STATISTICS

MORPHINE ACADEMY

لجان الدُفعات

"ملخص السلايد"

احتمالية الحادث

The Probability of an Event

⇐ احتمال وقوع أي حدث لازم يكون رقم بين 0 و 1.

بين ال 0 و 1 — 1 (certain) (always occur) — 0 impossible (never occur)

الاحتمال يقع بين الصفر والواحد

- P(A) must be between 0 and 1. ⇒ (impossible)
  - If event A can never occur, P(A) = 0. If event A always occurs when the experiment is performed, P(A) =1. (certain)

لا يمكن حدوثه (مستحيل)

حدوث مؤكد

- The sum of the probabilities for all simple events in S equals 1.

sample space

مجموع الاحتمالات لكل الحوادث = 1

Example

لما نرمي قطعة نقود

$\frac{1}{2}$ ⇐ احتمال الصورة

$\frac{1}{2}$ ⇐ احتمال الكتابة

مجموع الاحتمالات = 1

مثال

لما أعطيك شو احتمالية يطلع على حجر النرد عدد زوجي:

القيم الزوجية ⇐ 2,4,6

عدد

الاحتمال ⇐ $\frac{3}{6} = \frac{1}{2}$

- The **probability of an event A** is found by adding the probabilities of all the simple events contained in A.

Probability as a Numerical Measure of the Likelihood of Occurrence

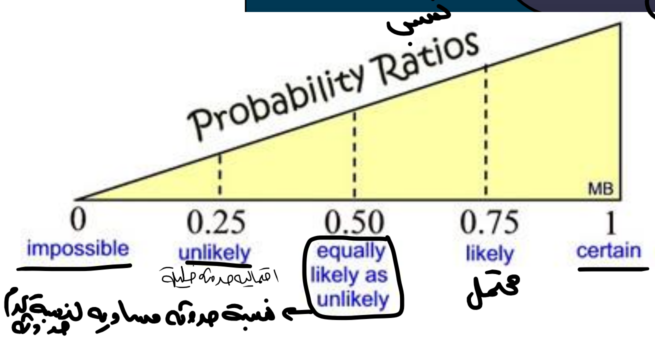
رقمي مقياس
احتمالية وتوقع حدث معين

Increasing Likelihood of Occurrence

مستحيل مؤكد

Probability: 0 — 0.5 — 1

0	0.5	1
The event is very unlikely to occur.	The occurrence of the event is just as likely as it is unlikely.	The event is almost certain to occur.
مستحيل	%50 likely %50 unlikely	مؤكد



The Probability of an Event

$P(C) \Rightarrow$ كيف أنهما $\Rightarrow$ قياس لقديش أنا متوقع أن يحدث الحدث (C)

قياسات   كم مرة

- The probability of an event A measures "how often" we think A will occur. We write **P(A)**.

توقع   أد ايش يكرر الحدث (A)

- Suppose that an experiment is performed n times. The relative frequency for an event A is

عدنا التجربة أكثر من مرة.

← تكرار نسبي.

عدد مرات التي حدث فيها A →

$$\frac{\text{Number of times A occurs}}{n} = \frac{f}{n}$$

← عدد مرات تكرار التجربة.

الحصول على عدد زوجي عند رمي حجر نرد ← (Example) $A = \{2, 4, 6\}$

$P(A) = P(2) + P(4) + P(6)$

$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$

$= 3/6 = \frac{1}{2}$

Let A be the event A = {o_1, o_2, …, o_k}, where o_1, o_2, …, o_k are k different outcomes. Then

نتائج   عدد هذه النتائج

$$P(A) = P(o_1) + P(o_2) + \cdots + P(o_k)$$

Finding Probabilities

- Probabilities can be found using كيف نجد الاحتمالات
  - تقديرات تجريبية دراسات
  - Estimates from empirical studies ⇒ تقديرات من الدراسات التجريبية.
  - بالمنطق تقديرات
  - Common sense estimates based on equally likely events. لما تكون كل النتائج ممكنة ومتساوية الاحتمال نستخدم المنطق

- **Examples:** احتمال أن يظهر صوره في كل تجربة $= \frac{1}{2}$
  - رمي
  - Toss a fair coin. **P(Head) = 1/2** (Common sense estimates)
  - 10% of the U.S. population has red hair. (Empirical studies) Select a person at random. **P(Red hair) = .10**

(Common sense estimates) Equally likely ⇐ رمي قطعة نقد

(Empirical studies) ⇐ اختيار شخص عشوائي لديه شعر أحمر في U.S

Example

- Toss two coins. What is the probability of observing at least one head?

