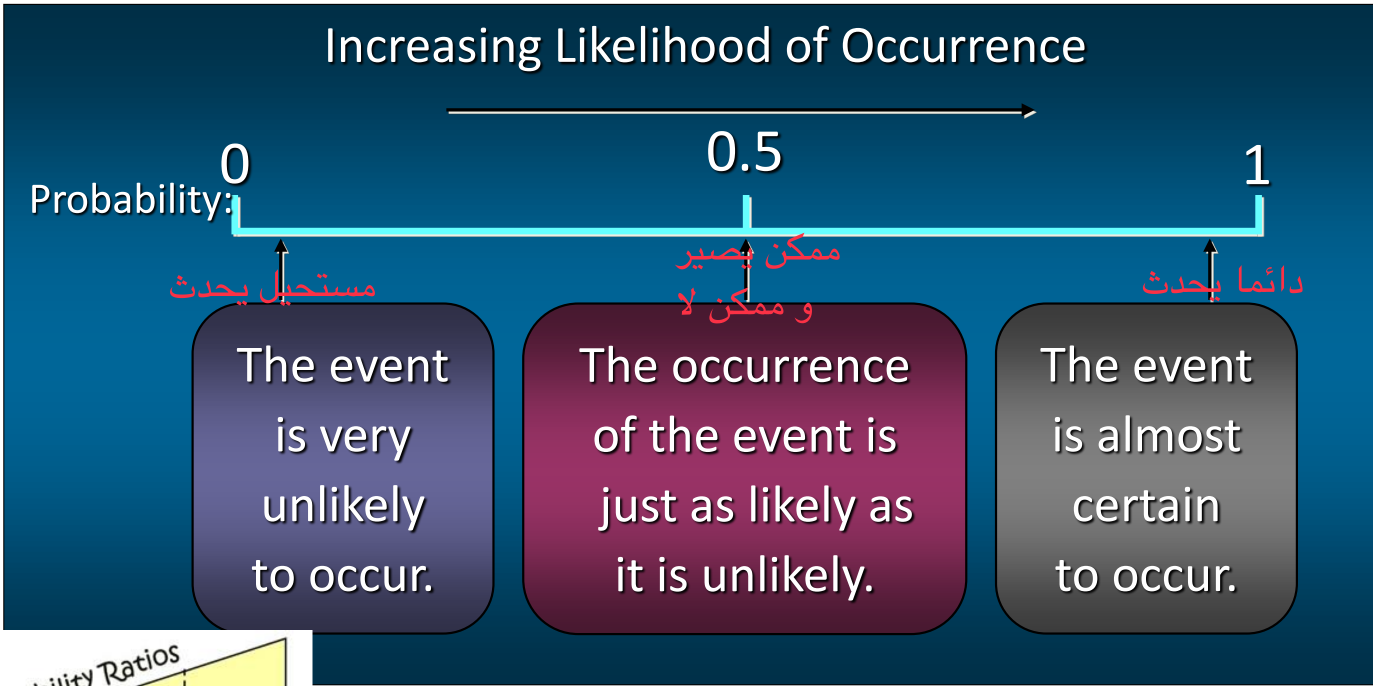
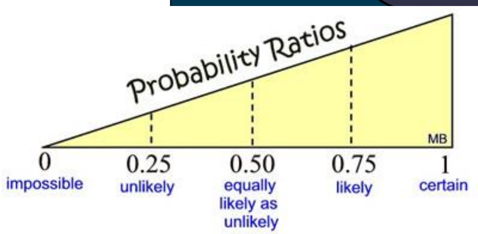
لجان الدُّفعات

STATISTICS

MORPHINE ACADEMY

Probability as a Numerical Measure of the Likelihood of Occurrence





The Probability of an Event

- The probability of an event A measures "how often" we think A will occur. We write **P(A).**
- Suppose that an experiment is performed *n* times. The relative frequency for an event A is

عدد إشي يلي صار عندي

$$\frac{\text{Number of times A occurs}}{n} = \frac{f}{n}$$

عدد الكلي

Let A be the event A = $\{o_1, o_2, \ldots, o_k\}$, where $o_1, o_2, \ldots, o_k$ are k different outcomes. Then

$$P(A) = P(o_1) + P(o_2) + \cdots + P(o_k)$$

Example

احنا خلينا نتعامل معاه
زي توجيهي بالوراثة

- Toss two coins. What is the probability of observing at least one head?

