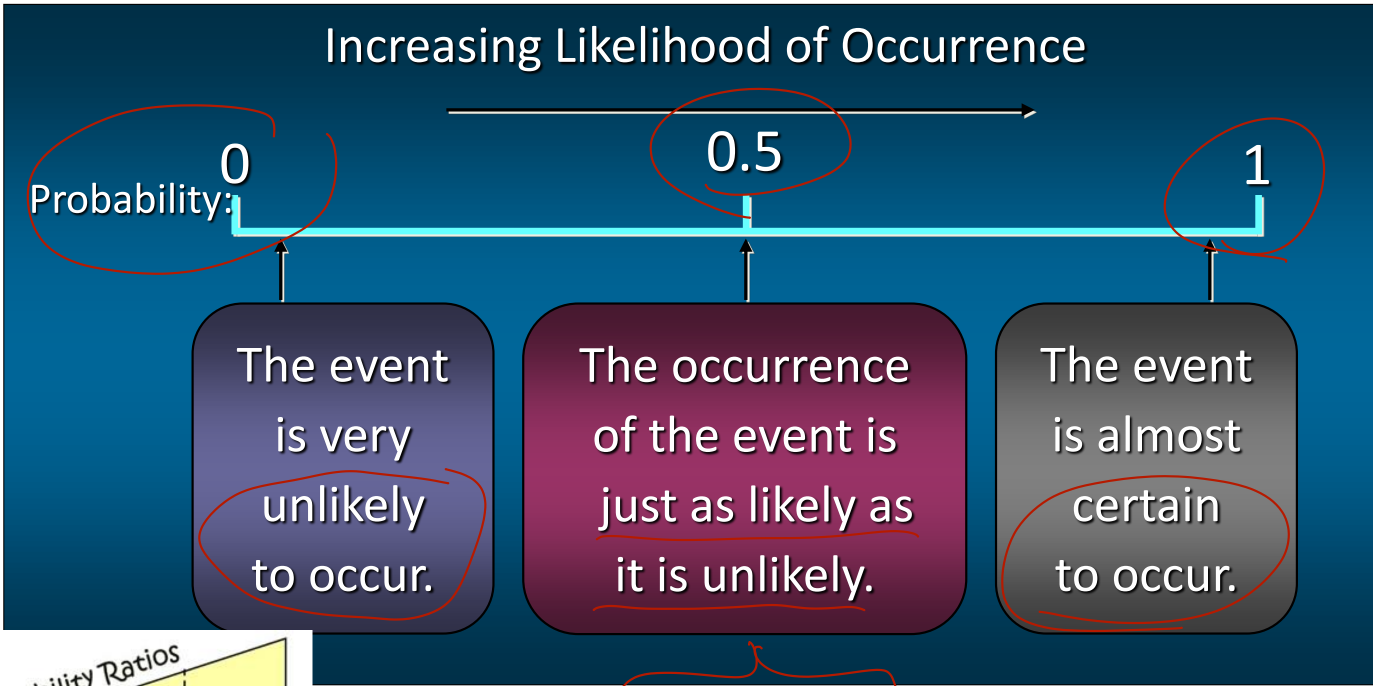
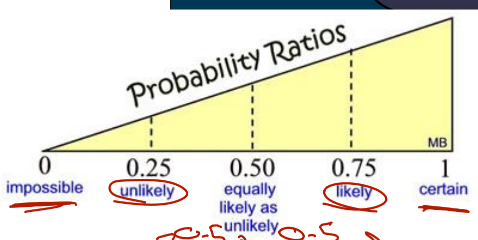
لجان الدفعات

STATISTICS

MORPHINE ACADEMY

Probability as a Numerical Measure of the Likelihood of Occurrence





$\frac{1}{2}$ فما تحت 0
قريب من الصفر.

$\frac{1}{2}$ فما فوق 1
قريب من الواحد.

