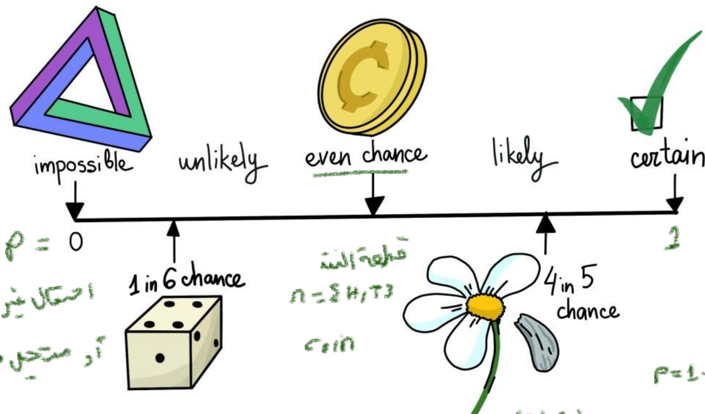
لجان الدفعات

STATISTICS

MORPHINE ACADEMY

مقدمة في الاحتمال

Introduction to Probability



P = 0

احتمال غير ممكن أو مستحيل حدوثه

قطعة النقد
Ω = {H, T}
coin

1

P=1 → certain
مؤكد حدوثه

احتماله أقل من 0.5
يعني حدوثه غير محتمل الحدوث

احتمال أكبر من 0.5
يعني هناك تحقق حدوثه

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قيم الاحتمال تنحصر من 0 إلى 1

Introduction to Probability

- **Experiments, Outcomes, Events and Sample Spaces**

simple event
نتيجة واحدة لكل حدث

compound
أكثر من نتيجة في الحدث

- **What is probability?**
- **Basic Rules of Probability**

مهمة قواعد

- **Probabilities of Compound Events**

حادث مركب: أكثر من نتيجة للحدث

Introduction to Probability

Why Learn Probability?

لا شيء مؤكد الحدوث

- Nothing in life is certain. In everything we do, we gauge the chances of successful outcomes, from business to medicine to weather.

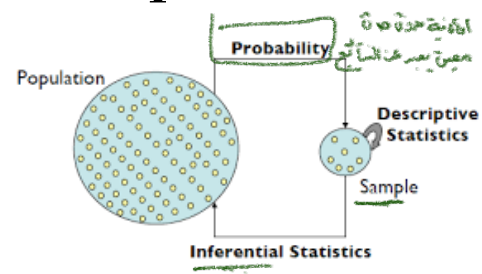
نقيس الفرص النتائج الناجحة

في جميع المجالات لا يوجد شيء مؤكد فقط نقيس فرص والنتائج الناجحة

- A probability provides a quantitative description of chances or likelihoods associated with various outcomes.

يزود الإحتمالات بوصف كمية للإحتمالات والفرص المرتبطة بالنتائج

- It provides a bridge between descriptive and inferential statistics.



الإحتمال: هو حلقة وصل بين الإحصاء الإستدلالي والوصفي.

Basic Probability Concepts

المفاهيم الأساسية للإحتمالات

Experiment

التجربة

- An **experiment** is the process by which an observation (or measurement) is obtained.

* عملية حساب أو الحصول على ملاحظات أو مقاييس

تنقسم التجربة الى نوعين :-

- **Deterministic** Vs **non-Deterministic experiment**

تجربة غير عشوائية (محدودة)

➢ Deterministic or Non-Random experiment

➢ Non-Deterministic or Random experiment

تجربة عشوائية (غير محدودة)

deterministic
experiment -تكون
نتيجته مؤكدة

non-determinsitic
expermint -تكون
نتيجته غير مؤكدة وتعتمد
على نتائج الحدث

Experiment

- **Deterministic** or **Non**-Random experiment

تجربة حتمية (مؤكدة)

- In **deterministic** experiment, the outcome can be predicted exactly in advance by using a mathematical mode that allows a perfect prediction of the phenomena's outcome.

رياضي نموذج

النتائج يمكن التنبؤ بها مسبقًا وبدقة عبر تعويض الكميات المقاسة بالمعادلات الرياضية والفيزيائية

- Many examples exist in physics and science.