لو رمي قطعتين نقود ما احتمالية
ظهور على الأقل رأس بغض النظ
كل موقعه:

1st Coin	2nd Coin	E_i	$P(E_i)$
H	H	HH	1/4
H	T	HT	1/4
T	H	TH	1/4
T	T	TT	1/4

or

or

Head is

Head is

Head is

Head ليس

$P(\text{at least 1 head})$

$= P(E_1) + P(E_2) + P(E_3)$

$= 1/4 + 1/4 + 1/4 = 3/4$

$P(\text{at least one head})$

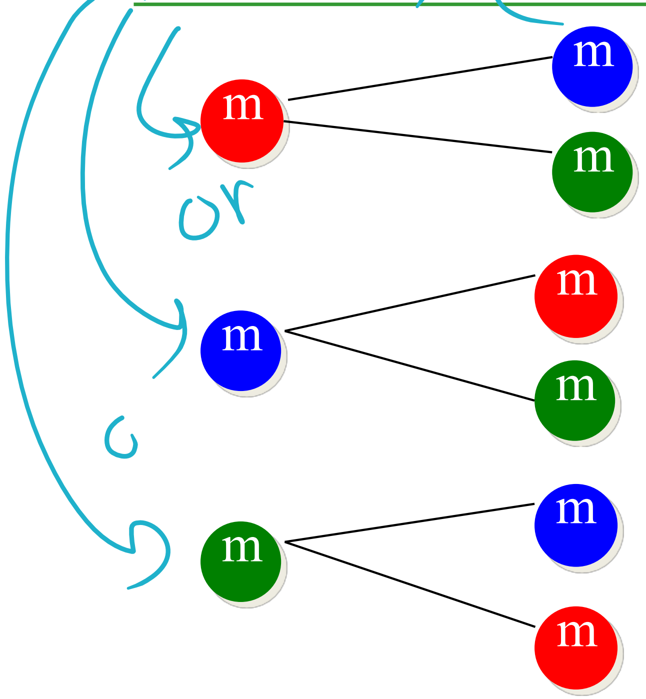
$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$

Example

- A bowl contains three M&Ms®, one red, one blue and one green. A child selects two M&Ms at random. What is the probability that at least one is red?

P (at least one red)

1st M&M 2nd M&M



E_i	$P(E_i)$
RB	1/6
RG	1/6
BR	1/6
BG	1/6
GB	1/6
GR	1/6

P(at least 1 red)

= P(RB) + P(BR)+ P(RG) + P(GR)

= 4/6 = 2/3

Counting Rules

احتمالية حدوثها متساوية.

- If the simple events in an experiment are **equally likely**, you can calculate

عدد نتائج تحقق A

$$P(A) = \frac{n_A}{N} = \frac{\text{number of simple events in A}}{\text{total number of simple events}}$$

العدد الكلي للنتائج

- You can use **counting rules** to find n_A and N.

Example

رمي حجر نرد → N : عدد النتائج الممكنة = 6 نتائج.

A: الحصول على عدد فردي → n_A : 3 نتائج

$A = \{1, 3, 5\}$

$P(A) = \frac{n_A}{N} = \frac{3}{6} = \frac{1}{2}$

(قاعدة العد) A Counting Rule for

(تجارب متعددة الخطوات) Multiple-Step Experiments

تتكون — عدة خطوات

If an experiment consists of a sequence of k steps in which there are n_1 possible results for the first step, n_2 possible results for the second step, and so on, then the total number of experimental outcomes is given by $(n_1)(n_2) \dots (n_k)$.

