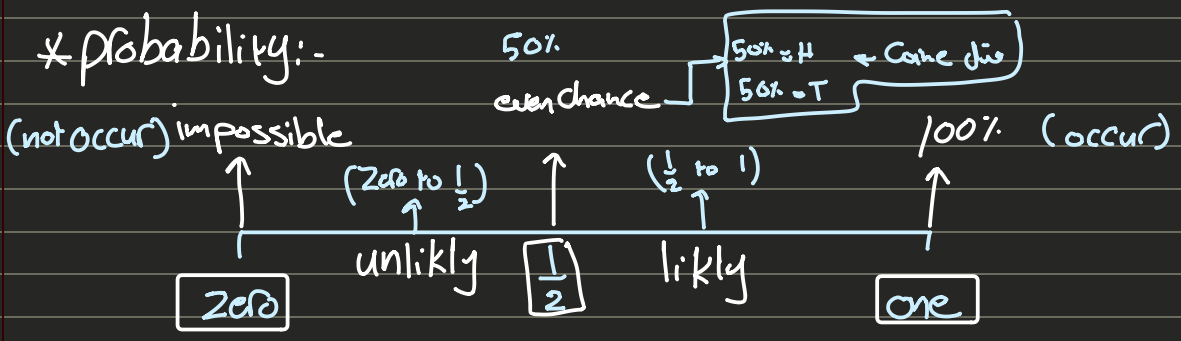
* probability :-



* probability الـ ← بتكون من (zero - one)

* probability

- provides quantitative description of Chance
- provides brige between descriptiv and inferential

Experiment → observation

→ 2 Type

Deterministic	non - Deterministic
→ Non random	→ Random
→ we are able predict out comes * متأكد من الاجابة	→ we are unable to predict out comes → They are not known exactly
(mathematical) مثل في	مثل: dice, Coin
Force = mass x acclearation لو كان معروف قيمتهم بقدر اعرف قيمة Force	على مدى طويل (long-run) ← بقدر اوصف احتمال الناتج

sample specc (S) :- (all) possible outcomes

- finite → معروف → في حدود الرد بقدر اعدهم
- infinite → Countably → مثل الاعداد الصحيحة (∞..-1, 0, 1...∞)
- infinite → uncontably → مثل الاعداد النسبية (كسور، جذور، اعداد عشرية)

Ex:-

1. Coine → Head $\frac{1}{2}$ = 50%
   → Tail $\frac{1}{2}$ = 50%

2. Die → {1, 2, 3, 4, 5, 6}

Number of outcomes $n(S) = 5^n$

Ex = 3 Coine

$n(S) = 2 \times 2 \times 2 = 8$ لان كل Coin الها احتمالين Head, Tial

* Evene :- any subset of sample space
→ (not necessrily all outcomems)

3 Type of Evenc

- impassible: ϕ — قبل حجر النرد فتعيل يطلع رقم 7 — no outcome
- Certain: (S) — في حجر النرد رقم اقل من 7 (S) — all outcomes always occur
- simple event: احتمال رقم 4 في حجر النرد — one simple (single)
- Compound event: يكون عبارة عن احتمال بحيث يكون عدد فردي {1, 3, 5} — more than one

* Union → (A ∪ B) (A or B) كل قيم A وقيم B

all sample point in A or B or both

تقاطع

* Intersection :- $A \cap B$ (A and B occur)