The Probability of an Event

ع بيقيس احتمال الحدث A مدى "توقعنا" لحدوثه ⇐ وبنكتب P(A)

- The probability of an event A measures "how often" we think A will occur. We write **P(A).**
- Suppose that an experiment is performed n times. The relative frequency for an event A is

$$\frac{\text{Number of times A occurs}}{n} = \frac{f}{n}$$

ع الحساب بناءً على التكرار:
إذا أجريت التجربة n مرة، فإن
الاحتمال النسبي للحدث A هو:

Let A be the event A = {$o_1, o_2, \ldots, o_k$}, where $o_1, o_2, \ldots, o_k$ are k different outcomes. Then

$$P(A) = P(o_1) + P(o_2) + \cdots + P(o_k)$$

Finding Probabilities

- Probabilities can be found using

تقديرات من دراسات تجريبية.

  – Estimates from empirical studies
  – Common sense estimates based on equally likely events.

تقديرات عقلانية بناءً على احداث متساوية الاحتمال.

- **Examples:**
  – Toss a fair coin. P(Head) = 1/2
  – 10% of the U.S. population has red hair. Select a person at random. P(Red hair) = .10

Example

- Toss two coins. What is the probability of observing at least one head?

1st Coin	2nd Coin	E_i	$P(E_i)$
H	H	HH	1/4
	T	HT	1/4
T	H	TH	1/4
	T	TT	1/4

P(at least 1 head)

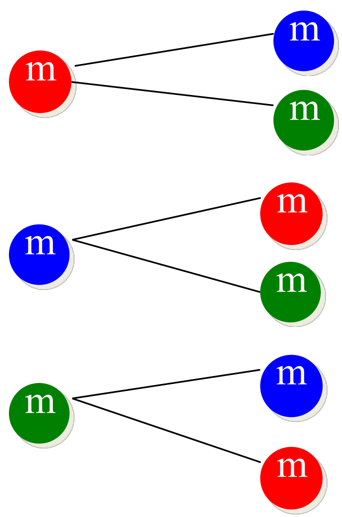
$= P(E_1) + P(E_2) + P(E_3)$

$= 1/4 + 1/4 + 1/4 = 3/4$

لأنه لا علاقة الها وجهين

Example

- A bowl contains three M&Ms®, one red, one blue and one green. A child selects two M&Ms at random. What is the probability that at least one is red?



1st M&M	2nd M&M	E_i	$P(E_i)$
		RB	1/6
		RG	1/6
		BR	1/6
		BG	1/6
		GB	1/6
		GR	1/6

P(at least 1 red)

= P(RB) + P(BR)+ P(RG) + P(GR)

= 4/6 = 2/3

لـ كل زوج الاحتمال 1/6 (لأن هناك في 3 الوان ويمكن اختيار 2 بترتيب)

العد

Counting Rules

اذا كانت الاحداث البسيطة في التجربة متساوية الاحتمال امكن حساب احتمال حدوث A بالعد

- If the simple events in an experiment are **equally likely,** you can calculate

$$\boxed{P(A) = \frac{n_A}{N} = \frac{\text{number of simple events in A}}{\text{total number of simple events}}}$$

