لجان الدفعات

STATISTICS

MORPHINE ACADEMY

Pharmaceutical Statistics

Lecture 3

Descriptive statistics

Indicators of central tendency

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هاي المحاضرة بتشرح كيف نوصف البيانات باستخدام أدوات احصائية خصوصًا
mean المتوسط
median الوسيط
mode المنوال

Measures of Central Tendency

- A measure of central tendency is a measure which indicates where the value at the center or middle of a data set is.
- The three most commonly used measures of central tendency are: Mean, Median, and Mode.

The Mean or Arithmetic Mean or Average

The number obtained by adding the values and dividing the total by the number of values

- **Notation**
  - **∑ (sigma) denotes the sum of a set of values**
  - ***x* is the variable usually used to represent the individual data values**
  - ***n* represents the number of values in a *sample***
  - ***N* represents the number of values in a *population***

- Mean of a set of sample values (read as x-bar)- formula

$$\bar{x} = \frac{\sum x}{n}$$ sample

- Mean of all values in a population (read as "mu")

$$\mu = \frac{\sum x}{N}$$ population

Mean

- **For the population mean of N values: $X_1, X_2, X_3, \ldots\ldots, X_N$, the mean is calculated as:**

$$\mu = \frac{\sum_{i=1}^{N} X_i}{N}$$

❖ **which is usually unknown, that's why we use the sample mean to estimate or approximate it.**

- **For the sample mean of n values (ungrouped or raw data) (: $X_1, X_2, X_3, \ldots\ldots, X_n$, the mean is calculated as:**

$$\overline{X} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Mean

Example:

Here is a sample of size 10 of ages, where

$\chi_1 = 42$, $\chi_2 = 28$, $\chi_3 = 28$, $\chi_4 = 61$, $\chi_5 = 31$, $\chi_6 = 23$, $\chi_7 = 50$, $\chi_8 = 34$, $\chi_9 = 32$,

$\chi_{10} = 37$.

Calculate the arithmetic mean?

$\overline{x}$ = (42 + 28 + … + 37) / 10 = 36.6

Sample mean of grouped or classified data (data that is placed in intervals)

Unlike listed data, the individual values for grouped data are not available and you are not able to calculate their sum. So to calculate the mean of grouped data in frequency table, the following procedure is used:

1. Find the midpoint for each class interval (X_i, i=1,2,…,K)
2. For each class multiply the frequency with each midpoint ($f_i * X_i$).
3. Find the sum $\sum (f_i * X_i)$
4. Divide the sum in step-3 by the sum of frequencies ($n=\sum f_i$) to get the sample mean as follow:
5. Mean= $\sum (f_i * X_i)/ n$

هاي الخطوات
هلأ بنشرحها
بالمثال

Worked Example

Calculating the Mean

In Tim's school, there are 25 teachers. Each teacher travels to school every morning in his or her own car. The distribution of the driving times (in minutes) from home to school for the teachers is shown in the table below:

Driving Times (minutes)	Number of Teachers
0 to less than 10	3
10 to less than 20	10
20 to less than 30	6
30 to less than 40	4
40 to less than 50	2
	25

The driving times are given for all 25 teachers, so the data is for a population. Calculate the mean of the driving times.

Worked Example

Solution

- **Step 1:** Determine the midpoint for each interval.
- For 0 to less than 10, the midpoint is 5. $\frac{0+10}{2}=5$
- For 10 to less than 20, the midpoint is 15. $\frac{10+20}{2}=15$
- For 20 to less than 30, the midpoint is 25. $\frac{20+30}{2}=25$
- For 30 to less than 40, the midpoint is 35.
- For 40 to less than 50, the midpoint is 45.

1 $\frac{\text{Lower}+\text{upper}}{2}=\text{midpoint}$

- **Step 2:** Multiply each midpoint by the frequency for the class.
- For 0 to less than 10, (5)(3)=15
- For 10 to less than 20, (15)(10)=150
- For 20 to less than 30, (25)(6)=150
- For 30 to less than 40, (35)(4)=140
- For 40 to less than 50, (45)(2)=90

2 midpoint * Frequency (التكرارات)

- **Step 3:** Add the results from Step 2 and divide the sum by 25.
- 15+150+150+140+90=545

3 نجمعهم

$\mu=545/25=21.8$

4 $\frac{\text{ناتج الجمع}}{\text{عدد التكرارات}}$

- Each teacher spends a mean time of 21.8 minutes driving from home to school each morning.