العلاقات الرياضية والفيزيائية: أمثلة على deterministic exp

- Example: Force = mass* acceleration. So if we are given values for mass and acceleration, we exactly know the value of force.

Experiment (Phenomena)

تجربة غير حتمية (عشوائية)

كمثال دراسة نشاط ـ توزيعات
او التوزيع الاحتمالي للبيانات
وهو يعتمد على results

- **Non**-Deterministic or **Random** experiment
- In random experiment, no mathematical model exists that allows a perfect prediction of the experiment's outcome.
- In this case, we are unable to predict the outcomes, or they are not known exactly. However, in the long-run, the outcomes exhibit a statistical regularity, so we can describe the probability of the possible outcomes.

في التجربة العشوائية لا يوجد علاقات رياضية تسمح بالتنبؤ الدقيق بالنتائج للتجربة

في هذه الحالة لا نقدر على التنبؤ بالنتائج أو معرفتها بدقة لكن على المدى الطويل سيظهر للنتائج انتظام إحصائي يجعلنا قادرين على وصف احتمالية حدوث النتائج المحتملة

Experiment (Phenomena)

➢ **Examples:**

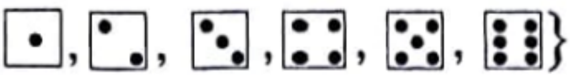
عملة نتائج التجربة (الاحتمالات) S ترمز الى جميع الاحتمالات الممكنة.

1. Tossing a coin, the outcomes S={**Head**, **Tail**}. In this case, we don't know exactly what we get, but in the long run we can calculate the probability and predict that 50% of the time we will get a head and 50% of the time we will get a tail.

قانون :مجموع الاحتمالات =1 ....يساعدنا في إيجاد احتمال مجهول لقيمة ما عبر طرح مجموع الاحتمالات من 1 فنجد الاحتمال المجهول

2. Rolling a die, so the outcomes

حجر النرد

S={}

S = تمثل جميع النتائج الممكنة

➢ We are unable to predict an outcome, but in the long run, we can determine that each outcome will occur 1/6 of the time.

احتمال ظهور الرقم 7 عند إلقاء حجر نرد=0 ،

احتمال ظهور رقم أقل من 7=1 اي 6/6=1

we guessing but not certain

لا نقدر على التنبؤ بالنتيجة لكن يمكن حساب احتمالية حدوثها ،

The Sample Space (S)

- The sample space (S or Ω) for a random phenomena (random experiment) is the set of all possible outcomes.

Ω الفضاء العيني: هو جميع عناصر الحادث الممكنة للتجربة العشوائية
the sample space

تقتصر دراستنا على النوع الأول

- The sample space S may contain:

الفضاء العيني يحتوي على:

محدود عدد من النواتج

1. A finite number of outcomes

نتائج محدودة مثل حجر النرد

2. A countably infinite number of outcomes, or

نستطيع عدها لكن غير محدودة مثل اختيار مجموعة اعداد من (1-100) , كتابة ال fraction الكسور إليها.

3. An uncountably infinite number of outcomes.

لا نستطيع عدّها , هي غير محدودة مثل كتابة ارقام حسابات على الآلة الحاسبة إلى اللانهاية


MORE EXAMPLES
PLEASE

The Sample Space (S)

هنا كل تجربة مستقلة + الإحتمال ثابت لكل عنصر

- **Examples of a sample space (S)**

1. Tossing a coin. Outcomes, $S = \{Head, Tail\} = \{H, T\}$
2. Tossing two coins once. Outcomes $S = \{HH, HT, TH, TT\}$. So number of outcomes $= n(S) = 2*2 = 4$.
3. Tossing three coins once. Then, outcomes:

   $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

   coins

   Number of outcomes $= n(S) = 2*2*2 = 8$ or S^n.
4. Rolling a die, then outcomes $S = \{1, 2, 3, 4, 5, 6\}$,

   So number of outcomes $= n(S) = 1*6 = 6$.

