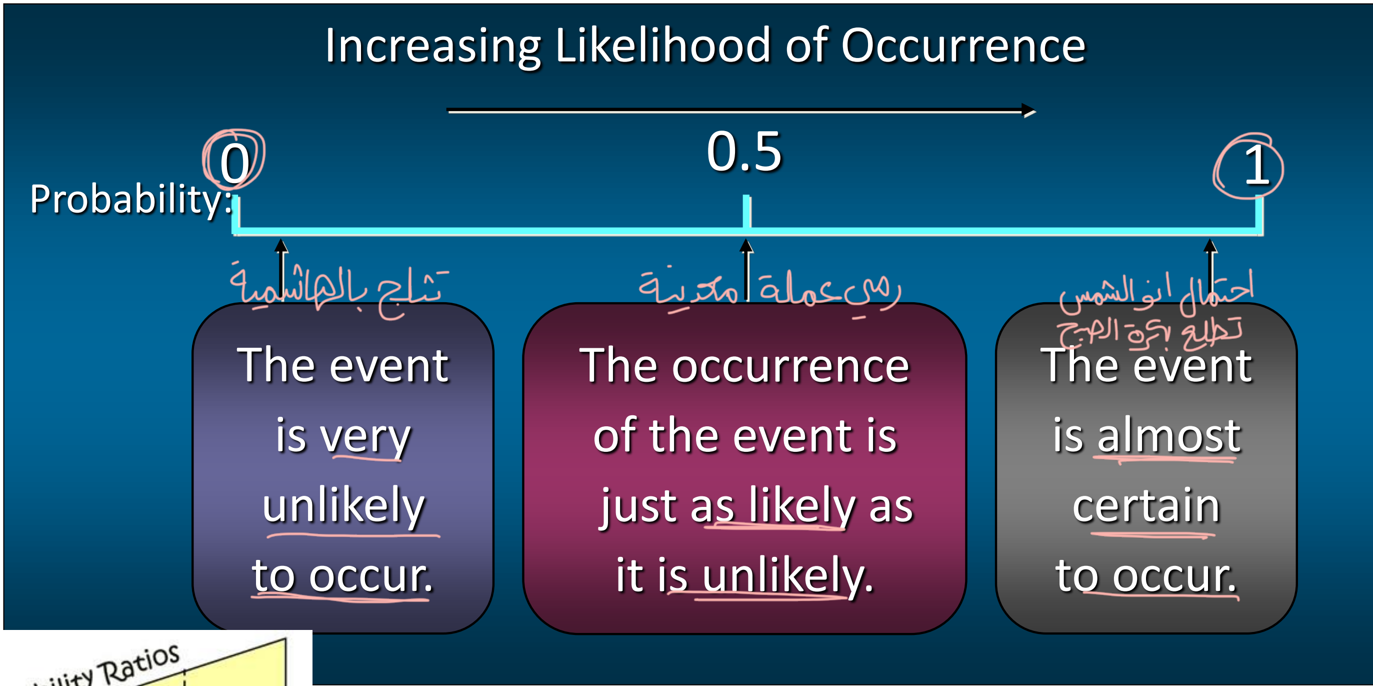
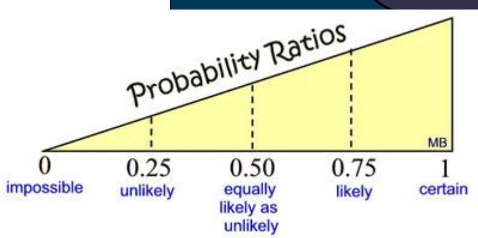
لجان الدفعات

STATISTICS

MORPHINE ACADEMY

Probability as a Numerical Measure of the Likelihood of Occurrence





Finding Probabilities

- Probabilities can be found using
  - Estimates from empirical studies تقديرًا من دراسات تجريبية
  - Common sense estimates based on equally likely events.

- **Examples:**
  - Toss a fair coin. $\boxed{P(\text{Head}) = 1/2}$
  - 10% of the U.S. population has red hair.
    Select a person at random. $\boxed{P(\text{Red hair}) = .10}$

الدكتورة قالت رح
يجي على نمطه بالسكند

Example

- Toss two coins. What is the probability of observing at least one head?