اذا حكالك عدد نتائج بدي
اضرب 2*2

1st Coin	2nd Coin	E_i	$P(E_i)$
H	H	HH	1/4
	T	HT	1/4
T	H	TH	1/4
	T	TT	1/4

P(at least 1 head)

$= P(E_1) + P(E_2) + P(E_3)$

$= 1/4 + 1/4 + 1/4 = 3/4$

2) يجي زيه :

خلينا نحله زي توجيهي :

1: H , T

2: H , T

1. (H, H) , $\frac{1}{2}, \frac{1}{2}$

2. (H, T) , $\frac{1}{2}, \frac{1}{2}$

3. (T, H) , $\frac{1}{2}, \frac{1}{2}$

(T, T) , $\frac{1}{2}, \frac{1}{2}$

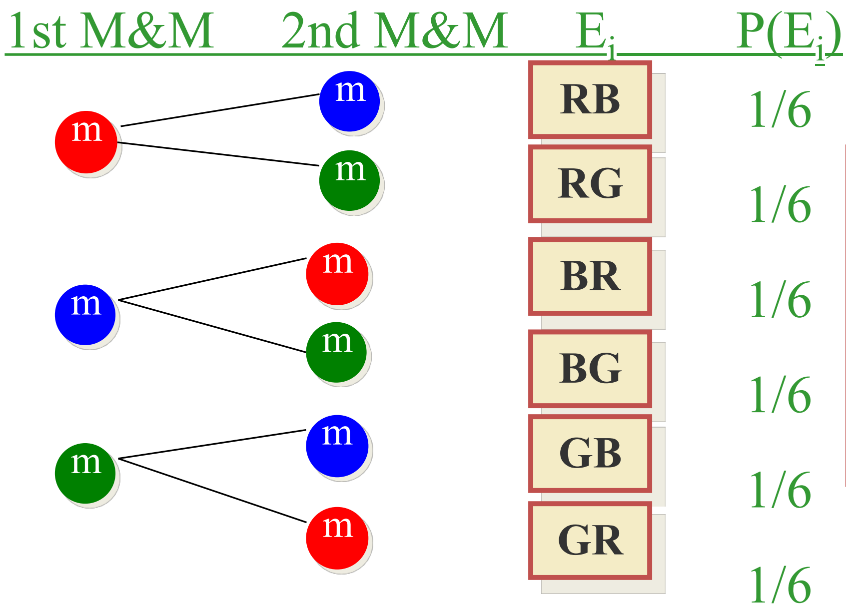
بالاحتمال انه يكون عندي واحدة H أو أكثر

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

الله أكبر

Example

- A bowl contains three M&Ms®, one red, one blue and one green. A child selects two M&Ms at random. What is the probability that at least one is red?



P(at least 1 red)

= P(RB) + P(BR)+ P(RG) + P(GR)

= 4/6 = 2/3

Counting Rules

- If the simple events in an experiment are **equally likely,** you can calculate

$$P(A) = \frac{n_A}{N} = \frac{\text{number of simple events in A}}{\text{total number of simple events}}$$

- You can use **counting rules** to find n_A and N.