يوجد عدد معين من النتائج

خطوة أولى : n_1 نتائج ممكنة.
خطوة ثانية : n_2 نتائج ممكنة.
خطوة ثالثة : n_3 نتائج ممكنة.
خطوة K : n_K نتائج ممكنة

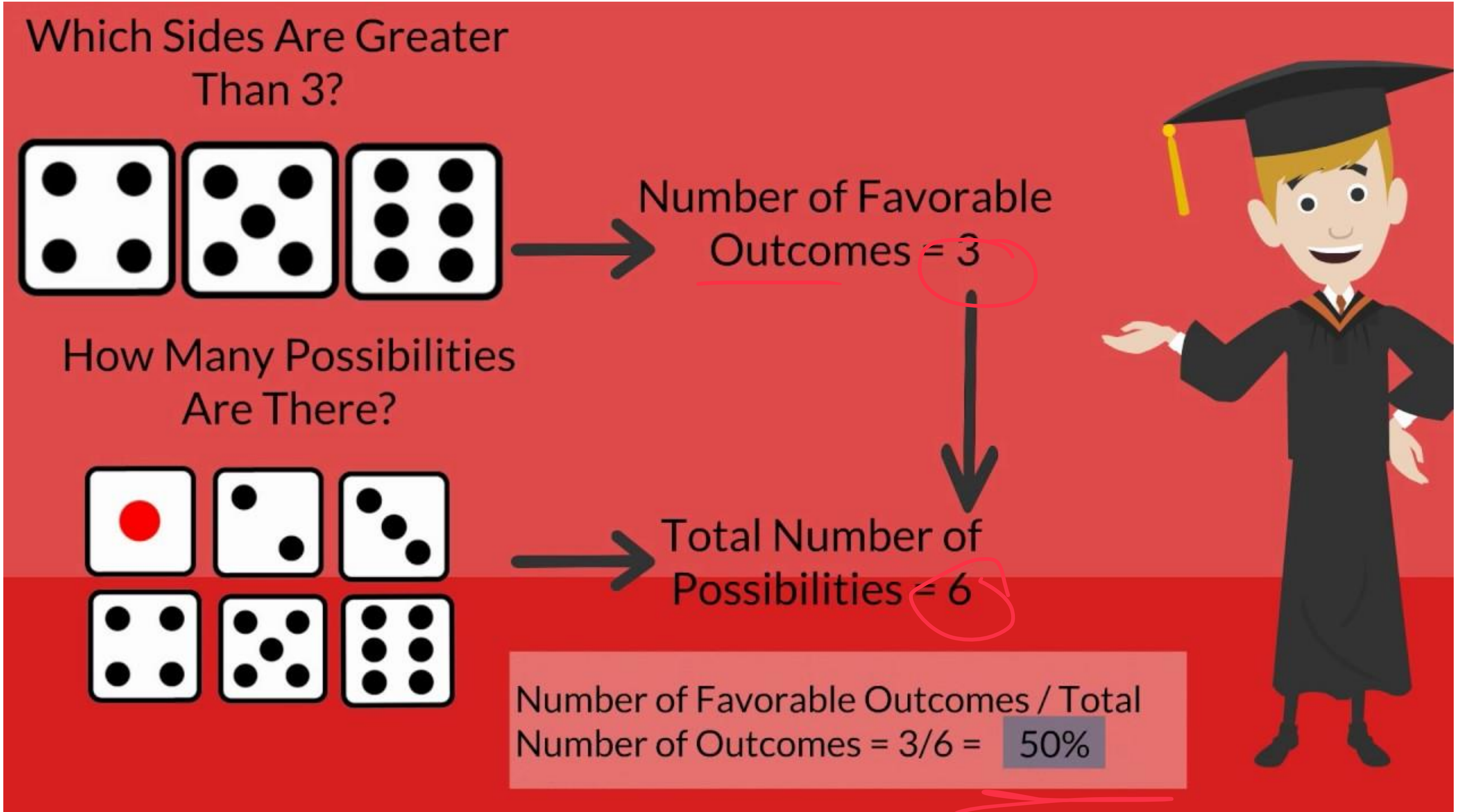
total outcomes $= n_1 * n_2 * \dots n_K$