قانون عدد النتائج S

Number of outcomes $= S^{(n)}$

تمثل n : عدد العملة الملقى

الاحتمال عبارة عن $= \dfrac{\text{عدد مرات ظهور العنصر}}{\text{عدد عناصر الفضاء العيني}}$

The Sample Space (S)

5. Rolling two dice, the outcomes are:

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

سؤال:
احتمال ظهور 1
في حجرين نرد هو
$P(A) = \frac{1}{36}$

So number of outcomes S = n (S) = 6*6 = 36

number of outcome = $s^n = 6^2 = 36$ outcomes
للتجربة المشتركة

s: عدد العناصر للحادث

n: عدد التكرار

The *mn* Rule

- If an experiment is performed in two stages, with ***m*** ways to accomplish the first stage and ***n*** ways to accomplish the second stage, then there are ***mn*** ways to accomplish the experiment.
- This rule is easily extended to ***k*** stages, with the number of ways equal to

$$n_1 n_2 n_3 \ldots n_k$$

Example: Toss two coins. The total number of simple events is:

$$2 \times 2 = 4$$

$s^n = 2^2 = 4$

نتائج الرمية ①

نتائج الرمية ②

Examples

Example: Toss three coins. The total number of simple events is:

حدث بسيط

$$2 \times 2 \times 2 = 8$$

Example: Toss two dice. The total number of simple events is:

$$6 \times 6 = 36$$

~~**Example:** Two M&Ms are drawn from a dish containing two red and two blue candies. The total number of simple events is:~~

$$\cancel{4 \times 3 = 12}$$

محذوف

The Event (E)

certain 100%
Propability = 1

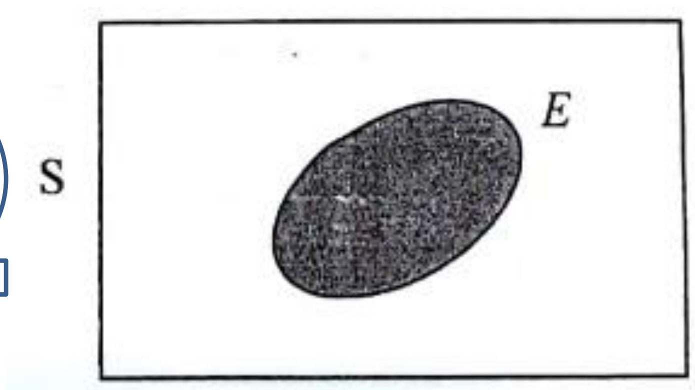
impossible 0%
propability = 0

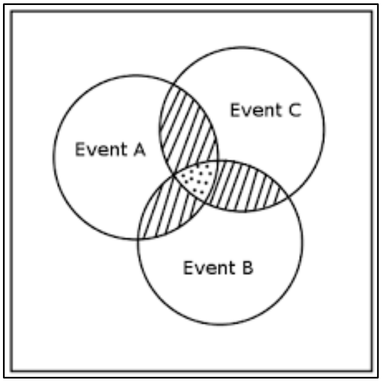
An Event, E

اي جزء من الفضاء العيني (هو حدث معين وله احتمال ونسبة)

- The event, which is denoted by E, is any subset of the sample space, S, so it is any set of outcomes (not necessarily all outcomes) of the random phenomena.
- The event, E, can be denoted by A, B, C, D,....

Venn diagram





Types of Events

أنواع الحوادث في الاحتمالات :-

- There are four types of events as follows:

حدث مستحيل

1. **Null (impossible) event (ϕ)**

رمز فاي ويرمز لاستحالة حدوث ال event

حدث مؤكد

2. **Entire (sure or certain) event (S)**

رمز S يعني تأكيد حدوثه

3. **Simple event**

حدث بسيط ← اي حدث له نتيجة واحدة كرمي حجر نرد وظهور العدد 1.

4. **Compound event**

حدث مركب ← اي حدث له اكثر من عنصر في النتيجة مثل رمي حجر نرد والحصول على اعداد فردية وبمثل هذه الحالة يكون الاحتمال

$P(A) = \frac{3}{6} = \frac{1}{2}$

ex : عند رمي حجر نرد :-

ظهور عدد اقل من 7 ← certain event

ظهور العدد 7 ← impossible event

Types of Events

1. The **null** event (the empty event) (ϕ), never occurs, impossible.

   $\phi = \{\ \} =$ the event that contains no outcomes

2. The **entire** (sure or certain) event, the sample space (S), S = the event that contains all outcomes

   So the sure event, S, always occurs.