ظهور وجه مرة على الأقل

1st Coin	2nd Coin	E_i	$P(E_i)$
H	H	HH	1/4
	T	HT	1/4
T	H	TH	1/4
	T	TT	1/4

مجموع احتمالات كل نتيجة من هاي النتائج

P(at least 1 head)

$= P(E_1) + P(E_2) + P(E_3)$

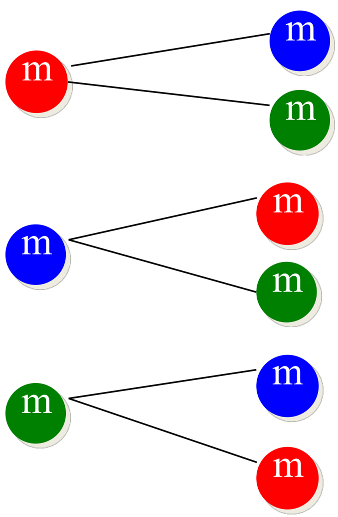
$= 1/4 + 1/4 + 1/4 = 3/4$

Example

- A bowl contains three M&Ms®, one red, one blue and one green. A child selects two M&Ms at random. What is the probability that at least one is red?

على الأقل وحدة حمراء

عند اختيار 2 M&Ms



1st M&M	2nd M&M	E_i	$P(E_i)$
R	B	RB	1/6
R	G	RG	1/6
B	R	BR	1/6
B	G	BG	1/6
G	B	GB	1/6
G	R	GR	1/6

P(at least 1 red)

= P(RB) + P(BR)+ P(RG) + P(GR)

= 4/6 = 2/3

Counting Rules

- If the simple events in an experiment are **equally likely**, you can calculate

$$P(A) = \frac{n_A}{N} = \frac{\text{number of simple events in A}}{\text{total number of simple events}}$$

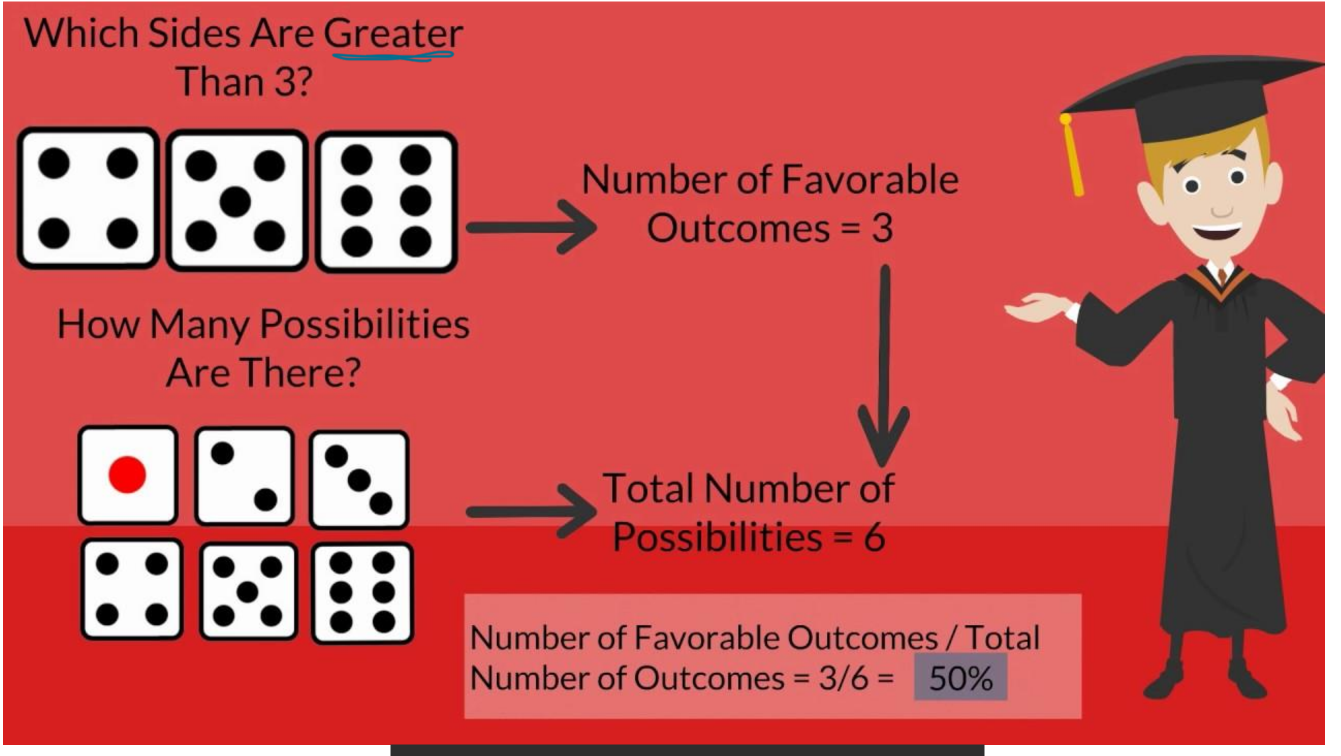
- You can use **counting rules** to find n_A and N.