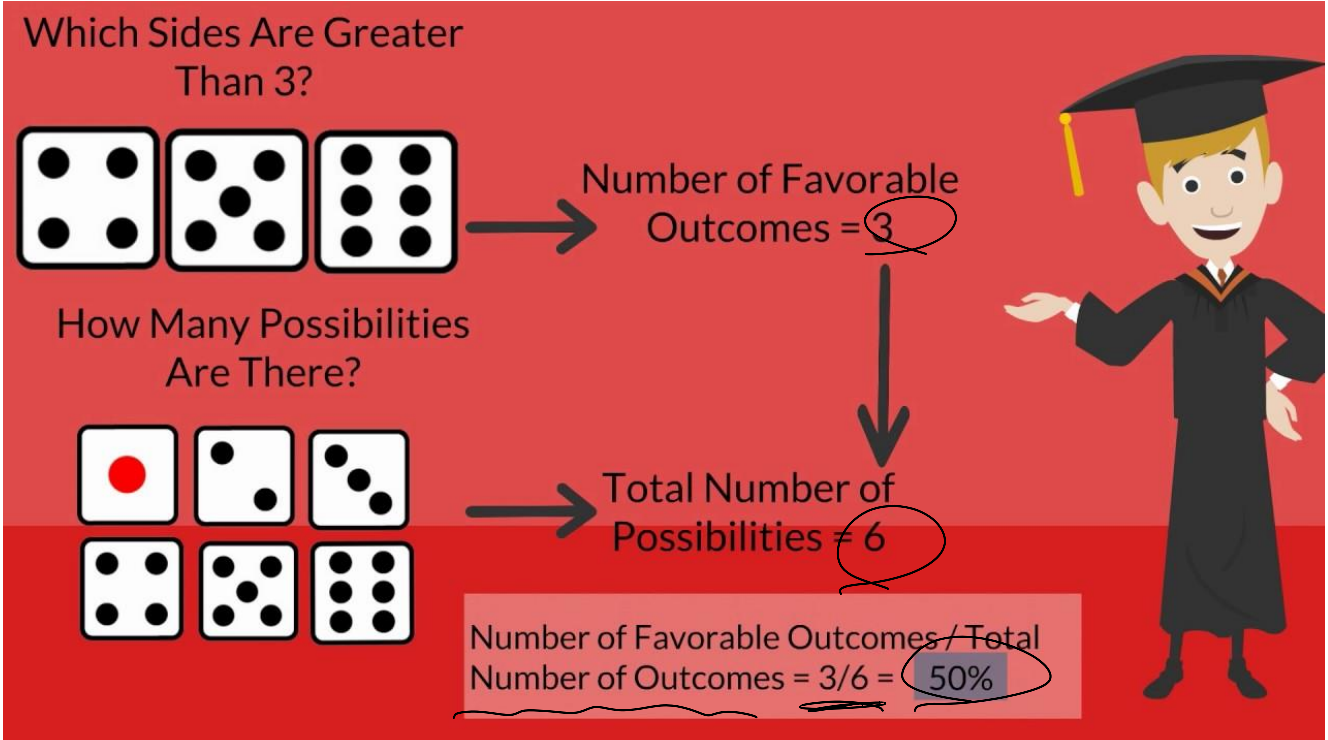
- You can use **counting rules** to find n_A and N.

A Counting Rule for Multiple-Step Experiments

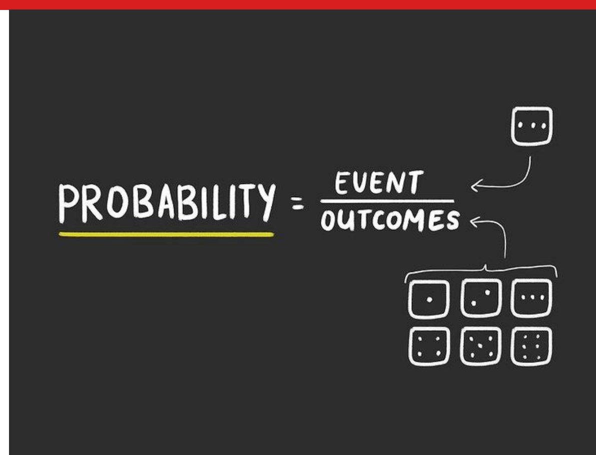
If an experiment consists of a sequence of k steps in which there are n_1 possible results for the first step, n_2 possible results for the second step, and so on, then the total number of experimental outcomes is given by $(n_1)(n_2) \dots (n_k)$.

A helpful graphical representation of a multiple-step experiment is a tree diagram.

يعني اذا كانت التجربة تتكون من عدة خطوات، فإن اجمالي العدد للنتائج يعتمد على عدد الاحتمالات في كل خطوة.



Which Sides Are Greater Than 3?
Number of Favorable Outcomes = 3
How Many Possibilities Are There?
Total Number of Possibilities = 6
Number of Favorable Outcomes / Total Number of Outcomes = 3/6 = 50%



PROBABILITY = EVENT / OUTCOMES

للترتيبات.

Counting Rule for Permutations

بنستخدم هاي القاعدة لما يكون ترتيب الاختيار مهم.

A second useful counting rule enables us to count the number of experimental outcomes when n objects are to be selected from a set of N objects, where the order of selection is important.

- Number of Permutations of N Objects Taken n at a Time

$$P_n^N = n!\binom{N}{n} = \frac{N!}{(N-n)!}$$

where: $N! = N(N-1)(N-2)\ldots(2)(1)$

$n! = n(n-1)(n-2)\ldots(2)(1)$

$0! = 1$

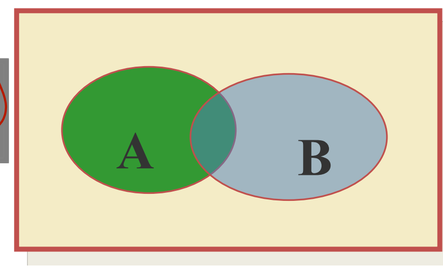
الهدف: هناك قواعد خاصة ستأخذ في حساب احتمالات الأحداث المركبة.