all sample points in both A and B

المشترك بين A , B

* Complement :- A^c , A' , $\bar{A}$

all sample points that are not in A

كل العناصر غير موجودة في A

even تمثل A قبل اذا كانت A

odd تمثل $\bar{A}$

Key words :-

1. Union → or , at least one
2. intersection → and , Both
3. Complement → not

Ex :- $\overline{A \cup B}$ → event A or B does not occur

$\bar{A} \cap \bar{B}$ → event A does not occur and event B does not occur

$A \cup \phi = A$

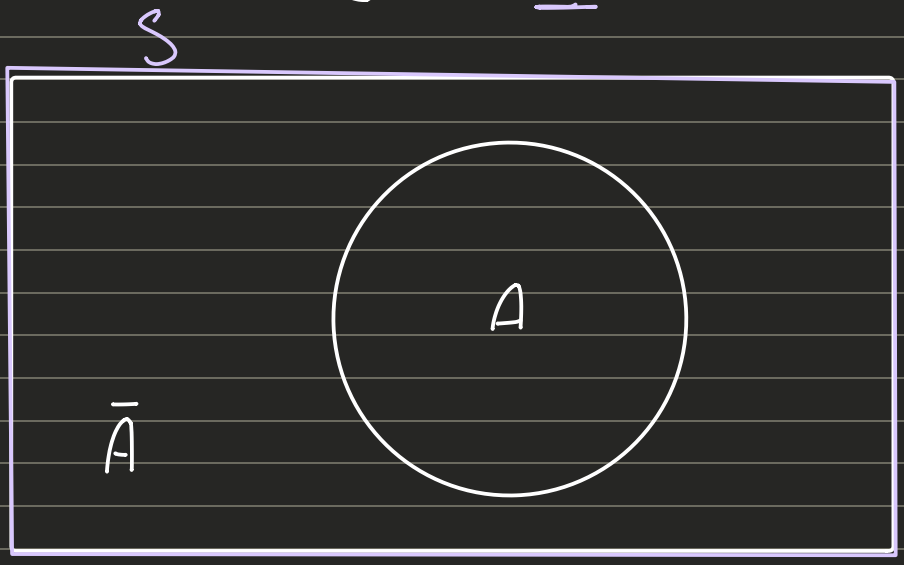
$A \cap \phi = \phi$

$A \cup S = S$

$A \cap S = A$

$A \cup \bar{A} = S$

$A \cap \bar{A} = \phi$



مجموع Sum ← Σ probability for all sample

event in S = 1

$S = \{1, 2, 3, 4, 5, 6\}$

probability $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6} = 1$

+ probabilites

اعتماداً على التحليل
او ملاحظات مسجلة

objectiv

- R.F — $1 = R.F$ مجموع
  - $P = \frac{n}{total}$
- Equally-likely
  - $P = \frac{1}{n}$
  - كل outcomes نفس probabilities

حسب الخبرة بتختلف
من شخص لشخص

subjectiv (judgment)

between (zero - one)

- never occurs
- always occurs

probability Rules:-

- $P(A) = \frac{n(A)}{n(S)}$ → عدد مرات حدوث A
- if $n(A) = \phi$ → $P(A) = \frac{n(A)}{n(S)} = \frac{zero}{n(S)} = 0$
- $P(S) = \frac{n(S)}{n(S)} = 1$

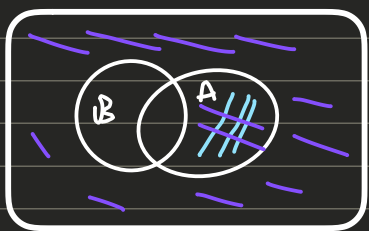
I. $P(A) + P(A') = 1$ → $P(A') = 1 - P(A)$

= مجموع ال probability

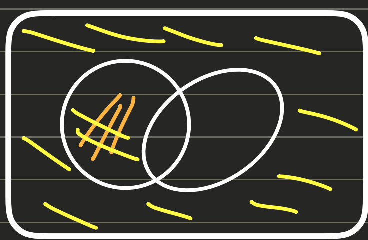
$P(A \cap \bar{A}) = 0$
$P(A \cup \bar{A}) = 1$

- red → just A
- blue → just B
- green → A and B
- pink → X A , X B (no A , no B)