Types of Events

3. A **simple** event is the outcome that is observed on a single repetition of the experiment.

- One and only one simple event can occur when the experiment is performed.
- Simple event is denoted by E with subscript, e.g. E_1, E_2, ..etc.

> **The set of all simple events of an experiment is called the sample space, S.**

الحادث البسيط هو النتيجة الواحدة التي ستظهر عند القيام بالتجربة مرة واحدة فقط

4. **Compound** event is the outcome that contains more than one simple event when the experiment is performed.

الحادث المركب يحتوي على اكثر من نتيجة عند القيام بتجربة

Example

- **The die toss:**
- Simple events:

الحادث البسيط
ظهور أي الأعداد عند
رمي النرد

Outcome	Simple event
1	E_1
2	E_2
3	E_3
4	E_4
5	E_5
6	E_6

Sample space:

جميع العناصر المحتمل ظهورها

$$S = \{E_1, E_2, E_3, E_4, E_5, E_6\}$$

S

$\bullet E_1$ $\bullet E_3$ $\bullet E_5$

$\bullet E_2$ $\bullet E_4$ $\bullet E_6$

Event

- An **event** is a collection of one or more **simple events.**

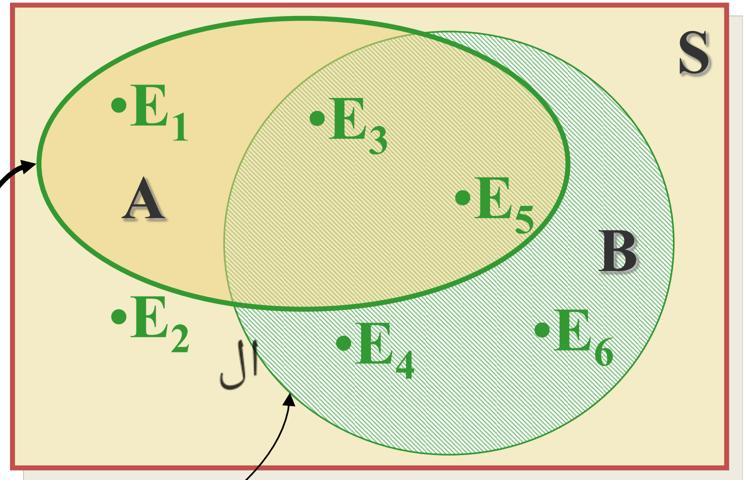
ال event هو واحد أو أكثر من الحوادث البسيطة.

Compound Event:

- **The die toss:**
  - –A: an odd number
  - –B: a number > 2

$A = \{E_1, E_3, E_5\}$

$B = \{E_3, E_4, E_5, E_6\}$



Examples of Event

إلقاء حجر النرد

- Rolling a die, then outcomes, S = {1, 2, 3, 4, 5, 6}

➢ **A** = The event that the number comes up is greater than 5 = {6} which is simple event.

➢ **B** = The event that the number comes up is an even number = {2, 4, 6}, which is a compound event. and odd numb

➢ **C** = Then event that the number comes is less than 7 = {1, 2, 3, 4, 5, 6}, which is a sure event.

➢ **D** = The event that the number comes up is greater than 6 = { } = ϕ, which is an impossible event.

Event Relations

Some Basic Relationships of Probability

There are some <u>basic probability relationships</u> that can be used to compute the probability of an event without knowledge of all the sample point probabilities.