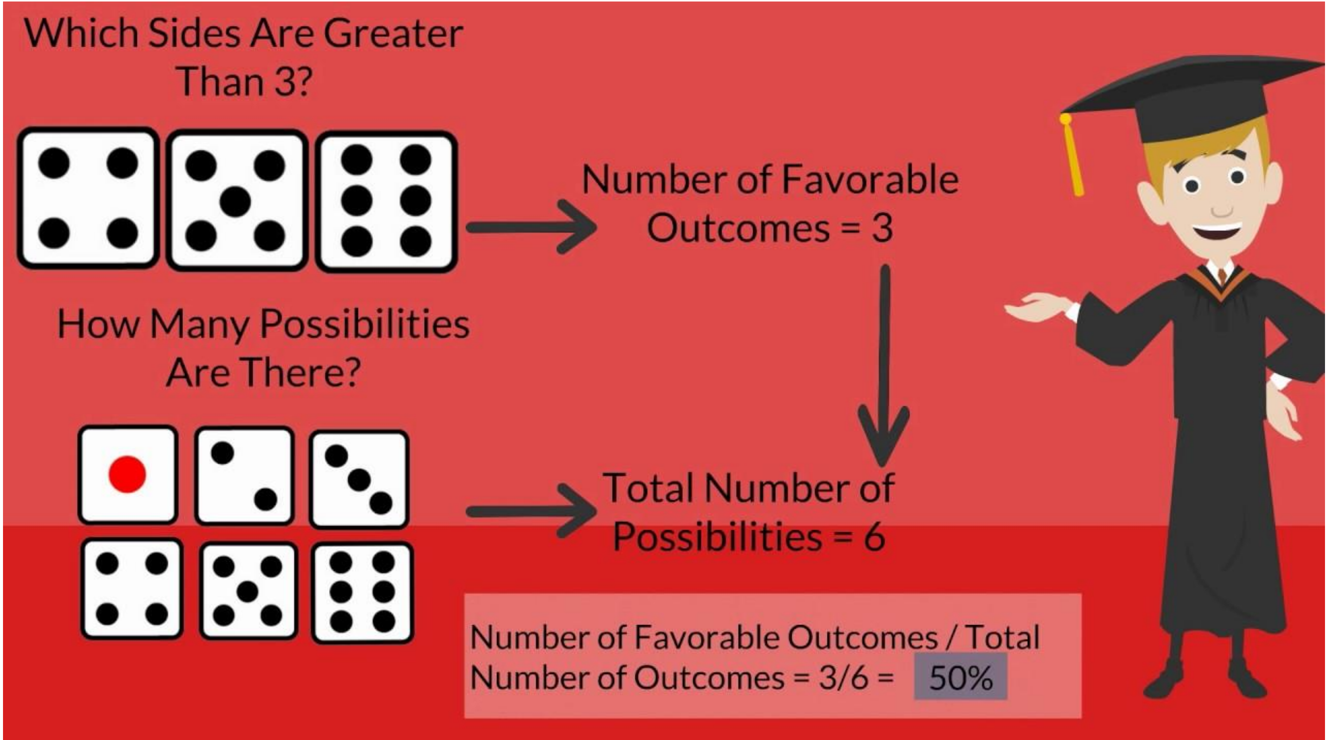
سبحان الله

A Counting Rule for Multiple-Step Experiments

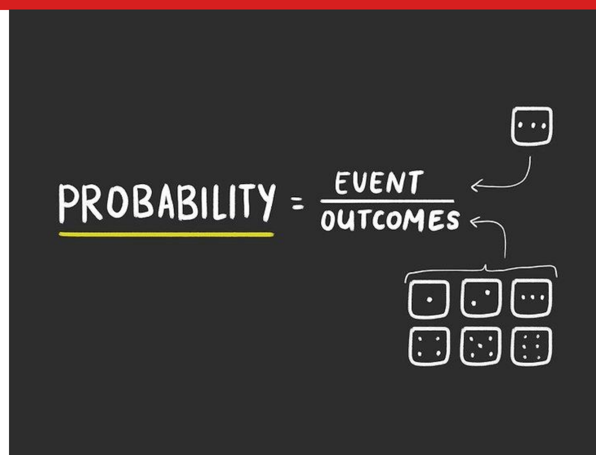
If an experiment consists of a sequence of k steps in which there are n_1 possible results for the first step, n_2 possible results for the second step, and so on, then the total number of experimental outcomes is given by $(n_1)(n_2) \ldots (n_k)$.

A helpful graphical representation of a multiple-step experiment is a <u>tree diagram</u>.

