لجان الدُفعات

STATISTICS

MORPHINE ACADEMY

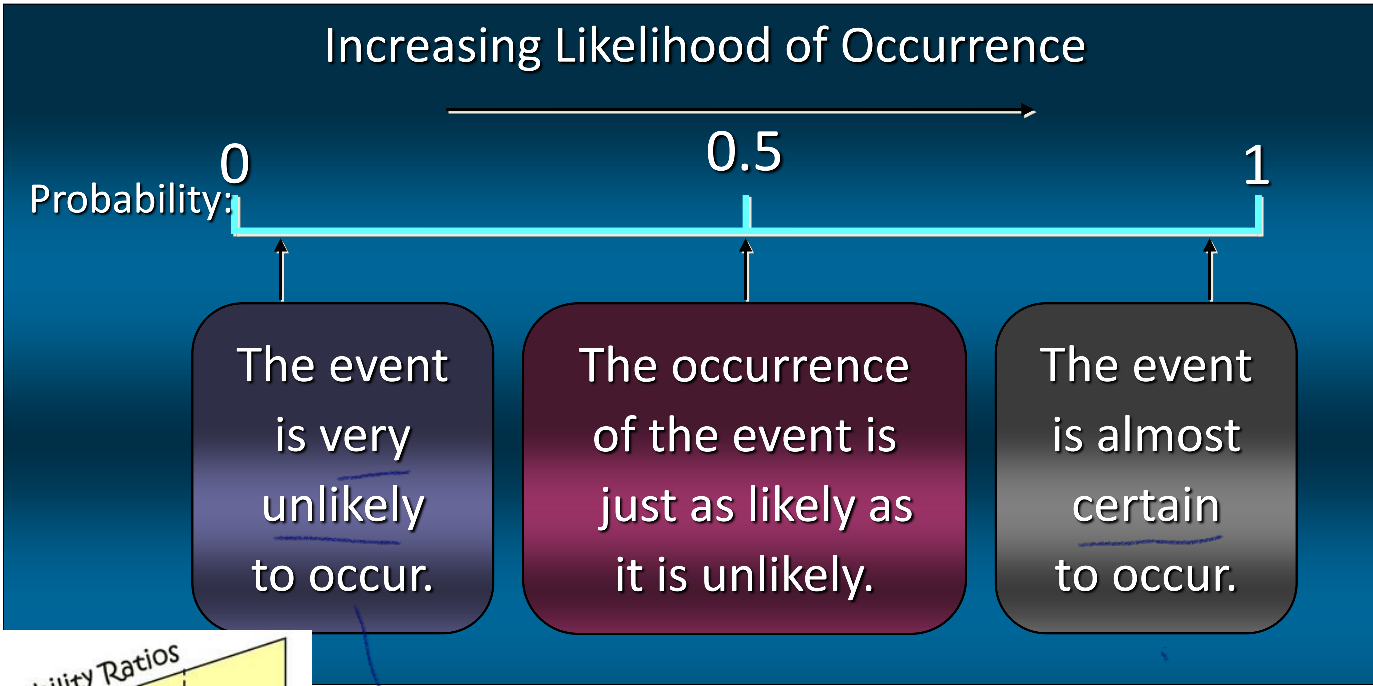
The Probability of an Event

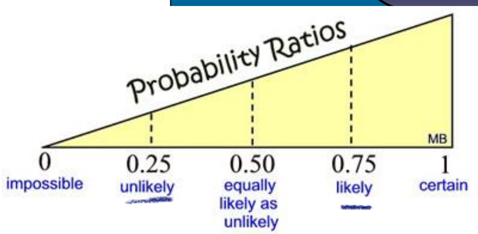
- P(A) must be between 0 and 1.
  - If event A can never occur, P(A) = 0. If event A always occurs when the experiment is performed, P(A) =1.
- The sum of the probabilities for all simple events in S equals 1.

$\Sigma P(A) = 1$

•The **probability of an event A** is found by adding the probabilities of all the simple events contained in A.