A Counting Rule for Multiple-Step Experiments

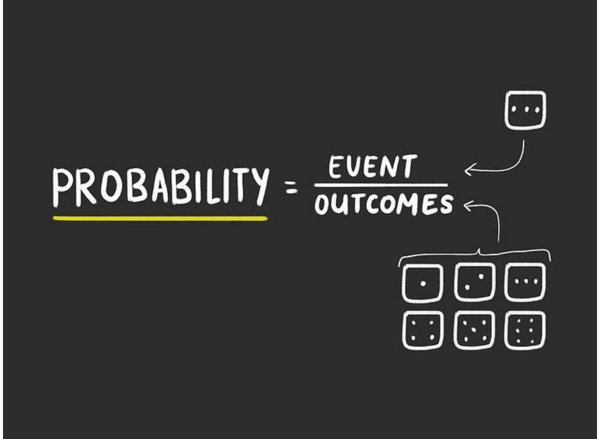
سلسلة من
الخطوات

If an experiment consists of a sequence of k steps in which there are n_1 possible results for the first step, n_2 possible results for the second step, and so on, then the total number of experimental outcomes is given by $(n_1)(n_2) \ldots (n_k)$.

A helpful graphical representation of a multiple-step experiment is a tree diagram.



Which Sides Are Greater Than 3?
Number of Favorable Outcomes = 3
How Many Possibilities Are There?
Total Number of Possibilities = 6
Number of Favorable Outcomes / Total Number of Outcomes = 3/6 = 50%



PROBABILITY = EVENT / OUTCOMES

محذوف من
هون

$$P_n^N = n!\binom{N}{n} = \frac{N!}{(N-n)!}$$

Permutations

x

- The number of ways you can arrange ***n*** distinct objects, taking them ***r*** at a time is

$$P_r^n = \frac{n!}{(n-r)!}$$

where $n! = n(n-1)(n-2)\ldots(2)(1)$ and $0! \equiv 1$.

Example: How many 3-digit lock combinations can we make from the numbers 1, 2, 3, and 4?

The order of the choice is important!

$$P_3^4 = \frac{4!}{1!} = 4(3)(2) = 24$$

X

Examples

Example: A lock consists of five parts and can be assembled in any order. A quality control engineer wants to test each order for efficiency of assembly. How many orders are there?

The order of the choice is important!

$$P_5^5 = \frac{5!}{0!} = 5(4)(3)(2)(1) = 120$$

$$C_n^N = \binom{N}{n} = \frac{N!}{n!(N-n)!}$$

Combinations

- The number of distinct combinations of ***n*** distinct objects that can be formed, taking them ***r*** at a time is

$$C_r^n = \frac{n!}{r!(n-r)!}$$

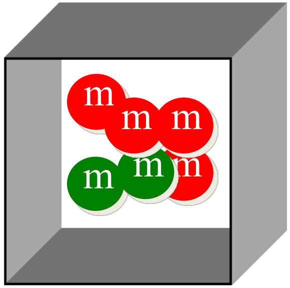
Example: Three members of a 5-person committee must be chosen to form a subcommittee. How many different subcommittees could be formed?

$$C_3^5 = \frac{5!}{3!(5-3)!} = \frac{5(4)(3)(2)1}{3(2)(1)(2)1} = \frac{5(4)}{(2)1} = 10$$

The order of the choice is not important!

لهون
محذوف

Example



- A box contains six M&Ms®, four red and two green. A child selects two M&Ms at random. What is the probability that exactly one is red?

The order of the choice is not important!

$$C_2^6 = \frac{6!}{2!4!} = \frac{6(5)}{2(1)} = 15$$

ways to choose 2 M & Ms.

$$C_1^2 = \frac{2!}{1!1!} = 2$$

ways to choose 1 green M & M.

$$C_1^4 = \frac{4!}{1!3!} = 4$$

ways to choose 1 red M & M.