رسم التخطيطي تمثيلا بيانيا مفيدا لتجربة متعددة الخطوت



الحمدلله



هون بحكيلك قديش عندي رقم
اكبر من ٣في حجر نرد =٣
طيب بدي احتمالية هسا
بقسم هاي ثلاث ع عدد
الكلب للاحتمالات يلي هو
ستة

Addition Law

- The addition law provides a way to compute the probability of event *A*, or *B*, or both *A* and *B* occurring.

- The law is written as:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

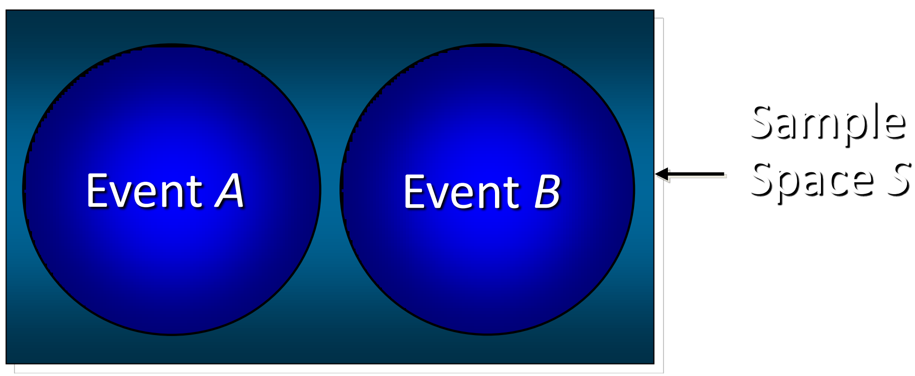
حديثين متنافين اذا ما كان عندي نقاط مشتركة

Two events are said to be <u>mutually exclusive</u> if the events have no sample points (outcomes) in common.

Two events are mutually exclusive if, when one event occurs, the other cannot occur (They can't occur at the same time. The outcome of the random experiment cannot belong to both A and B

If events *A* and *B* are mutually exclusive, $P(A \cap B) = 0$.

اذا كان حدثين متنافين اذا
احتمالية تقاطعهما صفر



Mutually Exclusive Events

- If events A and B are mutually exclusive, $P(A \cap B) = 0$.
- The addition law for mutually exclusive events is:

$$P(A \cup B) = P(A) + P(B)$$

there's no need to include "$- P(A \cap B)$"

بطل في عندي ناقص احتمالية
تقاطعهما لانو اصلا صفر

اللهم يا مقلب القلوب ثبت قلوبنا على دينك

- A: observe an odd number
- B: observe a number greater than 2

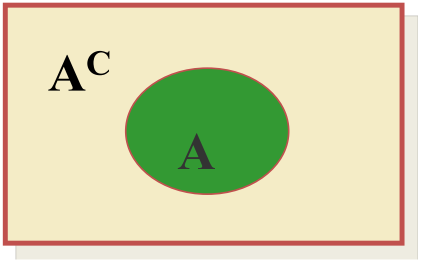
Not Mutually Exclusive

- C: observe a 6
- D: observe a 3

Mutually Exclusive

B and C?
B and D?

Calculating Probabilities for Complements



- We know that for any event **A:**

  $P(A \cap A^C) = 0$

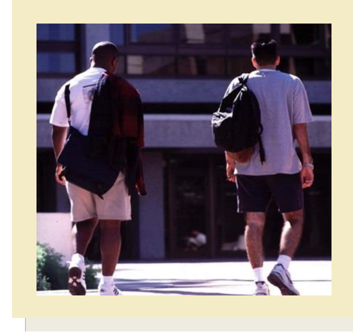
- Since either **A** or **A**^{C} must occur,

  $P(A \cup A^C) = 1$

- so that $P(A \cup A^C) = P(A) + P(A^C) = 1$

$$\boxed{P(A^C) = 1 - P(A)}$$

Example



Select a student at random from the classroom. Define:

A: male

$P(A) = 60/120$

B: female

	Brown	Not Brown
Male	20	40
Female	30	30

$P(A^c)$

A and B are complementary, so that

$P(B) = 1- P(A)$

$= 1- 60/120 = \cancel{40}/120$

60

انا عندي 60 زلمةشو متممة تبعته انه ما بيكون عندي ستين
زلمة يعني يكون عندي بنات

Calculating Probabilities for Intersections

- we can find $P(A \cap B)$ directly from the table. Sometimes this is impractical or impossible. The rule for calculating $P(A \cap B)$ depends on the idea of **independent and dependent events.**

Two events, **A** and **B**, are said to be **independent** if and only if the probability that event **A** occurs does not change, depending on whether or not event **B** has occurred.