- → A
- → $\bar{B}$



\# = $A \cap \bar{B}$



- → $\bar{A}$
- → B
- \# → $\bar{A} \cap B$

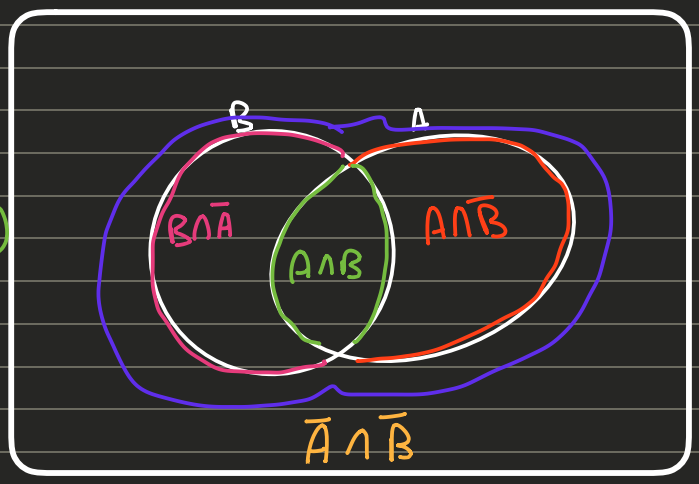
2. $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

3. $P(B \cap \bar{A}) = P(B) - P(A \cap B)$

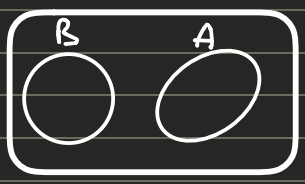
4. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

5. $(\bar{A} \cap \bar{B}) = \boxed{1 - P(A \cup B)}$

$P(\overline{A \cup B})$ نفسها



$P(A \cap B) = 0$ ← mutually exclusive الحدثان المتنافيان :note *



* $P(A \cup B) = P(A) + P(B) - \cancel{P(A \cap B)}$

zero =

مشروط

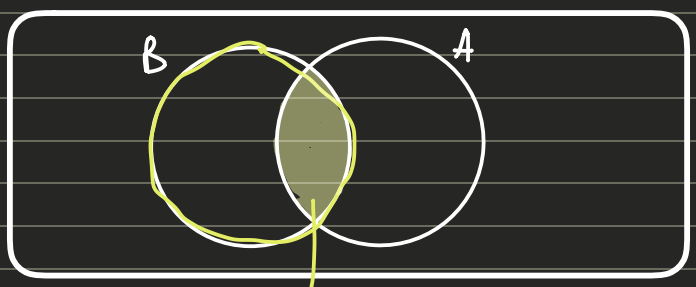
* Conditional probability:

صو احتمال حدث بناءً على وقوع حدث سابق

event B has occured

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(A|B) \cdot P(B)$$



Calculating $P(A \cap B)$

independent

$P(A \cap B) = P(B) \cdot P(A)$

dependent

$P(A \cap B) = P(A) \cdot P(A|B)$

Calculating $P(A|B)$

independent

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \rightarrow \frac{P(A) \cdot P(B)}{P(B)} = P(A)$$

$$\boxed{P(A|B) = P(A)}$$

dependent

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$$

* Random variable:.

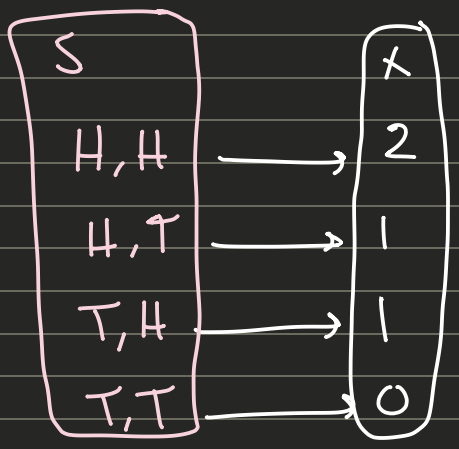
* الفكرة هو تحويل Sample space ← (Random variable) الى ارقام numeric outcome

مثال :- 2 Coins

$$S = \{(H,H), (H,T), (T,H), (T,T)\}$$

X = number of Head بمثل H

$X(H,H) = 2$, $X(H,T) = 1$, $X(T,H) = 1$, $X(T,T) = 0$



$P(H,H) = \frac{1}{4}$

$P(X=2) = \frac{1}{4}$

Random variable (Quantitativ variabl

Discrete

مجزية

- Set of outcome
  - finite
  - countably infinite
- fraction are not accepted

مثل في حجم النزل مستحيل يطلع رقم

1,5 or 2,5

Continuous

- any value
  - infinit uncontably
- fraction are accepted

مثل وزن الطلاب ممكن يكون

55,5

* الهدف حساب probability لـ Discrete RV

← من خلال تحويل لـ الى الرقم بقدر اقلها Histogram

or Table

or (X, P(x))