Probability as a Numerical Measure of the Likelihood of Occurrence





Impossible

The Probability of an Event

- The probability of an event A measures “how often” we think A will occur. We write **P(A).**
- Suppose that an experiment is performed n times. The relative frequency for an event A is

$$\frac{\text{Number of times A occurs}}{n} = \frac{f}{\underset{\text{الكل}}{n}} = \frac{\text{عدد التكرارات}}{\text{مجموع التكرارات}}$$

Let A be the event A = {o_1, o_2, …, o_k}, where o_1, o_2, …, o_k are k different outcomes. Then

$$P(A) = P(o_1) + P(o_2) + \cdots + P(o_k)$$

يحتوي الحدث على أكثر من نتيجة نجمع احتمالاتهم

Finding Probabilities

- Probabilities can be found using
  – Estimates from empirical studies
  – Common sense estimates based on equally likely events.

عندما نريد حساب احتمال أما أن تقدره من
البيانات التجريبية ك relative frequency أو من
الاحتمالات المتساوية لعناصر التجربة عبر جمعهم

- **Examples:**
  – Toss a fair coin. P(Head) = 1/2
  – 10% of the U.S. population has red hair. Select a person at random. P(Red hair) = .10

Example

- Toss two coins. What is the probability of observing at least one head?

n

$2^n = 2^2 = 4$

1st Coin	2nd Coin	E_i	$P(E_i)$
H	H	HH	1/4
	T	HT	1/4
T	H	TH	1/4
	T	TT	1/4

P(at least 1 head)

$= P(E_1) + P(E_2) + P(E_3)$

$= 1/4 + 1/4 + 1/4 = 3/4$

$P(A) = \dfrac{\text{عدد تكرار التجربة}}{\text{العدد الكلي في الفضاء العيني}}$

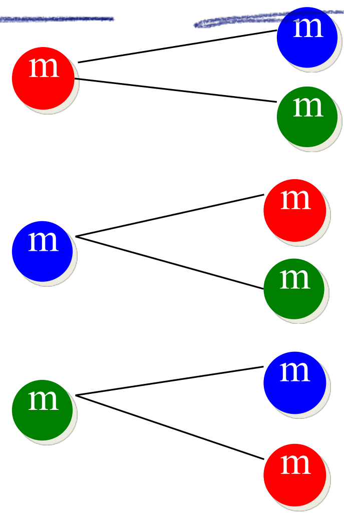
Example

- A bowl contains three M&Ms®, one red, one blue and one green. A child selects two M&Ms at random. What is the probability that at least one is red?

١.احدد الفضاء العيني

٢.احدد احتمال كل قطعة

٣.اجمع الاحتمالات لكل قطعة red



1st M&M	2nd M&M	E_i	$P(E_i)$
Red	Blue	RB	1/6
Red	Green	RG	1/6
Blue	Red	BR	1/6
Blue	Green	BG	1/6
Green	Blue	GB	1/6
Green	Red	GR	1/6

P(at least 1 red)

= P(RB) + P(BR)+ P(RG) + P(GR)

= 4/6 = 2/3

or: $P(A) = \dfrac{\text{عدد مرات تكرار التجربة}}{\text{عدد عناصر الفضاء العيني}}$

Counting Rules

- If the simple events in an experiment are **equally likely**, you can calculate

$$P(A) = \frac{n_A}{N} = \frac{\text{number of simple events in A}}{\text{total number of simple events}}$$