Conditional Probabilities

- The probability that A occurs, given that event B has occurred is called the **conditional probability** of A given B and is defined as

$$\boxed{P(A \mid B) = \frac{P(A \cap B)}{P(B)} \text{ if } P(B) \neq 0}$$

"given"

Conditional Probability

- The probability of an event given that another event has occurred is called a conditional probability.
- The conditional probability of *A given B* is denoted by $P(A|B)$.
- A conditional probability is computed as follows :

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Multiplication Law

The multiplication law provides a way to compute the probability of the intersection of two events.

The law is written as:

$$P(A \cap B) = P(B)P(A|B)$$

Dependent
بستخدم هذا
قانون

Independent سواء B صائزت و خلصت و لا لسا ما بأثر عشان هيك بستخدم
p(A)*p(B) هذا القانون

Independent Events

If the probability of event *A* is not changed by the existence of event *B*, we would say that events *A* and *B* are <u>independent</u>. اذا ما تغير احتمال A في وقوع B اذا الحدثين مستقلين

Two events *A* and *B* are independent if:

$P(A|B) = P(A)$ or $P(B|A) = P(B)$

P(A) does change, depending on whether B happens or not… → A and B are dependent!

Defining Independence

- We can redefine independence in terms of conditional probabilities:

Two events A and B are **independent** if and only if

$$P(A|B) = P(A) \quad \text{or} \quad P(B|A) = P(B)$$

Otherwise, they are **dependent**.

- Once you've decided whether or not two events are independent, you can use the following rule to calculate their intersection.

قاعدة ضرب

The Multiplicative Rule for Intersections

- For any two events, **A** and **B**, the probability that both **A** and **B** occur is

$$\boxed{\begin{aligned} P(A \cap B) &= P(A)\,P(B \text{ given that } A \text{ occurred}) \\ &= P(A)P(B|A) \end{aligned}}$$

- If the events **A** and **B** are independent, then the probability that both **A** and **B** occur is

$$\boxed{P(A \cap B) = P(A)\,P(B)}$$

N, H, N

NNH / HNN / NHN

$\frac{10}{100}$

Example 1

In a certain population, 10% of the people can be classified as being high risk for a heart attack. Three people are randomly selected from this population. What is the probability that exactly one of the three are high risk?

.9

Define H: high risk N: not high risk

P(exactly one high risk) = P(HNN) + P(NHN) + P(NNH)

= P(H)P(N)P(N) + P(N)P(H)P(N) + P(N)P(N)P(H)

$= (.1)(.9)(.9) + (.9)(.1)(.9) + (.9)(.9)(.1) = 3(.1)(.9)^2 = .243$

Example 2

$\frac{49}{100}$

Suppose we have additional information in the previous example. We know that only 49% of the population are female. Also, of the female patients, 8% are high risk. A single person is selected at random. What is the probability that it is a high risk female?

$\frac{8}{100}$

Define H: high risk F: female

From the example, P(F) = .49 and P(H|F) = .08.
Use the Multiplicative Rule:

$$P(\text{high risk female}) = P(H \cap F)$$

$$= P(F)P(H|F) = .49(.08) = .0392$$

طيب سؤال مهم : كيف عرفت انه سؤال بده
تقاطع مش اتحاد لانو جمعلي حدثين مع
بعض ياها انثى و عالية خطورة
و ما حكالي or
يعني بده سؤال يتحقق شرطين معا في نفس
شخص

هو بده ياه بـ سؤال انثى وتكون
معرضة لخطر

انثى = .49
عرضة للخطر = .08
اذا ضربت من بعض صح لوما
عرفت الا تون بربط بقدر
اطبق واحل

= .08 × .49