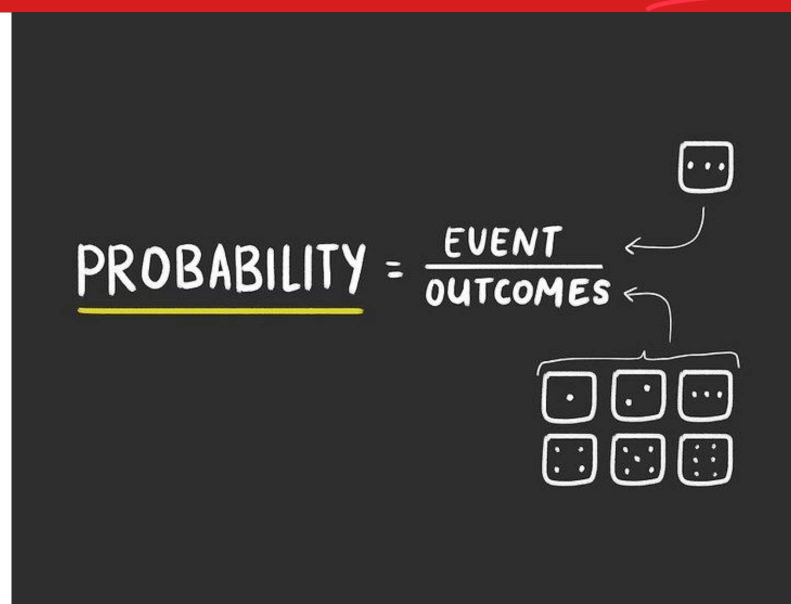
رسومية

A helpful graphical representation of a multiple-step experiment is a tree diagram.

شجرة الاحتمالات

طريقة تساعدك تعرف كل المسارات الممكنة في التجربة خطوة بخطوة.


Which Sides Are Greater Than 3?
Number of Favorable Outcomes = 3
How Many Possibilities Are There?
Total Number of Possibilities = 6
Number of Favorable Outcomes / Total Number of Outcomes = 3/6 = 50%


PROBABILITY = EVENT / OUTCOMES

$$P_n^N = n!\binom{N}{n} = \frac{N!}{(N-n)!}$$

Permutations

- The number of ways you can arrange ***n*** distinct objects, taking them ***r*** at a time is

$$P_r^n = \frac{n!}{(n-r)!}$$

where $n! = n(n-1)(n-2)...(2)(1)$ and $0! \equiv 1$.

Example: How many 3-digit lock combinations can we make from the numbers 1, 2, 3, and 4?

The order of the choice is important!

$$P_3^4 = \frac{4!}{1!} = 4(3)(2) = 24$$

Examples

Example: A lock consists of five parts and can be assembled in any order. A quality control engineer wants to test each order for efficiency of assembly. How many orders are there?

The order of the choice is important!

$$P_5^5 = \frac{5!}{0!} = 5(4)(3)(2)(1) = 120$$

$$C_n^N = \binom{N}{n} = \frac{N!}{n!(N-n)!}$$

Combinations

- The number of distinct combinations of ***n*** distinct objects that can be formed, taking them ***r*** at a time is

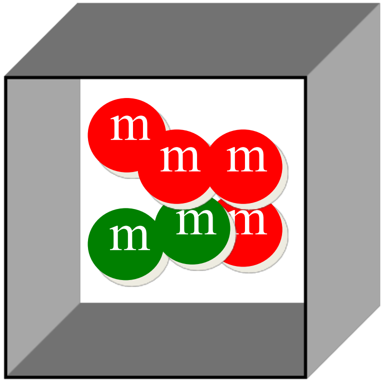
$$C_r^n = \frac{n!}{r!(n-r)!}$$

Example: Three members of a 5-person committee must be chosen to form a subcommittee. How many different subcommittees could be formed?

The order of the choice is not important!

$$C_3^5 = \frac{5!}{3!(5-3)!} = \frac{5(4)(3)(2)1}{3(2)(1)(2)1} = \frac{5(4)}{(2)1} = 10$$

Example



- A box contains six M&Ms®, four red and two green. A child selects two M&Ms at random. What is the probability that exactly one is red?

The order of the choice is not important!

$$C_2^6 = \frac{6!}{2!4!} = \frac{6(5)}{2(1)} = 15$$

ways to choose 2 M & Ms.

$$C_1^2 = \frac{2!}{1!1!} = 2$$

ways to choose 1 green M & M.

$$C_1^4 = \frac{4!}{1!3!} = 4$$

ways to choose 1 red M & M.

4 × 2 =8 ways to choose 1 red and 1 green M&M.