- Union of Two Events

اتحاد حادثين اي جميع العناصر ما عدا المشتركه

- Intersection of Two Events

التقاطع: العناصر المشتركة بين حادثين

- Complement of an Event

حادث متمم

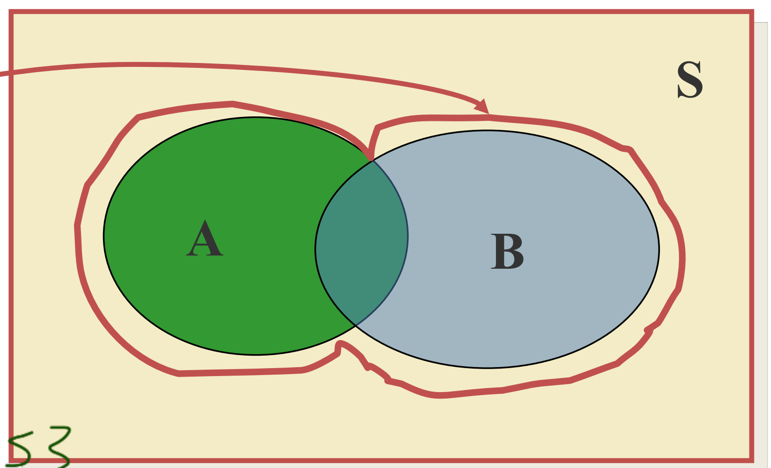
- Mutually Exclusive Events

حادثان مستحيل اجتماعهما

Union of Two Events

- The **union** of two events, A and B, is the event that either A **or** B **or both** occur when the experiment is performed. We write

A ∪ B



union

$A \cup B$

اتحاد

- example :

$A = \{1,2,3,4,5\}$

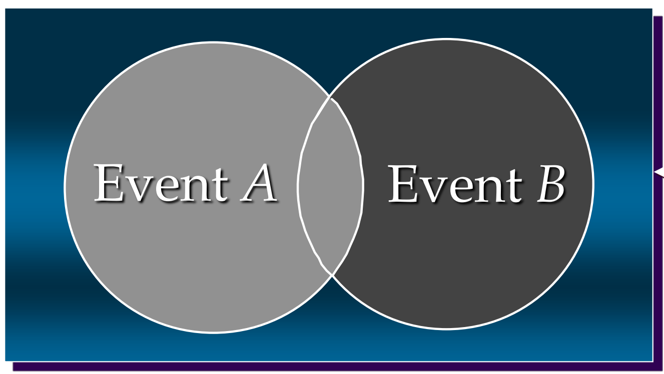
$B = \{4,5,6,7,8\}$

كل العناصر الموجودة بـ A و B بدون تكرار

أي $A \cup B = \{1,2,3,6,7,8\}$

Union of Two Events

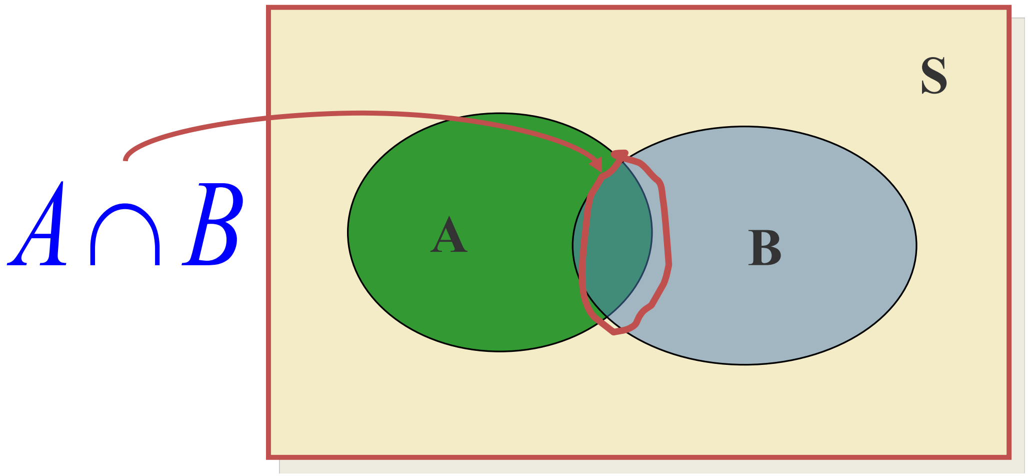
- The <u>union</u> of events A and B is the event containing all sample points that are in A or B or both.
- The union of events A and B is denoted by $A \cup B$.



Sample Space S

Intersection of Two Events

- The **intersection** of two events, **A** and **B**, the event that both A **and** B occur when the experiment is performed. We write **A ∩ B**.