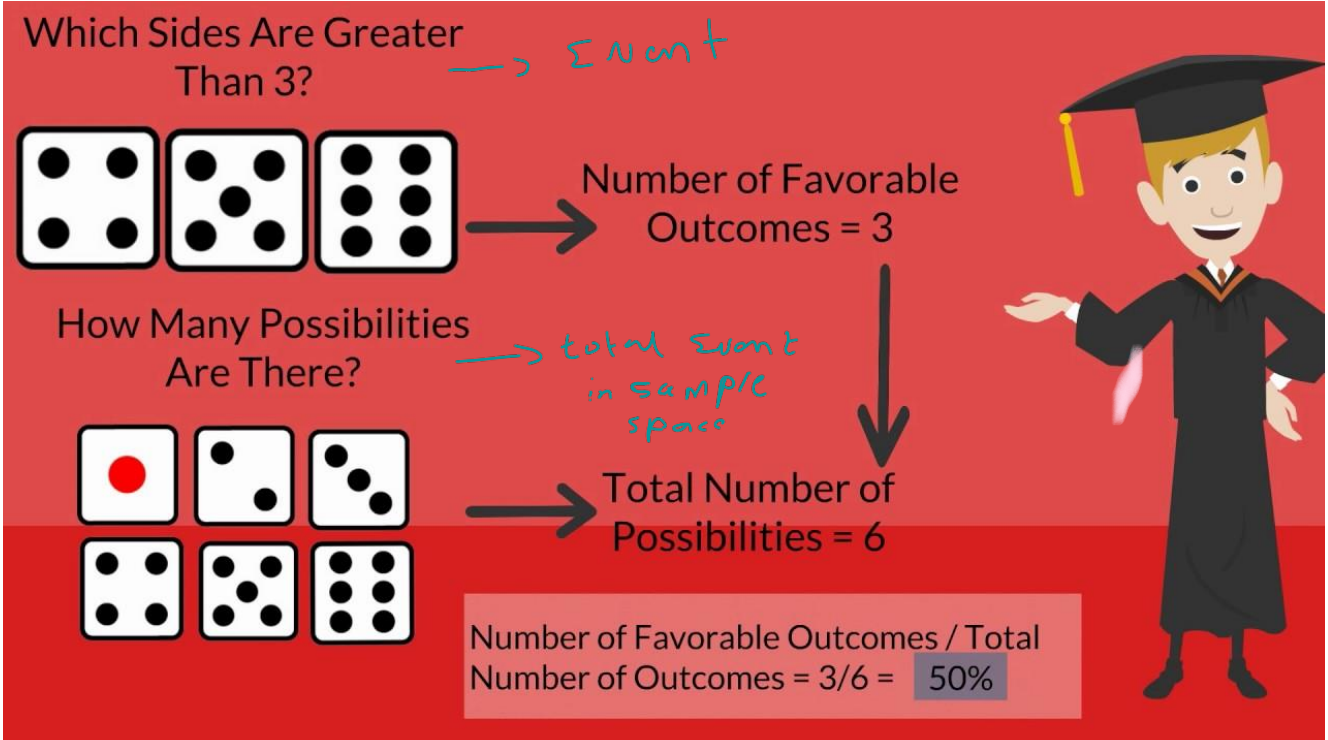
- You can use **counting rules** to find n_A and N.

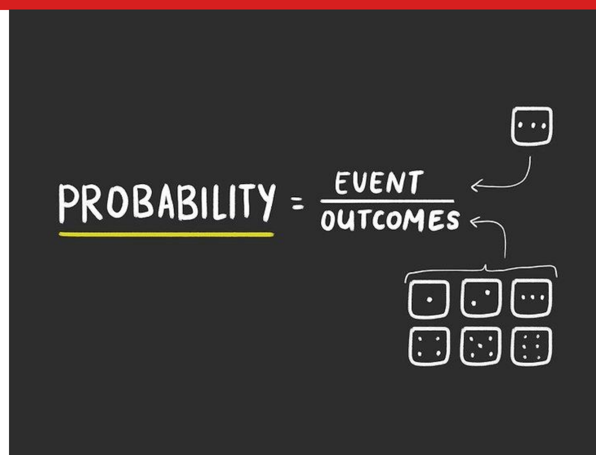
A Counting Rule for Multiple-Step Experiments

If an experiment consists of a sequence of k steps in which there are n_1 possible results for the first step, n_2 possible results for the second step, and so on, then the total number of experimental outcomes is given by $(n_1)(n_2) \ldots (n_k)$.

A helpful graphical representation of a multiple-step experiment is a <u>tree diagram</u>.

في مخططات نستعملها لفهم الاحتمالات


Which Sides Are Greater Than 3?
Event
Number of Favorable Outcomes = 3
How Many Possibilities Are There?
total Event in sample space
Total Number of Possibilities = 6
Number of Favorable Outcomes / Total Number of Outcomes = 3/6 = 50%


PROBABILITY = EVENT / OUTCOMES

Counting Rule for Permutations

A second useful counting rule enables us to count the number of experimental outcomes when n objects are to be selected from a set of N objects, where the order of selection is important.