Worked Example

To better represent the problem and its solution, a table can be drawn as follows:

Driving Times (minutes)	Number of Teachers f	Midpoint Of Class x	Product xf
0 to less than 10	3	$\frac{0+10}{2}=$ 5	mid.x Freq = 15
10 to less than 20	10	$\frac{10+20}{2}=$ 15	15x10 = 150
20 to less than 30	6	25	150
30 to less than 40	4	35	140
40 to less than 50	2	45	90

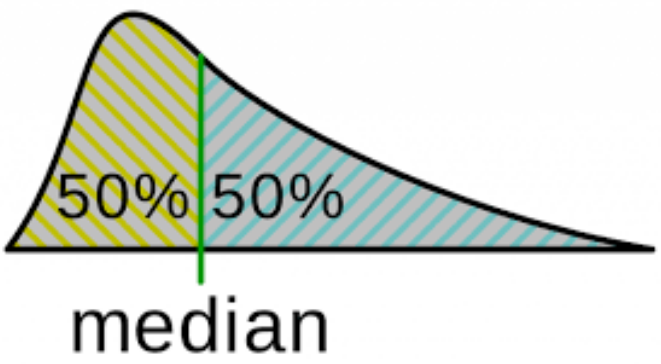
For the population, N=25 and ∑xf=545, so using the formula μ=∑xf/N, the mean would again be μ=545/25=21.8

Median (MD)

القيمة اللي بتقسم البيانات الى نصفين متساويين

- When ordering the data, it is the observation that divide the set of observations into two equal parts such that half of the data are before it and the other are after it (in other words, the number of values greater than the median is equal to number of values is less than the median).

- The median either will be a specific value in the data set or will fall between two values.



The Median of Ungrouped or (Raw) Data

- If n is odd, the median will be the middle of observations. It will be the $(n+1)/2^{th}$ ordered observation.

✓ E.g. when n= 11, then the median is the 6^{th} observation.

- If n is even, there are two middle observations. The median will be the mean of these two middle observations. It will be the $(n+1)/2^{th}$ ordered observation.

✓ When n = 12, then the median is the 6.5^{th} observation, which is an observation halfway between the 6^{th} and 7^{th} ordered observation.

Example:

- For this random sample, the ordered observations will be as:

  23, 28, 28, 31, 32, 34, 37, 42, 50, 61.

  Since n = 10, then the median is the 5.5^{th} observation,

  i.e. = (32+34)/2 = 33.

Properties of the Median:

- **Uniqueness:** For a given set of data there is one and only one median.
- **Simplicity.** It is easy to calculate.
- **Robust measure, it is not affected by extreme values (outliers)** as is the mean.

The Median of Grouped Data-Simple Frequency Table

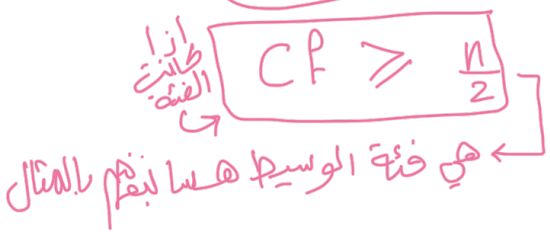
حساب الوسيط في جداول التكرار البسيطة

- To calculate the median for a simple frequency table, the following procedure is used:

Step-1: find the cumulative frequency (cf) التكرارات

Step-2: find the value of (n/2), where n is the total number of observations (the sum of frequencies)

Step-3: the median (MD) is the first value having cumulative frequency (cf) greater than or equal to (n/2)



The Median of Grouped Data-Simple Frequency Table

Example

The Gulf Pharmaceutical Industries Company in UAE planning to improve safety plan in its factory. For this, accident data for the last 50 weeks was compiled. These data were listed into the simple frequency table as shown below:

Number of Accidents (x)	2	7	12	17	22
Number of Weeks (f)	5	22	13	8	2

Calculate the median (MD) for the number of accidents per week?