تقاطع

العناصر المشتركة

بين A و B فقط

$A = \{2, 4, 6\}$

$B = \{2, 6\}$

- If two events A and B are **mutually exclusive,** then $P(A \cap B) = 0$.

$A \cap B = \{2, 6\}$

$P(A \cup B) = 1$

Intersection of Two Events

- The <u>intersection</u> of events A and B is the set of all sample points that are in both A and B.
- The intersection of events A and B is denoted by $A \cap B$.

مثال .. مهم جداً

A = male

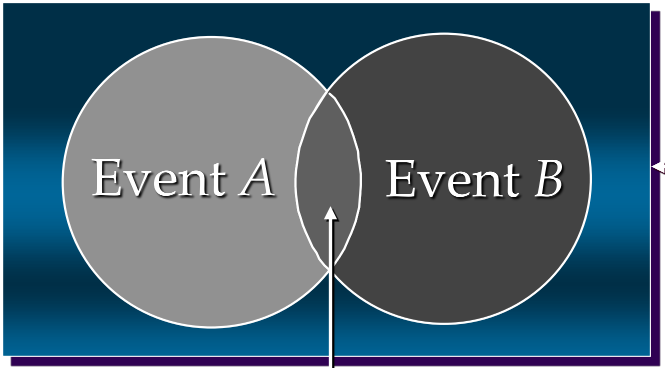
A^c = female

$A \cap A^c = \phi$

$P(A) = 0$

اجتماع صفة (الذكر والانثى)

Impossible صفر = قطعاً



Sample Space S

Intersection of A and B

اجتماع صفة الذكر والأنثى
أو إجتماع ظهور عدد
فردي وزوجي عند إلقاء
حجر النرد/أيضا
المتممة ..... أمثلة على
mutually exclusive

probability = 0

Complement of an Event

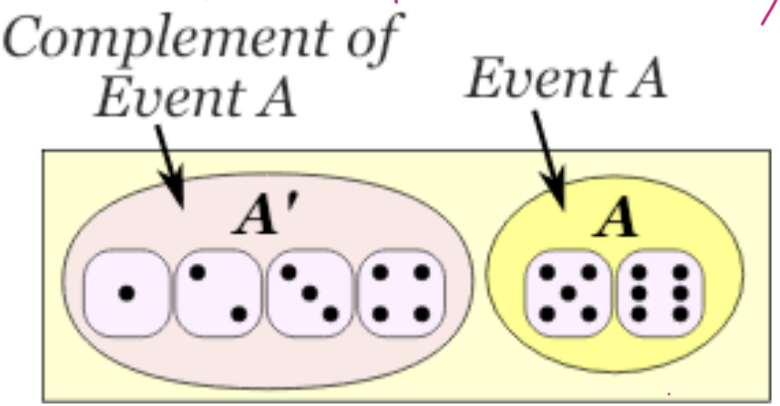
- The **complement** of an event **A** consists of all outcomes of the experiment that do not result in event A. We write $\mathbf{A^c}$ **or** $\bar{\mathbf{A}}$.

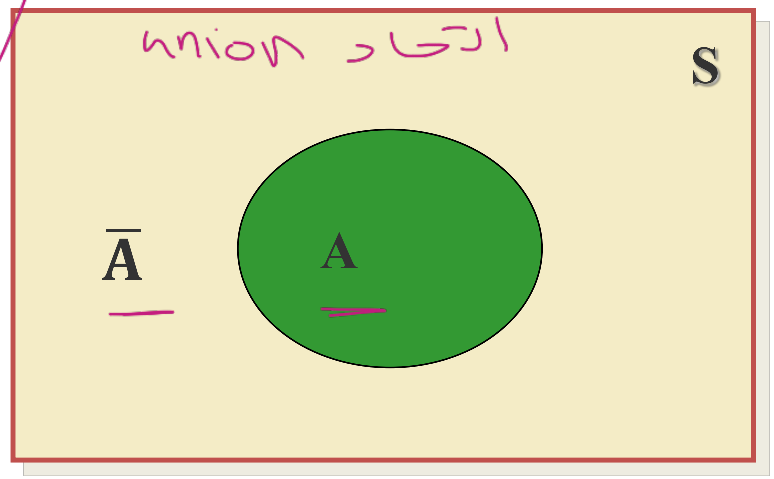
$A = \{1, 2\}$ / $A^c = \{3, 4, 5\}$

$A \cup A^c = S$ or 1 احتماله

$A \cap A^c = \phi$ or zero

union اتحاد





Complement of an Event

- The complement of event A is defined to be the event consisting of all sample points that are not in A.
- The complement of A is denoted by A^c.