- Number of Permutations of N Objects Taken n at a Time

$$P_n^N = n!\binom{N}{n} = \frac{N!}{(N-n)!}$$

where: $N! = N(N-1)(N-2)\ldots(2)(1)$

$n! = n(n-1)(n-2)\ldots(2)(1)$

$0! = 1$

Permutations

- The number of ways you can arrange ***n*** distinct objects, taking them ***r*** at a time is

$$P_r^n = \frac{n!}{(n-r)!}$$

where $n! = n(n-1)(n-2)...(2)(1)$ and $0! \equiv 1$.

Example: How many 3-digit lock combinations can we make from the numbers 1, 2, 3, and 4?

The order of the choice is important!

$$P_3^4 = \frac{4!}{1!} = 4(3)(2) = 24$$

Examples

Example: A lock consists of five parts and can be assembled in any order. A quality control engineer wants to test each order for efficiency of assembly. How many orders are there?

The order of the choice is important!

$$P_5^5 = \frac{5!}{0!} = 5(4)(3)(2)(1) = 120$$

Counting Rule for Combinations

A third useful counting rule enables us to count the number of experimental outcomes when n objects are to be selected from a set of N objects.

Number of Combinations of N Objects Taken n at a Time

$$C_n^N = \binom{N}{n} = \frac{N!}{n!(N-n)!}$$

where: $N! = N(N-1)(N-2)\ldots(2)(1)$

$n! = n(n-1)(n-2)\ldots(2)(1)$

$0! = 1$

Combinations

- The number of distinct combinations of ***n*** distinct objects that can be formed, taking them ***r*** at a time is

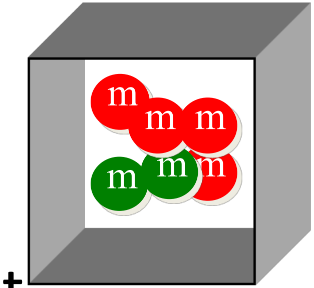
$$C_r^n = \frac{n!}{r!(n-r)!}$$

Example: Three members of a 5-person committee must be chosen to form a subcommittee. How many different subcommittees could be formed?

The order of the choice is not important!

$$C_3^5 = \frac{5!}{3!(5-3)!} = \frac{5(4)(3)(2)1}{3(2)(1)(2)1} = \frac{5(4)}{(2)1} = 10$$

Example



- A box contains six M&Ms®, four red and two green. A child selects two M&Ms at random. What is the probability that exactly one is red?

The order of the choice is not important!

$$C^6_2 = \frac{6!}{2!4!} = \frac{6(5)}{2(1)} = 15$$

ways to choose 2 M & Ms.

$$C^2_1 = \frac{2!}{1!1!} = 2$$

ways to choose 1 green M & M.

$$C^4_1 = \frac{4!}{1!3!} = 4$$

ways to choose 1 red M & M.

$4 \times 2 = 8$ ways to choose 1 red and 1 green M&M.

P(exactly one red) = 8/15

Calculating Probabilities for Unions and Complements

- There are special rules that will allow you to calculate probabilities for composite events.

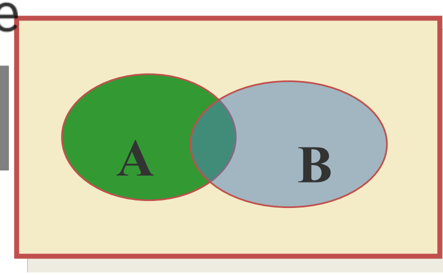
حوادث مركبة

- **The Additive Rule for Unions:**
- For any two events, **A** and **B**, the probability of their union, **P(A ∪ B)**, is

اول قانون للاتحاد بين حدثين=أذا كانوا الاحتماليين

$P(A \cap B) = 0$ سيكون mutually exclusive

1 $$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$



A→B Mutualy exclusive

$P(A \cup B) = P(A) + P(B)$

Addition Law

- The addition law provides a way to compute the probability of event *A*, or *B*, or both *A* and *B* occurring.
- The law is written as:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