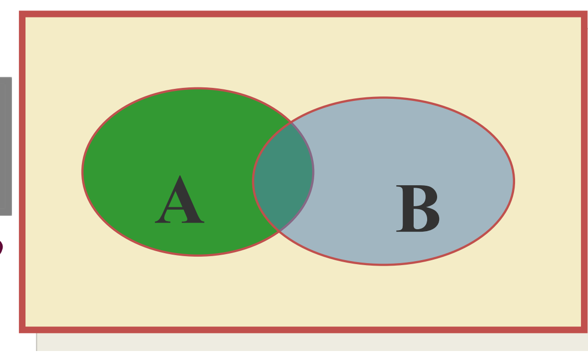
P(exactly one red) = 8/15

Calculating Probabilities for
الاتحادات المتممات
Unions and Complements

- There are special rules that will allow you to calculate probabilities for مركبة composite events.
  في قوانين خاصة لحساب الاحتمال.
  $A \cup B$ اتحاد union
  $A \cap B$ تقاطع
  complement متمم
- الجمع
  <u>The Additive Rule for Unions:</u>
- For any two events, **A** and **B**, the probability of their union, **P(A ∪ B)**, is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

احتمال وقوع A — احتمال وقوع B — احتمال وقوع A و B معاً.



Addition Law

- The addition law provides a way to compute the probability of event *A*, or *B*, or both *A* and *B* occurring.
- The law is written as:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

مثلاً أن يكون الانسان ذكر وأنثى معاً

أحداث متنافية

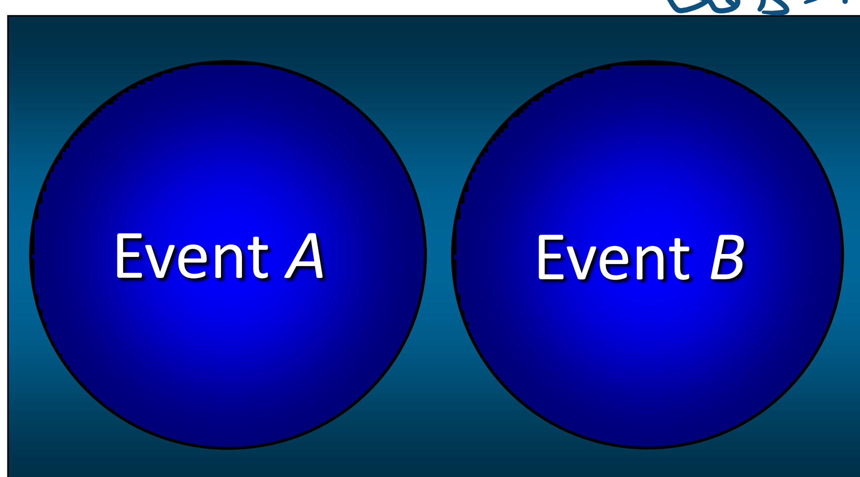
Two events are said to be <u>mutually exclusive</u> if the events have no sample points (outcomes) in common.

Two events are mutually exclusive if, when one event occurs, the other cannot occur (They can't occur at the same time. The outcome of the random experiment cannot belong to both A and B

لا يمكن حدوثهم بالوقت نفسه

If events A and B are mutually exclusive, $P(A \cap B) = 0$.

احتمال وقوع A و B معاً = zero



Sample Space *S*

Mutually Exclusive Events

- If events A and B are mutually exclusive, $P(A \cap B) = 0$.

- The addition law for mutually exclusive events is:

$$P(A \cup B) = P(A) + P(B) \quad + \text{zero}$$

there's no need to include "$- P(A \cap B)$"

$A: \{1,3,5\}$

$B: \{3,4,5,6\}$

$A \cap B = \{3,5\}$ not mutually exclusive

$C: \{6\}$

$D: \{3\}$

$C \cap D = \emptyset$ mutually Exclusive.

$B: \{3,4,5,6\}$

$C: \{6\}$

$B \cap C = \{6\}$ not mutually Exclusive

$B: \{3,4,5,6\}$

$D: \{3\}$

$B \cap D = \{3\}$

not mutually exclusive.

عادي يمكن يجتمع عدد فردي وأكبر من 2 .

Not Mutually Exclusive

–A: observe an odd number

–B: observe a number greater than 2

–C: observe a 6

–D: observe a 3

Mutually Exclusive

لا يمكن أن يظهر 3 و 6 معًا

لأنه رمي الحجر مرة واحدة .