Sample Space S

Venn Diagram

Example

- Select a student from the classroom and record his/her **hair color** and **gender**.
  - **A:** student has brown hair
  - **B:** student is female
  - **C:** student is male

في (A) لم يحدد ال gender

$B \cap C = \Phi$ or zero

حدثين مستحيل حدوثهما معًا

Mutually exclusive

B^c

Mutually exclusive; $B = C^C$

- What is the relationship between events **B** and **C**?

$A^c = \bar{A}$

- A^C: Student does not have brown hair

A^c or $\bar{A}$

- $B \cap C$: Student is both male and female = $\varnothing$
- $B \cup C$: Student is either male and female = all students = S

> complement B

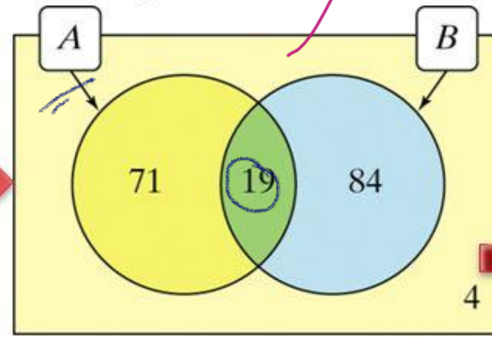
كل الشعبة

كل جزء في المستطيل يمثل كل عناصر الفضاء العيني S
لـ يقصد به الـ event

Venn Diagrams and Probability

Recall the example on gender and pierced ears. We can use a Venn diagram to display the information and determine probabilities.

	Pierced Ears?		
Gender	**Yes**	**No**	**Total**
Male	19	71	**90**
Female	84	4	**88**
Total	**103**	**75**	**178**



Define events *A*: is male and *B*: has pierced ears.

Coord.ngency table

Region in Venn diagram	In words	In symbols	Count
In the intersection of two circles	Male and pierced ears	$A \cap B$	19
Inside circle *A*, outside circle *B*	Male and no pierced ears	$A \cap B^C$	71
Inside circle *B*, outside circle *A*	Female and pierced ears	Female $A^C \cap B$	84
Outside both circles	Female and no pierced ears	$A^C \cap B^C$	4

هون احنا ذحسب الإحتمال

Event Relations

الاستدلال على رموز الاحتمالات من كلمات في السؤال

- Key words to recognise which event relation you should perform:

أو على الأقل

1. **Union**: if you see the word **or** or **at least one of** the two event occurs.

اتحاد

(و) كلا

2. **Intersection**: if you see the word **and** or **both events** occur.

تقاطع (المتكرر في الحدثين)

3. **Complement**: f you see the word **not**.

نفي الصفة (المتجهة لها) NO

De Moivre's Laws

قوانين الإحتمالات
ل ديمورقان

1. $\overline{A \cup B} = \bar{A} \cap \bar{B}$

The event A or B does not occur if the event A does not occur and the event B does not occur.

2. $\overline{A \cap B} = \bar{A} \cup \bar{B}$

The event A and B does not occur if the event A does not occur or the event B does not occur.

don't

Rules

- Roles involving the empty set (ϕ) and the entire event (S)

1. $A \cup \phi = A$
2. $A \cap S = A$
3. $A \cap \phi = \phi$
4. $A \cup S = S$
5. $A \cap \bar{A} = \phi$
6. $A \cup \bar{A} = S$

Events and Their Probabilities

An <u>event</u> is a collection of sample points.

The <u>probability of any event</u> is equal to the sum of the probabilities of the sample points in the event.

If we can identify all the sample points of an experiment and assign a probability to each, we can compute the probability of an event.

لا احتمال اي حدث هو مجموع احتمالات النتائج الفردية.