Solution

Step-1: We calculate the value of cumulative frequency (cf) as follows:

No. of Accidents (x)	Frequency, f	cf
2	5	5
7	22	27
12	13	40
17	8	48
22	2	50
Total	50	

Should be Equals

Step-2: The value of:

$$n/2 = 50/2 = 25$$

Step-3: The median (MD) is the first value having cumulative frequency (cf) greater than or equal to 25, then from the table, the value of the median is MD = 7.

$CF \geq \frac{n}{2} = 25$

اذًا 7 هي قيمة الوسيط

⟹ Median (MD) = 7 accidents per week.

The Median of Grouped Data-Frequency Table (Distribution)

- To calculate the median for a grouped frequency table (frequency distribution), the following procedure is used:
- Step-1: construct the cumulative frequency distribution.
- Step-2: decide the class that contain the median (class median).

➢ Class median: is the first class having cumulative frequency (cf) greater than or equal to (n/2).

- Step-3: find the median (MD) by using the following formula:

$$\text{Median} = L_m + \left(\frac{\frac{n}{2} - F}{f_m} \right) i$$

اذا كان عندي مرات

where:

n = the total frequency.
F = the cumulative frequency before class median.
f_m = the frequency of the class median.
i = the class width.
L_m = the lower boundary (limit) of the class median.

The Median of Grouped Data-Frequency Table (Distribution)

- Example:

The following frequency table represents the time in minutes of 50 pharmacists to their work selected from the records of a given Pharmaceutical Industries Company in Jordan:

Time to travel to work	Frequency
1 – 10	8
11 – 20	14
21 – 30	12
31 – 40	9
41 – 50	7

Calculate the median (MD) for the time traveled to the work for pharmacists of this Pharmaceutical Industries Company ?

Solution

1st Step: Construct the cumulative frequency distribution for the data as follows:

Time to Travel to work Class	Frequency (f)	Cumulative Frequency (cf)
1 – 10	8	8
11 – 20	14	22
21 – 30	12	34
31 – 40	9	43
41 – 50	7	50

CF ≥ 50/2 = 25 — القيمة الأقل

2nd Step: We find the class median as follows:

Class Median

$$\frac{n}{2}=\frac{50}{2}=25 \longrightarrow$$ class median is the 3rd class

MD الـ CF السابقة للـ — عدد f الـ MD — القيمة الأقل — 0.5 — اعلى − اقل = 30 − 21 = 9 + 1 = 10

So, $F = 22$, $f_m = 12$, $L_m = 20.5$ and $i = 10$

21 − 0.5 = 20.5

3rd Step: The median (MD) is:

$$\text{Median} = L_m + \left(\frac{\frac{n}{2} - F}{f_m}\right) i$$

$$= 20.5 + \left(\frac{25 - 22}{12}\right) 10$$

$$= 23$$

20.5 + (25 − 22)/12 × 10

Conclusion

Thus, 25 persons take less than 23 minutes to travel to work and another 25 persons take more than 23 minutes to travel to work.

The Mode of Raw Data and Simple Frequency Table

Example

A survey on the Ministry of Health showed the following distribution for the number of tablets sold in May 2018 for five types of medications used to treat blood pressure:

Medicine Name	Number of Tablets Sold
Almor	632
Lasix	1425
Aldacton	878
Indicardin	95
Diovan	471

Solution

Since the category with the highest frequency is Lasix, then the mode for the number of tablets sold in May 2018 for the five types of medications used to treat blood pressure is the Lasix drug.

The Mode of Grouped Data (Frequency Table)

- For the grouped data in a frequency table, the modal class is the class with the largest (highest) frequency). To find the mode for grouped data, use the following formula:

اذا كان السؤال فقرات

$$\text{Mode} = L_{mo} + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2} \right) i$$

where

- L_{mo} = Lower boundary (limit) of the modal class.
- Δ_1 = The difference of frequency between modal class and class before it.
- Δ_2 = The difference of frequency between modal class and class after it.
- i = class width.

هسا بنشرح بالمثال ⟹

Example

The following frequency table represents the time in minutes of 50 pharmacists to their work selected from the records of a given Pharmaceutical Industries Company in Jordan:

Time to travel to work	Frequency
1 – 10	8
11 – 20	14
21 – 30	12
31 – 40	9
41 – 50	7

اقل قيمة في حدود الفترة

الرقم الأكثر تكرارا "14" نحل عليها

$$Mode = L_{mo} + \left(\frac{\Delta_1}{\Delta_1+\Delta_2}\right) i$$

كما يلاحظ

Calculate the mode for the time traveled to the work for pharmacists of this Pharmaceutical Industries Company ?