note :-

$$0 \leq P(X=x) \leq 1$$

$$\Sigma P(X=x) = 1$$

Random variable ↵ ↳ Value of Random varible

:- probability جاهز وطلب Table ازاكنى ✗

X	0	1	2	3
P(X=x)	A	B	C	D

1. exactly 2 $\longrightarrow P_{(X=2)} = C$

2. less than 2 $\longrightarrow P(X<2) = P_{(X=0)} + P_{(X=1)}$
$A + B$

3. has 2 or less than 2 $\longrightarrow P_{(X \leq 2)} =$
$P_{(X=0)} + P_{(X=1)} + P_{X=2} = A+B+C$

4. more than 2 $\rightarrow P_{(X>2)} = P_{(X=3)} = D$

5. has 2 or more than 2 $\rightarrow P_{(X \geq 2)} = P_{(X=2)} + P_{X=3)} = C+D$

6. has more 1 and 3 or less than 3

$\rightarrow P(1 < X \leq 3) = P_{(X=2)} + P_{(X=3)} = C+D$

more ↵ ↳ 3 or more

Calculation :-

1. Mean (Expected value)

$$\mu = E(x) = \Sigma x\,p(x)$$

x	p(x)	x p(x)
x	=	
x	=	+
x	=	+
Total		$\Sigma x\,p(x)$

$\mu = E(x)$

[2] Standard deviation:-

$\sigma = \sqrt{\sigma^2}$ ← Variance

$\sigma^2 = \Sigma (X - M)^2 \, P(X)$

① حساب M ($E(X)$)

② نطرح الـ M من كل قيم X ← $(X_1 - M), (X_2 - M) \ldots$

③ تربيع القيم الناتجة من ② ← $(X_1 - M)^2, (X_2 - M)^2 \ldots$

④ ضرب كل القيم الناتج من ③ بـ $P(X)$ ← $(X_1 - M)^2 \times P_{(X_1)}$

$(X_2 - M)^2 \times P_{(X_2)} \ldots$

⑤ جمع كل القيم الناتجة من ④ تحت الجذر $\sqrt{\Sigma (X-M)^2 \times P_{(X)}}$

* Binomial probabability :-

- Characteristics :-

1. identical trials → كل مرة بعمل نفس الحدث، مثل رمي coines
2. only 2 possible outcomes. (H or T) ← Coine مثل

probability → p q

note :- $p + q = 1$

$q = 1 - p$

3. Trials are independent
4. $X \sim Bin(n, p)$ ← number of trials ، ← probability of s ، ← number of success

* حساب الـ probability ← Binomial

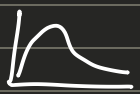
$$P(x) = \binom{n}{x} p^x q^{n-x}$$

n ← Number of trials

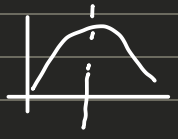
x ← Number of s

* Shape for distribution :-

1. $p < 0.5$ → positive



2. $p = 0.5$ → Symmetric



3. $p > 0.5$ → negative

Calculation :-

1. Mean → $M = np$
2. Standard deviation → $\sigma = \sqrt{npq}$

* اذا كانت table جاهزة طلب الـ probability :.

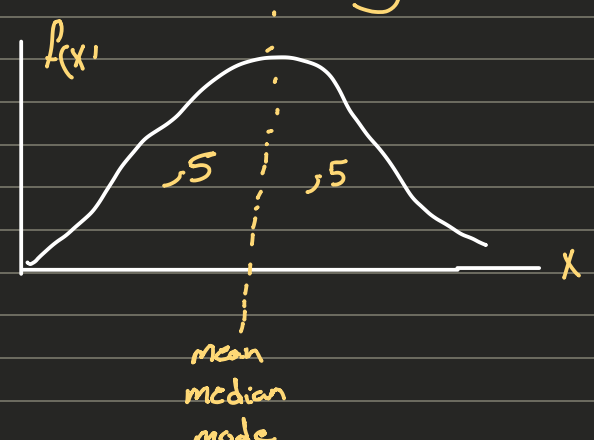
1. on one → $P_{(X=0)}$
2. exactly 3 → $P_{(X=3)}$
3. at least 2 → $P_{(X \geq 2)}$ (على الاقل 2)
4. most 2 → $P_{(X \leq 2)}$ (على الاكثر 2)

