$	$x\,p(x)$	$x - \mu$	$(x - \mu)^2$	$(x - \mu)^2 p(x)$
0	**.25**	**0**	**–1.00**	**1.00**	**.25**
1	**.50**	**.50**	**0**	**0**	**0**
2	**.25**	**.50**	**1.00**	**1.00**	**.25**
		$\mu = 1.0$			$\sigma^2 = .50$
					$\sigma = .71$

Example

- Toss a fair coin 3 times and record *x* the number of heads.

x	*p(x)*	*xp(x)*	$(x-\mu)^2 p(x)$
0	1/8	0	$(-1.5)^2(1/8)$
1	3/8	3/8	$(-0.5)^2(3/8)$
2	3/8	6/8	$(0.5)^2(3/8)$
3	1/8	3/8	$(1.5)^2(1/8)$

$$\mu = \sum xp(x) = \frac{12}{8} = 1.5$$

$$\sigma^2 = \sum (x-\mu)^2 p(x)$$

$$\sigma^2 = .28125 + .09375 + .09375 + .28125 = .75$$

$$\sigma = \sqrt{.75} = .688$$

يعني X هو عدد المرضى الذين تم شفاؤهم.

Example

- A university medical research centre finds out that treatment of skin cancer by the use of chemotherapy has a success rate of 70%. Suppose that 5 patients are treated with chemotherapy. The probability distribution of X successful cures of the five patients is given in the table below:

x	0	1	2	3	4	5
P(X = x)	0.002	0.029	0.132	0.309	0.360	0.168

1) Find μ?

2) Find σ?

Continued

Answer:

1) $\mu = E(x) = \Sigma x$

$P(x)$=(0)(0.002)+(1)(0.029)+…+(4)(0.360)+(5)(0.168)

= 3.5 patient

2) $\sigma^2 = E[(x-\mu)^2] = \Sigma\,(x-\mu)^2\,p(x)$

$=(0-3.3)^2(0.002)+(1-3.5)^2(0.029)+\ldots+(5-3.5)^2(0.168)$

=1.05

يعني الانتشار حول 3.5 مريض تكون حوالي 1.05

$\sigma = \sqrt{\sigma^2} = \sqrt{1.5} = 1.025$

SD

Continued

Conclusion

- **The value of $\mu = 3.5$ is the centre of the probability distribution. In other words, if the five cancer patients receive chemotherapy treatment, we expect that the number of them who are cured to be near 3.5.**
- **The standard deviation, which is 1.025 in this case, measures the spread of the probability distribution, that is, on the average, the number of patients receive chemotherapy treatment who are cured will be less than or more than 3.5 by 1.025.**

Example

Suppose that the number of cars, X, that pass through a car wash between 4:00 P.M and 5:00 P.M. on any sunny Friday has the probability distribution:

x	4	5	6	7	8	9
$P(X = x)$	0.083	0.083	0.250	0.250	0.167	0.167

الأجر المتوقع

Let $Y=g(X)=2X-1$ represents the amount of money in JD paid to the attendant by the manager. Find the attendant's expected earning for this particular time period?

- Solution

The attendant can expect to receive:

2x4 − 1 (etc)

$E(Y)=E(g(X))=E(2X-1)=\sum_{i=4}^{9}(2x-1)P(X=x_i)$

$=(7)(0.083)+(9)(0.083)+(11)(0.250)+(13)(0.250)+(15)(0167)+(17)(0.167) = 12.672$ JD