لجان الدفعات

STATISTICS

MORPHINE ACADEMY

Rules Summary

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A \cup B) = P(A) + P(B)$
- $P(A^C) = 1 - P(A)$
- $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$
- $P(A \cap B) = P(B).\,P(A/B)$
- $P(A/B) = P(A)$
- $P(B/A) = P(B)$
- $P(A \cap B) = P(B).P(A)$

- Additional rules:
  - $P(A \cap \bar{B}) = P(A) - P(A \cap B)$
  - $P(\bar{A} \cap B) = P(B) - P(A \cap B)$
  - $P(A \cup \bar{B}) = P(\bar{B}) + P(A \cap B)$
  - $P(\bar{A} \cup B) = P(\bar{A}) + P(A \cap B)$
  - De Morgan's Laws
  - $P(\overline{A \cup B}) = P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$
  - $P((\overline{A \cap B}) = P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B)$

Examples

For the random experiment of rolling two fair dice, suppose that we define the two events A and B as follows:

A: First die show 1

B: Second die show 1

Then find the value of $P(A \cup B)$?

<u>Solution</u>

The outcomes of the two events are as follows:

$$A = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)\} \Rightarrow P(A) = 6/36$$

$$B = \{(1,1), (2,1), (3,1), (4,1), (5,1), (6,1)\} \Rightarrow P(B) = 6/36$$

$$A \cap B = \{(1,1)\} \Rightarrow P(A \cap B) = 1/36$$

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 6/36 + 6/36 - 1/36 \\ &= 11/36 \\ &= 0.306 \end{aligned}$$

Example-Disjoint Events

- For random experiment of rolling two fair dice, suppose that we define the two events A and B as follows:

A: Sum of the two numbers comes up to 3

B: Sum of the two numbers comes up to 6

Then find $P(A \cup B)$?

$A=\{ (1,2), (2,1)\}$ $P(A)= 2/36$

$B=\{(1,5), (2,4), (3,3), (4,2),(5,1)\}$ $P(B)=5/36$

$P(A \cap B)=\phi$

So $P(A \cup B)= (2/36) + (5/36) = (7/36) = 0.194$

Examples

- Suppose that in the Gulf pharmaceutical industries company in UAE, there are two telephone lines 1 and II. Let A be the event that line I is busy and let B be the event that line II is busy.

$P(A)=0.55$ $P(B)=0.65$ and $P(A \cap B)=0.35$

Answer the following:

a) Find the probability that both lines are free?

Examples

Solution

P(Both lines are free) = $P(\bar{A} \cap \bar{B})$= $P(\overline{A \cup B})$

$=1 - P(A \cup B)$

$=1 - (P(A)+P(B) - P(A \cap B)$

$= 1 - (0.55+0.65 - 0.35)$

$= 1 - 0.85$

$= 0.15$

Continued

b) Find the probability that line I is busy and line II is free?

Solution

P(Line I is busy and line II is free)=$P(A\cap \overline{B})$

$=P(A\cap \overline{B})=P(A)-P(A\cap B)$

$=0.55 - 0.35$

$=0.20$

Continued

c) Find the probability that line I is free or line II is busy?

Solution

P(line I is free or line II is busy)=P($\bar{A}$ ∪ B)

$= P(\bar{A}) + P(A \cap B)$

$=1-P(A)+ P(A \cap B)$

$=1-(0.55+0.35)$

$= 0.8$

Additional Rules

- $P(A/\bar{B}) = \frac{P(A\cap\bar{B})}{P(\bar{B})} = \frac{P(A)-P(A\cap B)}{1-P(B)}$

$P(\bar{A}/B) = \frac{P(\bar{A}\cap B)}{P(B)} = \frac{P(B)-P(P\cap B)}{P(B)} = 1 - P(A/B)$

$P(\bar{A}/\bar{B}) = \frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})} = \frac{P(\overline{A\cup B})}{1-P(B)} = \frac{1-P(A\cup B)}{1-P(B)} = \frac{1-[P(A)+P(B)-P(A\cap B)}{1-P(B)}$

Example

For the random experiment of rolling two fair dice, suppose that we define the two events A and B as follows:

A: First die shows 1

B: Second die show 2

Then find the value of P(A/B)

Continued

Solution

- P(A/B) = $\frac{P(A \cap B)}{P(B)}$
- A={(1,1), (1,2), (1,3), (1,4),(1,5), (1,6)}
  B={(1,2), (2,2), (3,2), (4,2), (5,2), (6,2)}
  A∩B = {(1,2)} = P(A∩B) = 1/36

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/36}{6/36}$$

$$= \frac{1}{6}$$

$$= 0.167$$

Example

In the Queen Alia International Airport, suppose that the probability that a regularly scheduled flight departs on time is P(A) = 0.83 ; the probability that a regularly scheduled flight arrives on time is P(B) = 0.92 ; and the probability that it departs and arrives on time is P(A ∩ B) = 0.78. Find the probability that a plane

a) Arrives on time given that it departed on time, and

b) Departed on time given that it has arrived on time?

Solution

a) $P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{0.78}{0.83} = 0.94$

b) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.78}{0.92} = 0.85$

Example

Suppose that we have a drug box containing 20 tablets, of which 5 are defective, if 2 tablets are selected at random and removed from the box in succession without replacing the first, what is the probability that both tablets are defective?