$4 \times 2 = 8$ ways to choose 1 red and 1 green M&M.

P(exactly one red) = 8/15

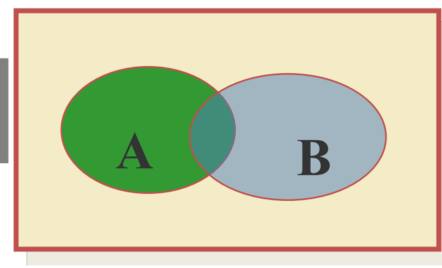
Calculating Probabilities for

اتحاد متممات

Unions and Complements

- There are special rules that will allow you to calculate probabilities for composite events.
- **<u>The Additive Rule for Unions:</u>**
- For any two events, **A** and **B**, the probability of their union, **P(A ∪ B)**, is

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$



Addition Law

- The <u>addition law</u> provides a way to compute the probability of event *A*, or *B*, or both *A* and *B* occurring.

- The law is written as:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

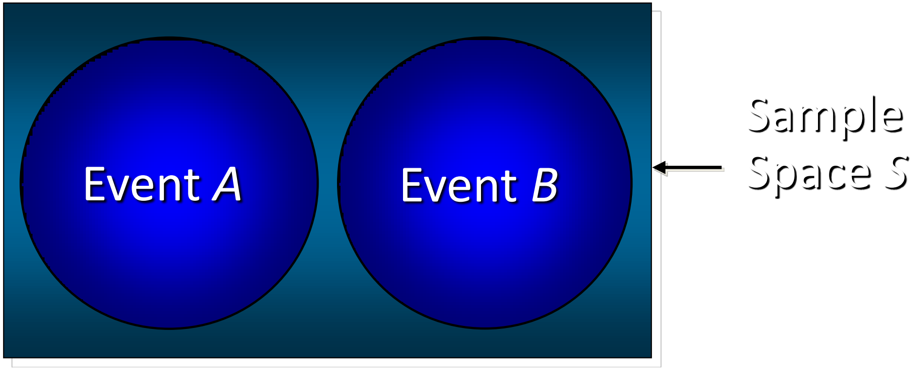
إذا كان الحدثين ما عندهم أي نقطة مشتركة بالنتائج (ما بصيروا مع بعض) بنقول انهم متنافيين

Two events are said to be mutually exclusive if the events have no sample points (outcomes) in common.

Two events are mutually exclusive if, when one event occurs, the other cannot occur (They can't occur at the same time. The outcome of the random experiment cannot belong to both A and B

تقاطعهم صفر

If events A and B are mutually exclusive, $P(A \cap B) = 0$.



Mutually Exclusive Events

- If events A and B are mutually exclusive, $P(A \cap B) = 0$.
- The addition law for mutually exclusive events is:

$$P(A \cup B) = P(A) + P(B)$$

there's no need to include "$- P(A \cap B)$"

احنا أصلاً هاي بنحطها
عشان مايصير تكرار

- A: observe an odd number
- B: observe a number greater than 2

Not Mutually Exclusive

- C: observe a 6
- D: observe a 3

Mutually Exclusive

B and C?
B and D?

Example: Additive Rule

Example: Suppose that there were 120 students in the classroom, and that they could be classified as follows:

A: brown hair

$P(A) = 50/120$

B: female

$P(B) = 60/120$

	Brown	Not Brown
Male	20	40
Female	30	30

تقاطع

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 50/120 + 60/120 - 30/120$$
$$= 80/120 = 2/3$$

A Special Case

When two events A and B are **mutually exclusive,** $P(A \cap B) = 0$ and $P(A \cup B) = P(A) + P(B)$.