عدد أكبر من 2 — ظهور العدد 6

B and C? نعم تحدث

B and D? نعم تحدث

عدد أكبر من 2 — يظهر العدد 3

Example: Additive Rule

Example: Suppose that there were $\boxed{120}$ students in the classroom, and that they could be classified as follows:

A: brown hair

(20+30)

P(A) = 50/120

B: female

30+30

P(B) = 60/120

	Brown	Not Brown
Male	20	40
Female	30	30

$P(A \cap B) = \frac{30}{120}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 50/120 + 60/120 - 30/120$
$= 80/120 = 2/3$

A Special Case

When two events A and B are **mutually exclusive**, $P(A \cap B) = 0$ and $P(A \cup B) = P(A) + P(B)$.

Ex

A: male with brown hair

$P(A) = 20/120$

B: female with brown hair

$P(B) = 30/120$

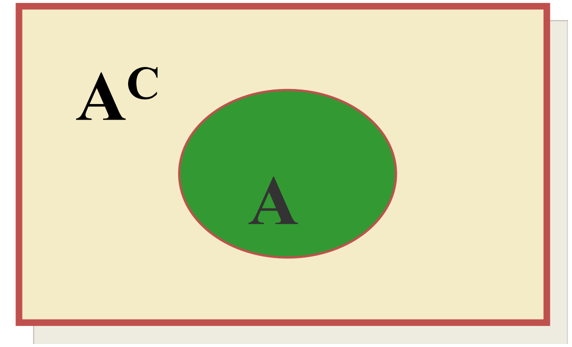
	Brown	Not Brown
Male	20	40
Female	30	30

A and B are mutually exclusive, so that →

$P(A \cup B) = P(A) + P(B)$
$= 20/120 + 30/120$
$= 50/120$

$P(A \cap B) = \text{zero}$

ما يكون الانسان ذكر وأنثى في نفس الوقت.



Calculating Probabilities for Complements

المتمام .

- We know that for any event **A:**

A حدث معين اسمه   كل شيء عدا A

$$\mathbf{P(\underline{A} \cap \underline{A^C}) = 0}$$

- Since either **A** or **A^C** must occur,

A حدث معين اسمه   كل شيء عدا A

$$\mathbf{P(A \cup A^C) = 1}$$

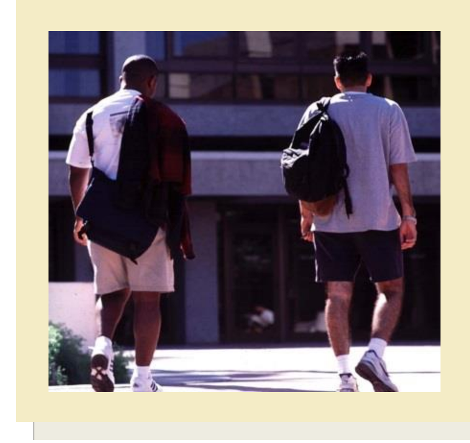
- so that $\mathbf{P(A \cup A^C) = P(A) + P(A^C) = 1}$

$+ P(\underbrace{\cancel{A \cap A^C}}_{\text{zero}})$

$$\boxed{\mathbf{P(A^C) = 1 - P(A)}}$$

احتمال المتممة الحدث A   احتمال وقوع A

Example



Select a student at random from the classroom. Define:

A: male (20+40)/120

P(A) = 60/120

B: female

	Brown	Not Brown
Male	20	40
Female	30	30

A and B are complementary, so that

P(B) = 1- P(A)

= 1- 60/120 = ~~[illegible]~~

$= \frac{120}{120} - \frac{60}{120} = \frac{60}{120}$

Calculating Probabilities for Intersections

التقاطع

(∩) ⇒ احتمال حدوث حدثين معاً.

- we can find P(A ∩ B) directly from the table. Sometimes this is impractical (غير عملي) or impossible (مستحيل). The rule for calculating P(A ∩ B) depends on the idea of **independent and dependent events.**

المستقلة — غير مستقلة (تعتمد على غيرها)

> Two events, **A** and **B**, are said to be **independent** if and only if the probability that event **A** occurs does not change, depending on whether or not event **B** has occurred.

اذا كان احتمال A لا يتغير لو فرضنا أن B حدث أو لم يحدث.