Solution

We shall let:

A: the event that the first tablet is defective

B: the event that the second tablet is defective

$A \cap B$: the event that A occurs, and then B occurs after A has occurred (both events occur)

Continued

- The probability of first removing a defective tablet is 5/20 = ¼, then the probability of removing a second defective tablet from the remaining 4 is 4/19. hence:
- $P(A \cap B) = P(B).\ P(A/B)$

$$= (1/4).\ (4/19)$$

$$= 1/19 = 0.053$$

Additional Rules

Definition of Independence

Two events A and B are said to be (statistically) independent if any one of the following equivalent conditions holds:

1) $P(A \cap B) = P(A)P(B)$
2) $P(A/B) = P(A)$
3) $P(B/A) = P(B)$

Otherwise, they are said to be dependent events

Then,

1) A and not B are independent, that is: $P(A \cap \bar{B}) = P(A)P(\bar{B})$
2) Not A and B are independent, that is: $P(\bar{A} \cap B) = P(\bar{A})P(B)$
3) Not A and not B are independent, that is: $P(\bar{A} \cap \bar{B}) = P(\bar{A})P(\bar{B})$

Example

Two dice are rolled. Suppose that we define the two events A and B as follows:

A: First die show 1.

B: The sum of the two numbers comes up on the two dice is 7.

Are the two events A and B independent?

Solution

$A = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)\}$

$B = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

$A \cap B = \{(1,6)\}$

(1) $P(A) = (\frac{6}{36})$ *and* $P(B) = (\frac{6}{36})$

then $P(A)P(B) = \frac{1}{36}$

(2) $P(A \cap B) = \frac{1}{36}$

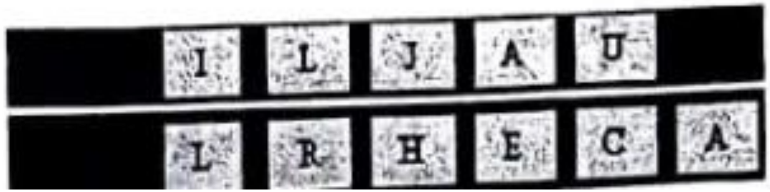
Then we get that $P(A \cap B) = \frac{1}{36} = P(A)P(B)$

Example

Two sets of cards with a letter on each card as follows are placed into separate bags :

Bag Number I

Bag Number II



Sara randomly picked one card from each bag. Find the probability that:

a) She picked the letter J and R?

b) Both letters are L?

c) Both letters are vowels?

Solution

(a) Probability that she picked J and R
$= (1/5)(1/6) = 1/30 = 0.033$

(b) Probability that both letters are L
$= (1/5)(1/6) = 1/30 = 0.033$

(c) Probability that both letters are vowels
$= (3/5)(2/6) = 6/30 = 0.2$

Contingency Table

A contingency table provides a different way of calculating probabilities. The table helps in determining conditional probabilities quite easily. The table displays sample values in relation to two different variables that may be dependent or contingent on one another. The two variables are divided into several categories with their frequencies.

Example

- A random sample of size 200 adults are classified below according to the sex and the level of education attained:

Education	Male (M)	Female (F)	Total
Elementary (E)	38	45	83
Secondary (S)	28	50	78
Higher (H)	22	17	39
Total	88	112	200

Continued

Suppose that a person is selected at random from this group, then find the probability that:

(a) the person is male?

Solution

P(the person is male) = P(M) = 88 / 200 = 0.44

(b) the person has a secondary education?

Solution

P(the person has a secondary education)

= P(S) = 78 / 200 = 0.39

(c) the person is not a male?

Solution

P(the person is not a male) = 1- P(the person is male)

= 1 - P(M) = 1 – 0.44 = 0.56

OR

P(the person is not a male) = P(the person is female)

= P(F) = 112 / 200 =

Continued

d) The person is a male or has higher education?

Solution:

P(the person is a male or has higher education)

$P(M \cup H)=P(M)+P(H)-P(M \cap H)=(88/200)+(39/200)-(22/200)=105/200=0.525$

e) The person is female and not has an elementary education

Solution

P(the person is a female and not has an elementary education)

$P(F \cap \overline{E}) = P(F)-P(F \cap E)= (112/200)-(45/200)=67/200=0.335$

f) The person is a male or has a secondary education?

P(the person is not a female or has a secondary education)

$P(\overline{F} \cup S)=P(\overline{F})+P(F \cap S)=(88/200)+(50/200)=(138/200)=0.69$

Continued

g) The person is neither a male nor has a higher education?

solution#

P(the person is neither a male nor has a higher education)

$=P(\bar{M} \cap \bar{H})$

$=(P(\overline{M \cup H})$

$=1-P(M\cup H)$

$=1-[P(M)+P(H)-P(M\cap H)$

$=1-[88/200+39/200-22/200]=1-(105/200)= 0.475$

h) The person is a male given that the person has a secondary education?

Solution

P(the person is a male given that the person has a secondary education)

$P(M/S) =P(M\cap S)/P(S)=(28/200)/(78/200)=28/78=0.359$

Continued

i) The person does not have a higher education degree given that the person is a female?

Solution

P(the person does not have a higher education degree given that the person is a female)

$$P(\overline{H}/F) = \frac{P(\bar{H} \cap F)}{P(F)} = \frac{P(F) - P(H \cap F)}{P(F)} = 1 - P(H/F) =$$

$$= 1 - \frac{P(H \cap F)}{P(F)} = 1 - ((17/200)/(112/200)) = 1 - (17/112) = 0.848$$