Solution

The modal class is the interval **11 – 20** because it has the largest frequency, then we have:

$L_{mo} = 10.5$, $\Delta_1 = (14 - 8) = 6$, $\Delta_2 = (14 - 12) = 2$ and $i = 10$

F F الى قبل F بعد

11 - 0.5

20 - 11 = 9 + 1 = 10

The Mode of Grouped Data (Frequency Table)

- Use the below formula to calculate the mode:

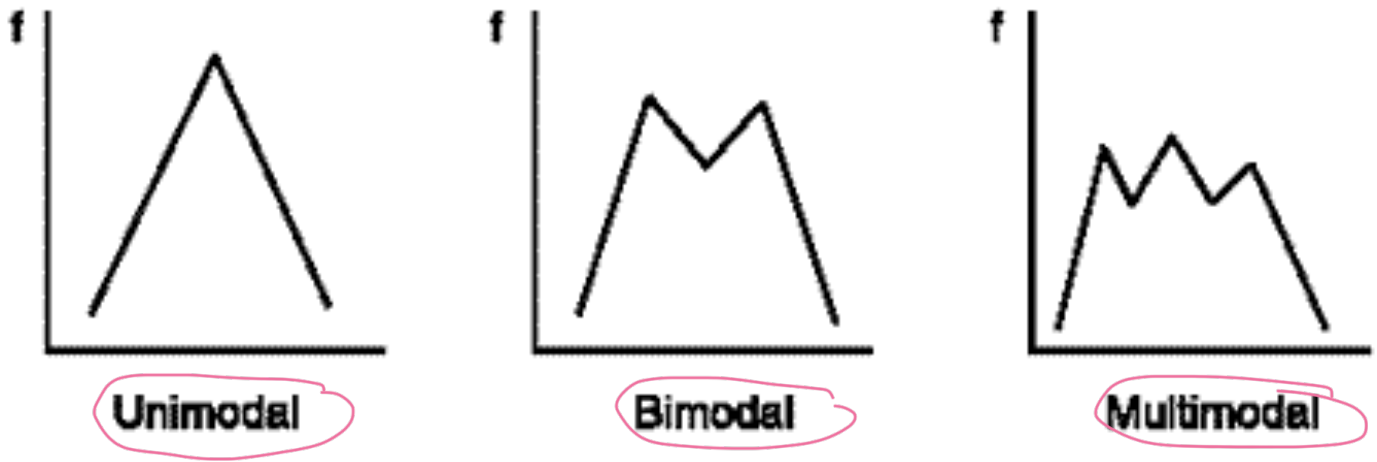
$$\text{Mode} = L_{mo} + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2} \right) i$$