منفصل

Mutually Exclusive (Disjoint) Events

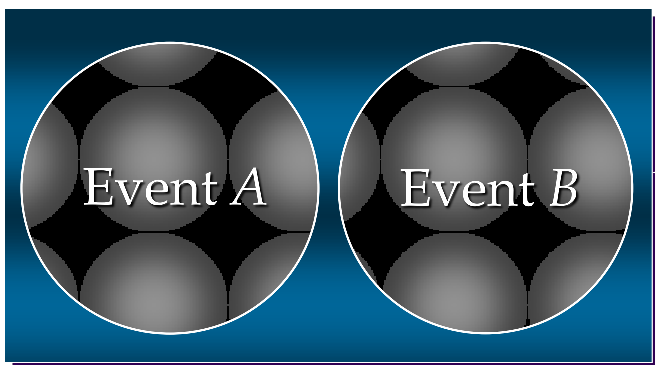
Two events are said to be mutually exclusive if the events have no sample points (outcomes) in common.

Two events are mutually exclusive if, when one event occurs, the other cannot occur (They can't occur at the same time. The outcome of the random experiment cannot belong to both A and B

If events A and B are mutually exclusive, $P(A \cap B) = 0$.

Mutualy exclusive event

حدثان لا يمكن اجتماعهما



Sample Space S

لا يوجد تقاطع

Mutually Exclusive Events

- If events A and B are mutually exclusive, $P(A \cap B) = 0$.
- The addition law for mutually exclusive events is:

$$P(A \cup B) = P(A) + P(B)$$

there's no need to include "$-\ P(A \cap B)$"

Mutually Exclusive (Disjoint) Events

- **Two events are mutually exclusive if, when one event occurs, the other cannot, and vice versa.**

- Experiment: Toss a die
  - A: observe an odd number
  - B: observe a number greater than 2
  - C: observe a 6
  - D: observe a 3

Not Mutually Exclusive

Mutually Exclusive

B and C?
B and D?

$P(C \cap B) = P(D \cap B) = \frac{1}{6}$

$P(C \cap D) = 0$ Mutualy exclusive

Example: Additive Rule

$S = 120$

Example: Suppose that there were 120 students in the classroom, and that they could be classified as follows:

A: brown hair

$P(A) = 50/120$ < male / female

B: female

$P(B) = 60/120$

	Brown	Not Brown
Male	20	40
Female	30	30

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 50/120 + 60/120 - 30/120$$
$$= 80/120 = 2/3$$

احسب احتمال انه يكون
انثى و شعرها بني
A B

A Special Case

حالة خاصة

When two events A and B are **mutually exclusive,** $P(A \cap B) = 0$ and $P(A \cup B) = P(A) + P(B)$.

A: male with brown hair

$P(A) = 20/120$

B: female with brown hair

$P(B) = 30/120$

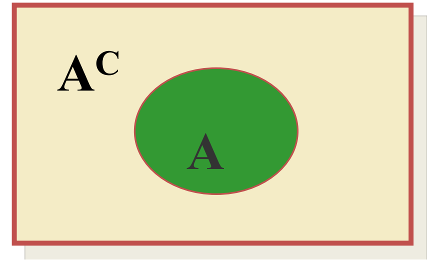
	Brown	Not Brown
Male	20	40
Female	30	30

A and B are mutually exclusive, so that

$P(A \cup B) = P(A) + P(B)$
$= 20/120 + 30/120$
$= 50/120$

$(A \cap B) = 0$

Calculating Probabilities for Complements



- We know that for any event **A:**

  $P(A \cap A^C) = 0$

- Since either **A** or **A^C** must occur,

  $P(A \cup A^C) = 1$

- so that **$P(A \cup A^C) = P(A) + P(A^C) = 1$**

$$P(A \cup A^c) = P(A) + P(A^c) - \underset{\text{zero}}{\cancel{P(A \cap A^c)}}$$

$$= 1$$

$$P(A) + P(A^c) = 1$$

نحسب القيمة المجهولة

للإحتمال

$$\boxed{P(A^C) = 1 - P(A)}$$

مجموع الإحتمالات

Example

Select a student at random from the classroom. Define:

A: male

P(A) = 60/120

B: female

	Brown	Not Brown
Male	20	40
Female	30	30

A and B are complementary, so that

P(B) = 1- P(A)
= 1- 60/120 = 40/120

Calculating Probabilities for Intersections

- we can find P(A ∩ B) directly from the table. Sometimes this is impractical or impossible. The rule for calculating P(A ∩ B) depends on the idea of **independent and dependent events.**

يمكن حساب التقاطع بين الحوادث مباشرة لكن هذا الشيء غير عملي ومستحيل بالغالب .. فالقانون يعتمد على استقلالية الحادث من عدمه

Two events, **A** and **B**, are said to be **independent** if and only if the probability that event **A** occurs does not change, depending on whether or not event **B** has occurred.