مثال

اذا الأم جابت ولد بأول حمل الها هل هذا بيأثر بأي شي جنس الطفل بالحمل الثاني طبعًا لا.

الاحتمالات الشرطية

Conditional Probabilities

- The probability that A occurs, given that event B has occurred is called the **conditional probability** of A given B and is defined as

شو يعني احتمال شرطي:

*ماهو احتمال أن يكون الطالب ذكر اذا عرفنا أن شعره بني؟

أنا عارف من قبل أن الطالب شعره بني وبدي أعرف من هدول يلي شعرهم بني كم نسبة الذكور منهم.

$$\boxed{P(A \mid B) = \frac{P(A \cap B)}{P(B)} \text{ if } P(B) \neq 0}$$

"given"

Conditional Probability

الاحتمال الشرطي

احتمال وقوع حدث A :
بشرط أننا نعرف أن حدث B
قد وقع فعلاً.

$P(A/B)$

معطى B

- The probability of an event given that another event has occurred is called a conditional probability.
- The conditional probability of *A given B* is denoted by $P(A|B)$.
- A conditional probability is computed as follows :

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Multiplication Law

(قانون الضرب للتقاطع)

حساب احتمال حدوث حدثين معًا (التقاطع) حتى لو لم يكونا مستقلان.

- The multiplication law provides a way to compute the probability of the intersection of two events.

- The law is written as:

$$P(A \cap B) = P(B)P(A|B)$$

احتمال حدوث A و B معًا — احتمال حدوث B — احتمال حدوث A بافتراض أن B حدث فعلاً.

Example 1

- Toss a fair coin twice. Define
  - A: head on second toss صوره بالرمية الثانية
  - B: head on first toss كتابة بالرمية الأولى

HH	1/4
HT	1/4
TH	1/4
TT	1/4

$P(A|B) = \frac{1}{2}$

$P(A|\text{not } B) = \frac{1}{2}$

P(A) does not change, whether B happens or not…

A and B are independent!

(المستقلة) Independent Events

If the probability of event *A* is not changed by the existence of event *B*, we would say that events *A* and *B* are <u>independent</u>.

Two events *A* and *B* are independent if:

$P(A|B) = P(A)$ or $P(B|A) = P(B)$

معرفة أن B حصل أولا ما بتغير من احتمال حدوث (A).

اذا كان احتمال
حدوث A يتأثر
باحتمال حدوث B.

P(A) does change, depending on whether B happens or not…

A and B are dependent!

غير مستقلين

Defining Independence

- We can redefine independence in terms of conditional probabilities:

Two events A and B are **independent** if and only if

$$\mathbf{P(A|B) = P(A)} \quad \text{or} \quad \mathbf{P(B|A) = P(B)}$$

Otherwise, they are **dependent**.

- Once you've decided whether or not two events are independent, you can use the following rule to calculate their intersection.

Multiplication Law for Independent Events

- The multiplication law also can be used as a test to see if two events are independent.
- The law is written as:

$$P(A \cap B) = P(A)P(B)$$

The Multiplicative Rule for Intersections

- For any two events, **A** and **B**, the probability that both **A** and **B** occur is

$$P(A \cap B) = P(A)\,P(B \text{ given that } A \text{ occurred}) = P(A)P(B|A)$$

- If the events **A** and **B** are independent, then the probability that both **A** and **B** occur is

$$P(A \cap B) = P(A)\,P(B)$$

Example 1

In a certain population, 10% of the people can be classified as being high risk for a heart attack. Three people are randomly selected from this population. What is the probability that exactly one of the three are high risk?

Define H: high risk N: not high risk

P(exactly one high risk) = P(HNN) + P(NHN) + P(NNH)

= P(H)P(N)P(N) + P(N)P(H)P(N) + P(N)P(N)P(H)

$= (.1)(.9)(.9) + (.9)(.1)(.9) + (.9)(.9)(.1) = 3(.1)(.9)^2 = .243$

Example 2

Suppose we have additional information in the previous example. We know that only 49% of the population are female. Also, of the female patients, 8% are high risk. A single person is selected at random. What is the probability that it is a high risk female?

Define H: high risk F: female

From the example, $P(F) = .49$ and $P(H|F) = .08$.
Use the Multiplicative Rule:

$$P(\text{high risk female}) = P(H \cap F)$$

$$= P(F)P(H|F) = .49(.08) = .0392$$