$$\text{Mode} = 10.5 + \left(\frac{6}{6+2} \right) 10 = \text{ 18 minutes}$$

Some Definitions

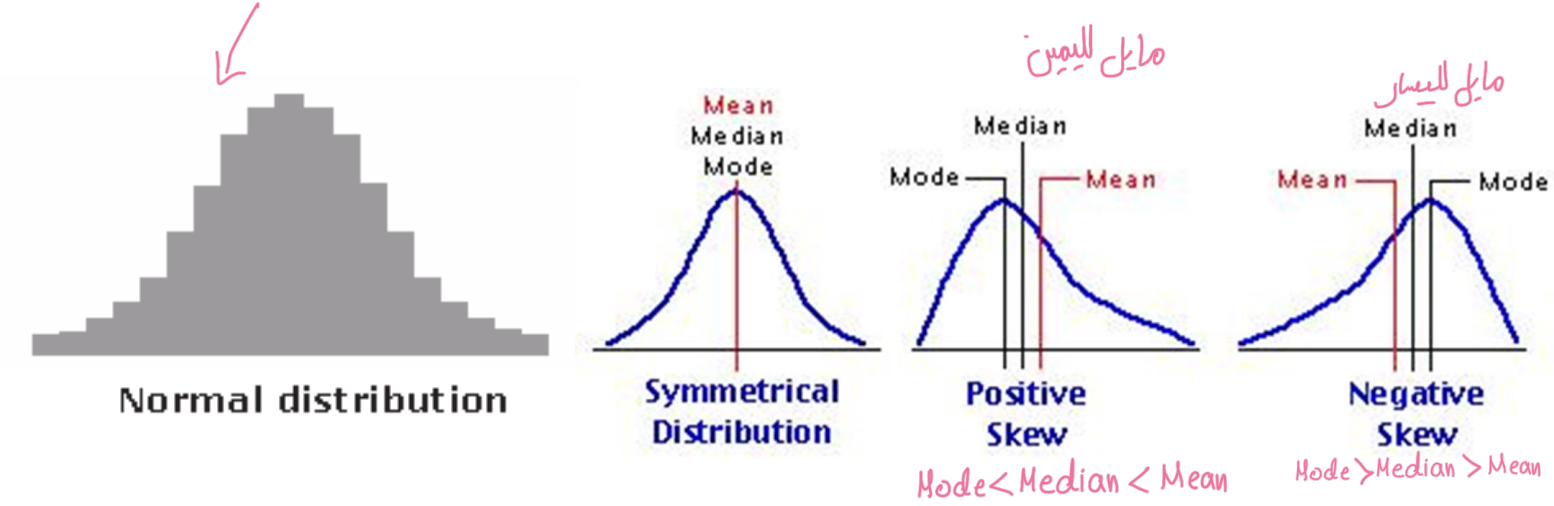
انواع التوزيع

- **Distributions** – (of a variable) tells us what values the variable takes and how often it takes these values.
- Unimodal- having a single peak
- Bimodal-having two peak
- Symmetric-left and right half are mirror images



How Mean, Median, and Mode are related?

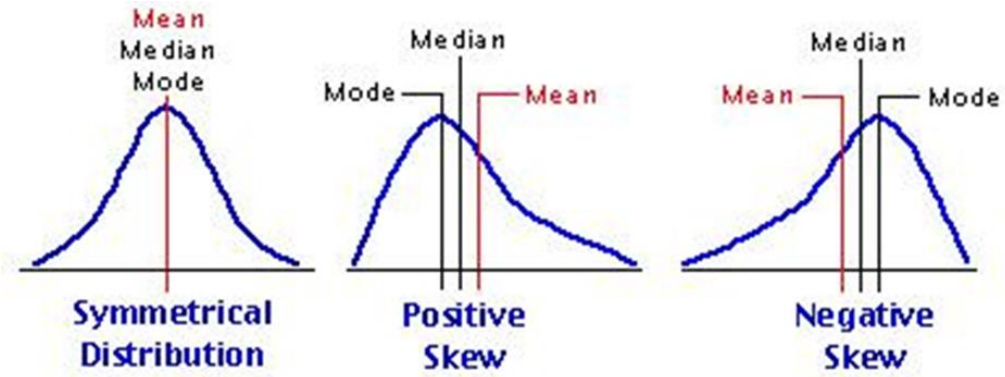
- A comparison of the mean, median, and mode can reveal information about the distribution shape
- A bell-shaped (normal) distribution is **symmetric**
  - Data values are evenly distributed on both sides of the mean
  - Unimodal (one peak)
  - Mean ≈ Median ≈ Mode



How Mean, Median, and Mode are related?

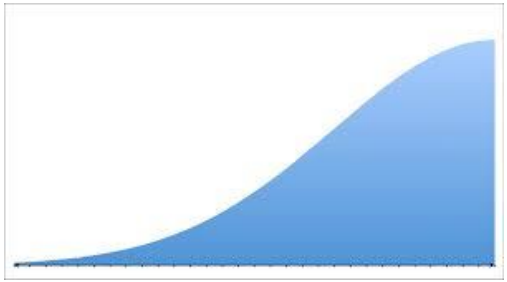
- Right-skewed (or positively) distribution has the majority of data values fall to the left of the mean and cluster at the lower end of the distribution; the "tail" is to the right
- Mode < Median < Mean
- Median is the "center" point