Assigning Probabilities to Events

Probability of an event P(E): "Chance" that an event will occur

- Must lie between 0 and 1
- "0" implies that the event will not occur
- "1" implies that the event will occur

الاحتمال يقع بين 0-1

الحدث المستحيل وقوعه احتماله 0

الحدث احتماله 1 مؤكد حدوثه

Types of Probability:

انواع وطرق حساب الاحتمالات

- **Objective**
  - ✓ Relative Frequency Approach تكرار نسبي = $\frac{\text{عدد تكرار قيمة حدث}}{\text{مجموع التكرارات}} = \frac{f}{\Sigma f}$
  - ✓ Equally-likely Approach في حدث ما لكل عنصر احتمال متساوي =

- **Subjective** → يعتمد على التنبؤ والخبرة <- subjective/prediction

Relative Frequency Approach: Relative frequency of an event occurring in an infinitely large number of trials

عدد كبير غير محدد

Time Period	Number of Male Live Births	Total Number of Live Births	Relative Frequency of Live Male Birth
1965	1927.054	3760.358	0.51247
1965-1969	9219.202	17989.360	0.51248
1965-1974	17857.860	34832.050	0.51268

$R.F = \frac{f}{n} = \frac{f}{\Sigma f}$

Equally-likely Approach: If an experiment must result in n equally likely outcomes, then each possible outcome must have probability 1/n of occurring.

يكون كل حدث له احتمال متساوي

Examples:

1. Roll a fair die $= \frac{1}{6}$
2. Select a SRS of size 2 from a population

اختيار عنصرين من عينة المجتمع

اختيار عينة عشوائية بسيطة

Subjective Probability: A number between 0 and 1 that reflects a person's degree of belief that an event will occur predictive

تنبؤات الطقس

احتمال يعتمد على الاعتقاد الشخصي

Example: Predictions for rain

Assigning Probabilities

Classical Method

Assigning probabilities based on the assumption of equally likely outcomes

Relative Frequency Method

Assigning probabilities based on experimentation or historical data

بيانات تجريبية

بيانات تاريخية

Subjective Method

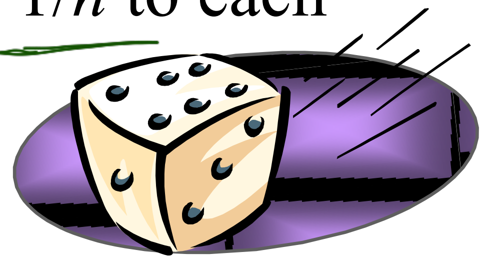
Assigning probabilities based on judgment

لا نستخدم الحسابات

Classical Method

If an experiment has n possible outcomes, this method

would assign a probability of $1/n$ to each outcome.



- Example
- Experiment: Rolling a die
- Sample Space: $S = \{1, 2, 3, 4, 5, 6\}$

Probabilities: Each sample point has a $1/6$ chance of occurring

مثل حجر النرد

الطريقة التقليدية ..لتجربة لها عدد نتائج محددة تكون

الاحتمالات متساوية لكل عنصر في التجربة =n/1

Relative Frequency Method

Each probability assignment is given by dividing the frequency (number of days) by the total frequency (total number of days).

Number of Polishers Rented	Number of Days	Probability (relative frequency)
0	$f =$ 4	.10
1	6	.15
2	18	.45
3	10	.25
4	2	.05
	$\Sigma f =$ 40	1.00

4/40

$$R.f. = \frac{f}{\Sigma f}$$

relative frequency = $\dfrac{\text{no. of days}}{\text{total no. of days}}$

Subjective Method

مثل البورصة

predictive depend on experence not Calculation

Applying the subjective method, an analyst made the following probability assignments.

Exper. Outcome	Net Gain or Loss	Probability
(10, 8)	\$18,000 Gain	.20
(10, –2)	\$8,000 Gain	.08
(5, 8)	\$13,000 Gain	.16
(5, –2)	\$3,000 Gain	.26
(0, 8)	\$8,000 Gain	.10
(0, –2)	\$2,000 Loss	.12
(–20, 8)	\$12,000 Loss	.02
(–20, –2)	\$22,000 Loss	.06