التقاطع يعتمد على نوع الحادث هل هو independent event أو dependent

Conditional Probabilities

يعني الحادث B وقع

المطلوب في السؤال

- The probability that A occurs, given that event B has occurred is called the **conditional probability** of A given B and is defined as

تقاطع

بعد السلاش (|) يكون في المقام دائماً

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \text{ if } P(B) \neq 0$$

"given"

اي أنه حدث

لما اقول حادثين مستقلين independent event ...فهذا يعني حدوث الاول لا يؤثر بقيمة الاحتمال للحادث الثاني مثل الحمل
ولما اقول dependent event يعني حدوث الحدث يؤثر على قيمة احتمال الحادث الثاني مثل سحب الكرات بشكل عشوائي دون إرجاع

Conditional Probability

- The probability of an event given that another event has occurred is called a <u>conditional probability</u>.
- The conditional probability of <u>A given B</u> is denoted by $P(A \mid B)$.
- A conditional probability is computed as follows :

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Multiplication Law

- The multiplication law provides a way to compute the probability of the intersection of two events.
- The law is written as:

$$P(A \cap B) = P(B)P(A \mid B)$$

ضرب تبادلي لإيجاد التقاطع
لأي حدثين

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Example 1

القاء العملة مرتين

- Toss a fair coin twice. Define
  - A: head on second toss
  - B: head on first toss

$P(A|B) = \frac{1}{2}$

$P(A|\text{not } B) = \frac{1}{2}$

Independ event

HH 1/4

HT 1/4

TH 1/4

TT 1/4

P(A) does not change, whether B happens or not…

A and B are independent!

حادثين مستقلين

Independent Events

- If the probability of event A is not changed by the existence of event B, we would say that events A and B are independent.

- Two events A and B are independent if:

  $$P(A \mid B) = P(A) \quad \text{or} \quad P(B \mid A) = P(B)$$

P(A) does change, depending on whether B happens or not…

A and B are dependent!

Defining Independence

- We can redefine independence in terms of conditional probabilities:

Two events A and B are **independent** if and only if

$$P(A|B) = P(A) \quad \text{or} \quad P(B|A) = P(B)$$

Otherwise, they are **dependent**.

- Once you've decided whether or not two events are independent, you can use the following rule to calculate their intersection.

Multiplication Law for Independent Events

- The multiplication law also can be used as a test to see if two events are independent.
- The law is written as:

$$P(A \cap B) = P(A)P(B)$$

The Multiplicative Rule for Intersections

- For any two events, **A** and **B**, the probability that both **A** and **B** occur is

$$\boxed{\begin{aligned} P(A \cap B) &= P(A)\,P(B \text{ given that } A \text{ occurred}) \\ &= P(A)P(B|A) \end{aligned}}$$

- If the events **A** and **B** are independent, then the probability that both **A** and **B** occur is

$$\boxed{P(A \cap B) = P(A)\,P(B)}$$

Example 1

In a certain population, 10% of the people can be classified as being high risk for a heart attack. Three people are randomly selected from this population. What is the probability that exactly one of the three are high risk?

Define H: high risk N: not high risk

P(exactly one high risk) = P(HNN) + P(NHN) + P(NNH)

= P(H)P(N)P(N) + P(N)P(H)P(N) + P(N)P(N)P(H)

$= (.1)(.9)(.9) + (.9)(.1)(.9) + (.9)(.9)(.1) = 3(.1)(.9)^2 = .243$

Example 2

Suppose we have additional information in the previous example. We know that only 49% of the population are female. Also, of the female patients, 8% are high risk. A single person is selected at random. What is the probability that it is a high risk female?

Define H: high risk F: female

From the example, $P(F) = .49$ and $P(H|F) = .08$.

Use the Multiplicative Rule:

وانها ايضا انثى

female

$$P(\text{high risk female}) = P(H \cap F)$$

$$= P(F)P(H|F) = .49(.08) = .0392$$