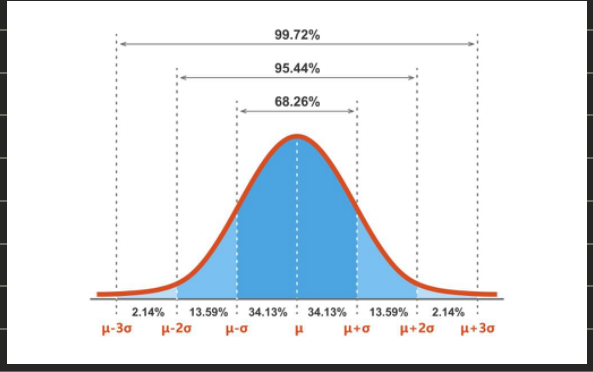
* Continuous probability Density function (pdf) :-

_ Normal Distribution :-

1. Bell - Shaped Symmetrical
2. Mean = Median = Mode
3. Total areas under curve = probability = 1
4. probability = areas

$$X \sim N(\mu, \sigma^2)$$





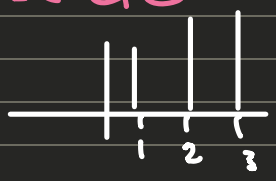
68% → $\mu \pm 1\sigma$

95% → $\mu \pm 2\sigma$

99,7% → $\mu \pm 3\sigma$

*Comparison :-

Discrete

① 

كل رقم له احتمال

② $P(X=x)$

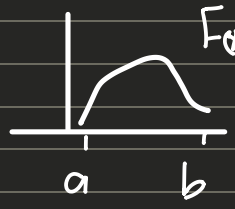
③ $P(X \leq a) \neq P(X < a)$

$P(X \leq a)$ ← احتمال a داخل

$P(X < a)$ ← لا يدخل احتمال a

④ $\sum P(x) = 1$

Continuous

① 

يكون الاحتمال على فترة

probability = areas under curve

② $P(a \to b) = \int_a^b f(x)$

③ $P(X \leq a) = P(X < a)$

← لا يدخل احتمال

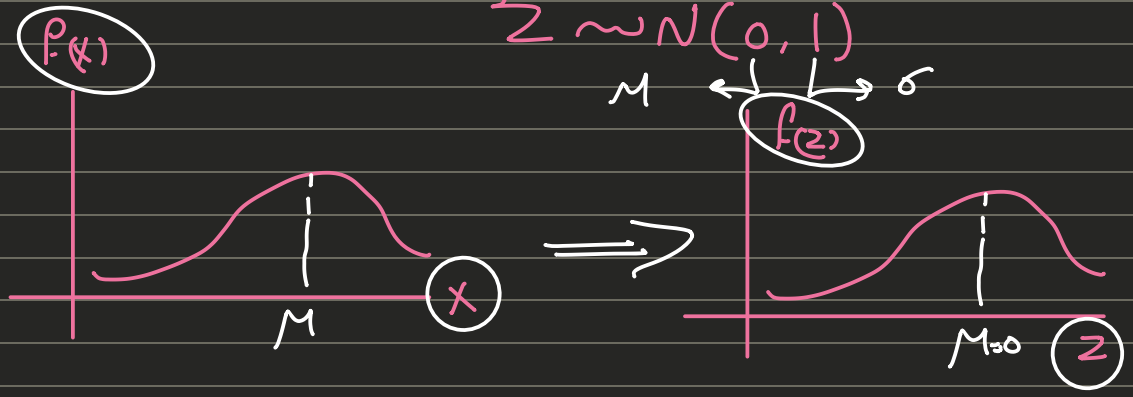
$P(x = a) = \int_a^a f(x) = 0$ لأن

④ كل مساحة تحت المنحنى = 1