A: male with brown hair

$P(A) = 20/120$

B: female with brown hair

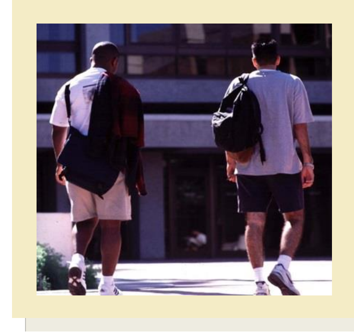
$P(B) = 30/120$

	Brown	Not Brown
Male	20	40
Female	30	30

A and B are mutually exclusive, so that

$$P(A \cup B) = P(A) + P(B)$$
$$= 20/120 + 30/120$$
$$= 50/120$$

Example



Select a student at random from the classroom. Define:

A: male

$P(A) = 60/120$

B: female

ما في تقاطع

	Brown	Not Brown
Male	20	40
Female	30	30

A and B are complementary, so that

$P(B) = 1 - P(A)$

$= 1 - 60/120 = 60/120 = \frac{1}{2}$

$\frac{120}{120}$

Calculating Probabilities for Intersections

التقاطع

- we can find P(A ∩ B) directly from the table. Sometimes this is impractical or impossible. The rule for calculating P(A ∩ B) depends on the idea of **independent and dependent events.**

> Two events, **A** and **B**, are said to be **independent** if and only if the probability that event **A** occurs does not change, depending on whether or not event **B** has occurred.

Conditional Probability

- The probability of an event given that another event has occurred is called a conditional probability.
- The conditional probability of *A given B* is denoted by $P(A|B)$.
- A conditional probability is computed as follows :

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Multiplication Law

الضرب

- The multiplication law provides a way to compute the probability of the intersection of two events.

- The law is written as:

$$P(A \cap B) = P(B)P(A|B)$$

احتمال وقوع الحدث A بشرط انه B صار

القانون بساعدنا نحسب احتمال تقاطع حدثين A وB

(يعني احتمال حدوث الحدثين مع بعض)

Example 1

- Toss a fair coin twice. Define
  - A: head on second toss
  - B: head on first toss

HH	1/4
HT	1/4
TH	1/4
TT	1/4

$P(A|B) = ½$

$P(A|\text{not } B) = ½$

P(A) does not change, whether B happens or not…

A and B are independent!

Independent Events

If the probability of event *A* is not changed by the existence of event *B*, we would say that events *A* and *B* are <u>independent</u>.

Two events *A* and *B* are independent if:

$P(A|B) = P(A)$ or $P(B|A) = P(B)$

اذا احتمال A مابيتأثر بوجود وحدوث B والأمر نفسه مع B فالأحداث مستقلة

P(A) does change, depending on whether B happens or not… → A and B are dependent!

Defining Independence

- We can redefine independence in terms of conditional probabilities:

مستقلين

Two events A and B are **independent** if and only if

$$\mathbf{P(A|B) = P(A)} \quad \text{or} \quad \mathbf{P(B|A) = P(B)}$$

غير مستقل

Otherwise, they are **dependent**.

- Once you've decided whether or not two events are independent, you can use the following rule to calculate their intersection.

Multiplication Law for Independent Events

The multiplication law also can be used as a test to see if two events are independent.

The law is written as:

$$P(A \cap B) = P(A)P(B)$$

The Multiplicative Rule for Intersections

- For any two events, **A** and **B**, the probability that both **A** and **B** occur is

$$\boxed{\begin{aligned} P(A \cap B) &= P(A)\, P(B \text{ given that } A \text{ occurred}) \\ &= P(A)P(B|A) \end{aligned}}$$

- If the events **A** and **B** are independent, then the probability that both **A** and **B** occur is

$$\boxed{P(A \cap B) = P(A)\, P(B)}$$

Example 1

In a certain population, 10% of the people can be classified as being high risk for a heart attack. Three people are randomly selected from this population. What is the probability that exactly one of the three are high risk?

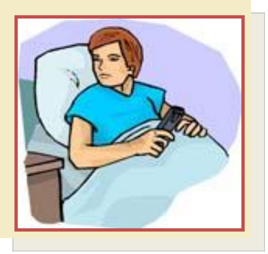
Define H: high risk N: not high risk

P(exactly one high risk) = P(HNN) + P(NHN) + P(NNH)

= P(H)P(N)P(N) + P(N)P(H)P(N) + P(N)P(N)P(H)

$= (.1)(.9)(.9) + (.9)(.1)(.9) + (.9)(.9)(.1) = 3(.1)(.9)^2 = .243$

Example 2



Suppose we have additional information in the previous example. We know that only 49% of the population are female. Also, of the female patients, 8% are high risk. A single person is selected at random. What is the probability that it is a high risk female?

Define H: high risk F: female

From the example, $P(F) = .49$ and $P(H|F) = .08$.
Use the Multiplicative Rule:

$$P(\text{high risk female}) = P(H \cap F)$$

$$= P(F)P(H|F) = .49(.08) = .0392$$