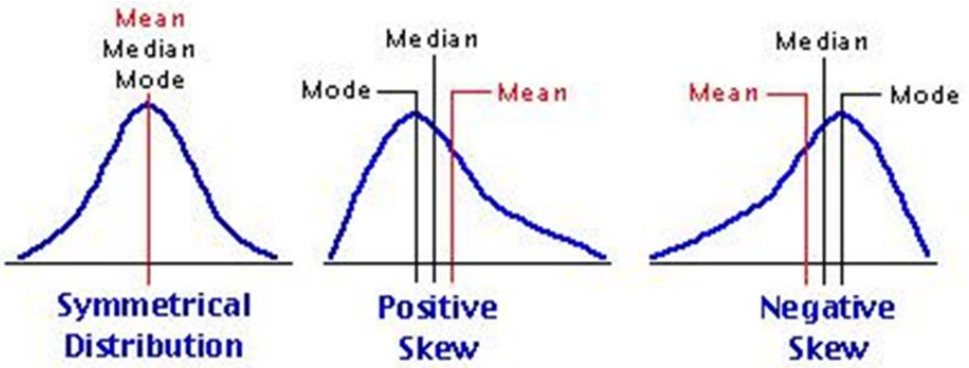
Right-skewed distribution



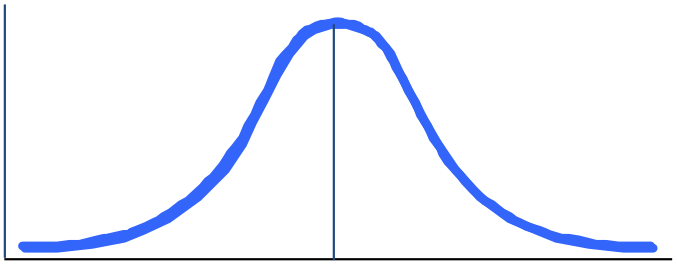
How Mean, Median, and Mode are related?

- Left-skewed (or negatively) distribution has the majority of data values to the right of the mean and cluster at the upper end of the distribution, with the tail to the left
- Mean < Median < Mode



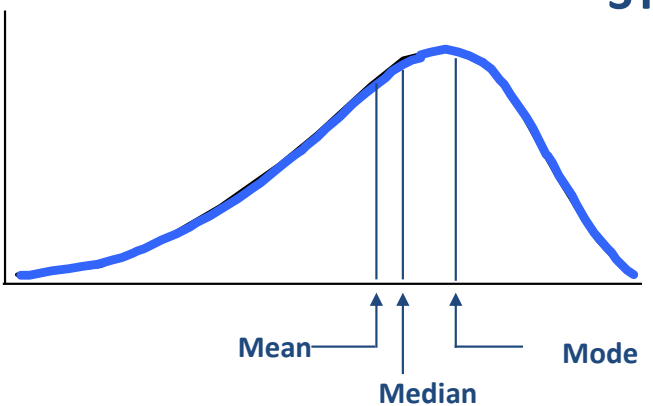


Skewness

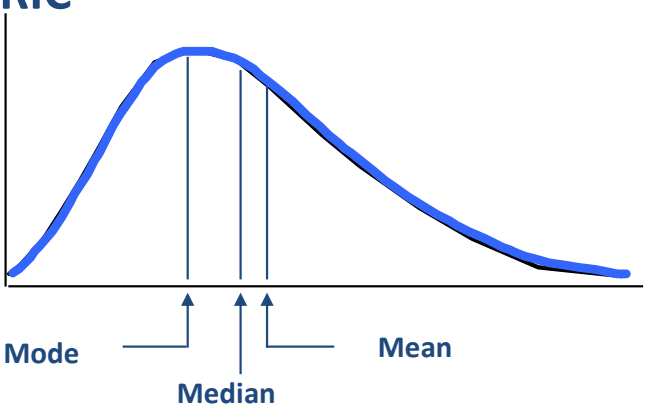


Mode = Mean = Median

SYMMETRIC



SKEWED LEFT
(negatively)



SKEWED RIGHT
(positively)

Which Measure of Central Tendency is best?

- There is no single best answer to that question because there are no objective criteria for determining the most representative measure for all data sets
- Avoid the term “average” , instead use the actual measure of central tendency that is calculated (mean, median, mode)
- Use the advantages and disadvantages stated above to decide which measure of central tendency is best.
- Large sample values tend to inflate the mean. This will happen if the histogram of the data is right-skewed.
- The median is not influenced by large sample values and is a better measure of centrality if the distribution is skewed.
- If mean=median=mode then the data are said to be symmetrical