* حتى احسابي areas under curve بدون تكامل

$X \sim$ Normal ⟶ standard normal Distribution

$Z \sim N(0,1)$



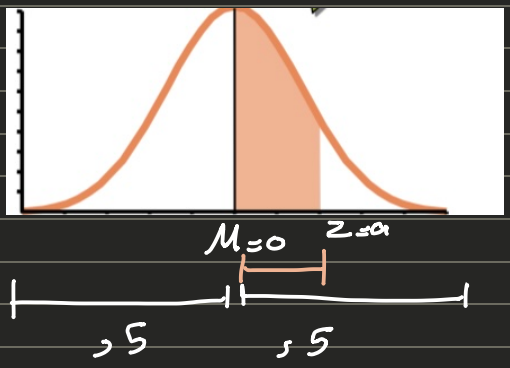
① تحديد σ , M

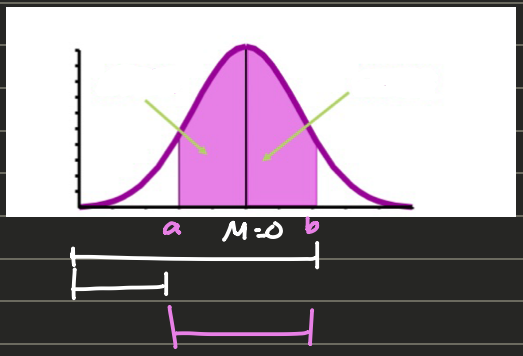
② تحويل x ← z

$$z = \frac{x - M}{\sigma}$$

③ في الجدول بحسب Probability to z

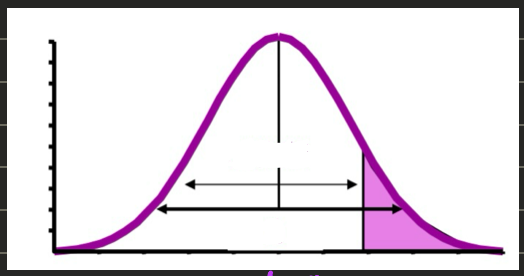
* الأفكار :-

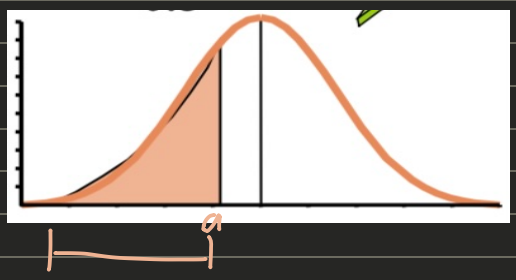




① $P(0 < z < a)$ P = Value of B from table - Value of a from table

= Value of a from table - ,5





$P(a < z) = 1$ - Value of a from table (probability of A)

$P(z < a) = P(A)$ Value of a from table

* Approximating a Binomial Distribution with a Normal distribution :-

Use when it would be impractical to use Binomial Distribution to find probability

$X \sim Bin(n, p)$

$E(X)\ (\mu) = n \cdot p$

$\sigma = \sqrt{npq}$

if :-

① $n \geq 30$ (large)