Calculating Probabilities for Unions and Complements

- There are special rules that will allow you to calculate probabilities for composite events.
- **<u>The Additive Rule for Unions:</u>**
- For any two events, **A** and **B**, the probability of their union, **P(A ∪ B)**, is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

احتمال الاتحاد



Addition Law

- The addition law provides a way to compute the probability of event *A*, or *B*, or both *A* and *B* occurring.
- The law is written as:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

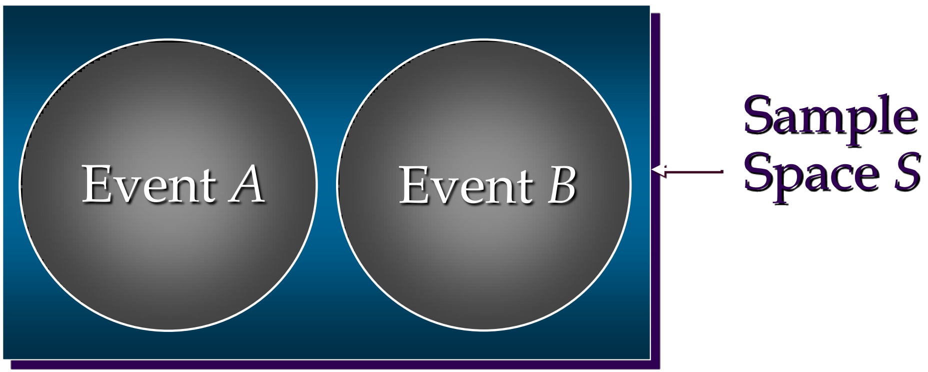
Mutually Exclusive (Disjoint) Events

مرينا متنافيه اذا لم يشتركا في اي نتيجة مشتركة (مستحيل يصيروا)

Two events are said to be <u>mutually exclusive</u> if the events have no sample points (outcomes) in common.

Two events are mutually exclusive if, when one event occurs, the other cannot occur (They can't occur at the same time. The outcome of the random experiment cannot belong to both A and B

If events A and B are mutually exclusive, $P(A \cap B) = 0$.



Mutually Exclusive Events

- If events A and B are mutually exclusive, $P(A \cap B) = 0$.
- The addition law for mutually exclusive events is:

$$P(A \cup B) = P(A) + P(B)$$

there's no need to include "$- P(A \cap B)$"

Mutually Exclusive (Disjoint) Events

- **Two events are mutually exclusive if, when one event occurs, the other cannot, and vice versa.**

- Experiment: Toss a die
  - A: observe an odd number
  - B: observe a number greater than 2
  - C: observe a 6
  - D: observe a 3

Not Mutually Exclusive

Mutually Exclusive

B and C?
B and D?

Example: Additive Rule

Example: Suppose that there were 120 students in the classroom, and that they could be classified as follows:

	Brown	Not Brown
Male	20	40
Female	30	30

A: brown hair

$P(A) = 50/120$

B: female

$P(B) = 60/120$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 50/120 + 60/120 - 30/120$$
$$= 80/120 = 2/3$$

A Special Case

When two events A and B are **mutually exclusive,** $P(A \cap B) = 0$ and $P(A \cup B) = P(A) + P(B)$.

A: male with brown hair

$P(A) = 20/120$

B: female with brown hair

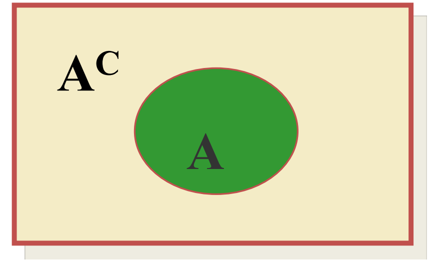
$P(B) = 30/120$

	Brown	Not Brown
Male	20	40
Female	30	30

A and B are mutually exclusive, so that

$P(A \cup B) = P(A) + P(B)$
$= 20/120 + 30/120$
$= 50/120$

Calculating Probabilities for Complements



- We know that for any event **A:**

  $P(A \cap A^C) = 0$

- Since either **A** or **A**^{C} must occur,

  $P(A \cup A^C) = 1$

- so that $P(A \cup A^C) = P(A) + P(A^C) = 1$

$$P(A^C) = 1 - P(A)$$

Example

Select a student at random from the classroom. Define:

A: male

$P(A) = 60/120$

B: female

	Brown	Not Brown
Male	20	40
Female	30	30

A and B are complementary, so that

$P(B) = 1 - P(A)$
$= 1 - 60/120 = 40/120$

Conditional Probabilities

- The probability that A occurs, given that event B has occurred is called the **conditional probability** of A given B and is defined as

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \text{ if } P(B) \neq 0$$

"given"

يعني احتمال
يعتمد ع حدث
آخر : $P(A|B)$

Conditional Probability

- The probability of an event given that another event has occurred is called a conditional probability.
- The conditional probability of A given B is denoted by $P(A \mid B)$.
- A conditional probability is computed as follows :

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Multiplication Law

يوفر طريقة لحساب احتمال التداخل P(A∩B)

- The <u>multiplication law</u> provides a way to compute the probability of the intersection of two events.
- The law is written as:

$$\boxed{P(A \cap B) = P(B)P(A \mid B)}$$

الصيغة : احتمال حدوث A و B يعتمد على احتمال B مضروبًا

في الاحتمال الشرطي لـ A بشرط B.

Example 1

- Toss a fair coin twice. Define
  - A: head on second toss
  - B: head on first toss

$P(A|B) = ½$

$P(A|\text{not } B) = ½$

HH	1/4
HT	1/4
TH	1/4
TT	1/4

P(A) does not change, whether B happens or not…

A and B are independent!

Independent Events

لا يعني مش عم بتأثر بالتالي

- If the probability of event *A* is not changed by the existence of event *B*, we would say that events *A* and *B* are <u>independent</u>.

- Two events *A* and *B* are independent if:

  $$\boxed{P(A \mid B) = P(A)} \quad \text{or} \quad \boxed{P(B \mid A) = P(B)}$$

P(A) does change, depending on whether B happens or not… → A and B are dependent!

Multiplication Law for Independent Events

- The multiplication law also can be used as a test to see if two events are independent.
- The law is written as:

$$P(A \cap B) = P(A)P(B)$$

يمكن استخدام قانون الضرب لحساب التداخل

طبعًا هاد بعد التحديد اذا كانوا مستقلين او لأ.

The Multiplicative Rule for Intersections

- For any two events, **A** and **B**, the probability that both **A** and **B** occur is

$$\boxed{\begin{aligned} P(A \cap B) &= P(A)\, P(B \text{ given that } A \text{ occurred}) \\ &= P(A)P(B|A) \end{aligned}}$$

- If the events **A** and **B** are independent, then the probability that both **A** and **B** occur is

$$\boxed{P(A \cap B) = P(A)\, P(B)}$$

Example 1

In a certain population, 10% of the people can be classified as being high risk for a heart attack. Three people are randomly selected from this population. What is the probability that exactly one of the three are high risk?

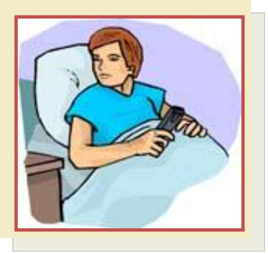
Define H: high risk N: not high risk

P(exactly one high risk) = P(HNN) + P(NHN) + P(NNH)

= P(H)P(N)P(N) + P(N)P(H)P(N) + P(N)P(N)P(H)

$= (.1)(.9)(.9) + (.9)(.1)(.9) + (.9)(.9)(.1) = 3(.1)(.9)^2 = .243$

Example 2



Suppose we have additional information in the previous example. We know that only 49% of the population are female. Also, of the female patients, 8% are high risk. A single person is selected at random. What is the probability that it is a high risk female?

Define H: high risk F: female

From the example, $P(F) = .49$ and $P(H|F) = .08$.
Use the Multiplicative Rule:

$$P(\text{high risk female}) = P(H \cap F)$$

$$= P(F)P(H|F) = .49(.08) = .0392$$