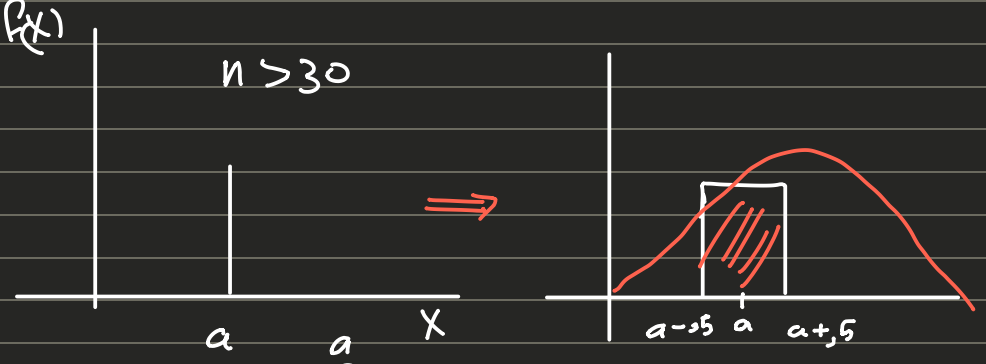
② $n \cdot p > 5$

③ $nq > 5$

$(1-p)$

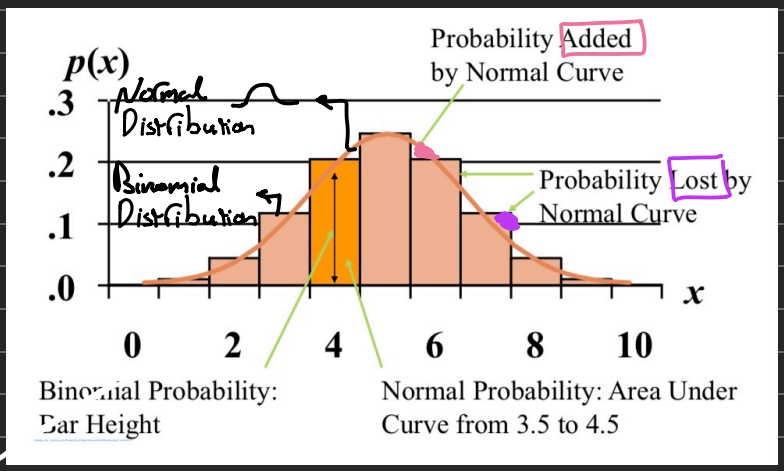
Normal Approximation to Binomial Distributions

* Continuity Correction : $\pm > 5$ improves accuracy



$P(X = a) = \int_a^a f(x) = 0$ ✗

$P(a-,5 < a < a+,5)$ = areas under curve



Binomial ⟶ Normal ⟶ Z-value

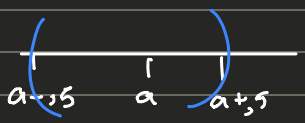
① Calculat the interval :-

if interval in Range (Zero to n) ✓

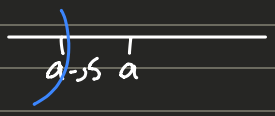
$$\mu \pm 3\sigma = np \pm 3\sqrt{np(1-p)}$$

② P (Binomial) ⟶ P(Normal

- $P(X=a) \longrightarrow P(a-,5 < X < a+,5)$

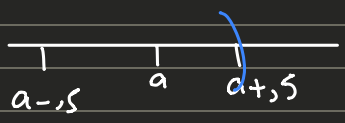


- $P(X<a) \longrightarrow P(X < a-,5)$

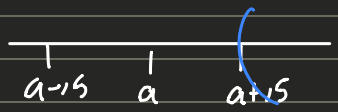


- $P(X \leq a) \longrightarrow P(X < a+,5)$

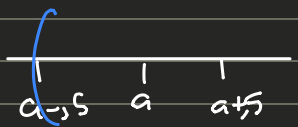
= اذا كان في a تدخل



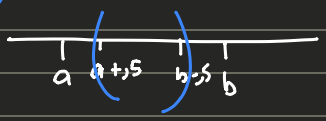
- $P(X>a) \longrightarrow P(X > a+,5)$



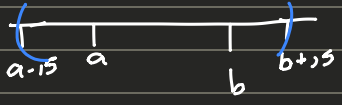
- $P(X \geq a) \longrightarrow P(X > a-,5)$



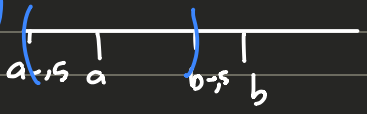
- $P(a < X < b) \longrightarrow P(a+,5 < X < b-,5)$



- $P(a \leq X \leq b) \longrightarrow P(a-,5 < X < b+,5)$



- $P(a \leq X < b) \longrightarrow P(a-,5 < X < b-,5)$



- $P(a < X \leq b) \longrightarrow P(a+,5 < X < b+,5)$

③ $Z = \frac{X - \mu}{\sigma}$