

Saved Photos

Date: / /



لجان الرُفعات



تفريغ إحصاء صيدلاني

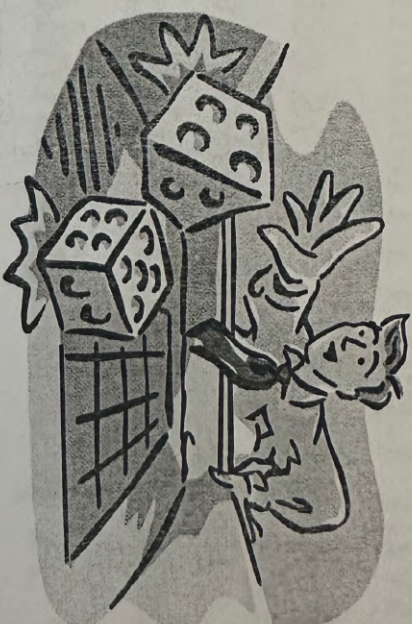
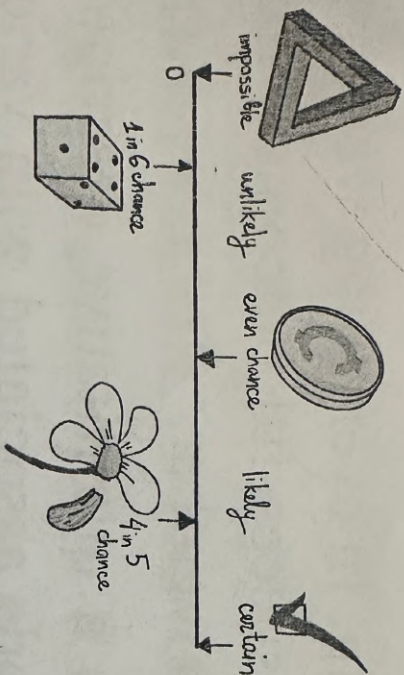


موضوع المحاضرة: Introduction to Probability
Slide 1-50

رقم المحاضرة: سكند - (I)

إعداد الصيدلانيه: تولين الدلابيح

Introduction to Probability



Prepared and Presented by:

Dr Muna Ogal

بہنو نے
میں (میں) ایک
• (11)

ایز اے
ایضاً = میں،
بہنو نے
impossible
(مستحکماً حدوتہ)

Introduction to Probability

- Experiments, Outcomes, Events and Sample Spaces
- What is probability?
- Basic Rules of Probability
- Probabilities of Compound Events

اذا كانت الاحتمال أكثر من 0.5 (مرجح) *likely* اذا كانت أكبر من 0.5 (مفر) و أقل من 0.5 (مرجح) *unlikely*

اذا كانت الاحتمال 50% *Pregnancy* مثال ان الحمل في احتمال جنس الجنين يكون 50% (male) 50% (female)

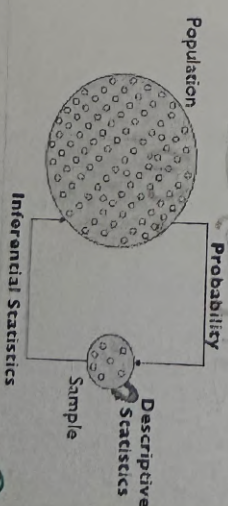
اذا كانت الاحتمال 100% : مثال : كل انسان فينا رج يولد مع 100% حاد الاحتمال

Introduction to Probability

Why Learn Probability?

- Nothing in life is certain. In everything we do, we gauge the chances of successful outcomes, from business to medicine to weather.
- A probability provides a quantitative description of chances or likelihoods associated with various outcomes.
- It provides a bridge between descriptive and inferential statistics.

الاحتمال سے مدد جو دہ فی کل الجبال سے بالحیاء سہاء
الاحتمالية أو الاحتمالية أو غيرها



الاجتماعية أو الاقتصادية غير ها .

Inferential Statistics

3

Basic Probability Concepts

* مثال على الاحتمالات:

حالة الطقس

مثلاً احتمال سقوطه لا

الاحتمال 70 %



* في منه ناحية العلم

احتمال كل العلم ودراساتنا

منه على حسابات

وفي افور بتعمد

على الاحتمالات

بتحليلنا الدافخ

لانه نعمل على

الدراسة لانه

احتمال تملح

النتيجة هي

فهو تحليلنا منك

جس أو حلته وملك

ما در descriptive analysis

و اد inferential statistics

لحت أو ملك لا ستنتاج منه . مثلاً انه في

There is a high risk factor

لانه أكل الحرام يزيد من العوامل الخطرة



Experiment

ہی التجربہ الی نقہ ۱

• An experiment is the process by which an observation (or measurement) is obtained.

field ۱ ما ۱ Random حایہ می حتم (محداتی) خطہ

• Deterministic Vs non-Deterministic experiment

→ Deterministic or Non-Random experiment

→ Non-Deterministic or Random experiment

معنا ته یا نه هو نه ما فی عسوی ائیہ ، هو نه بنکو نه متاک نه نه من الی حاجہ ،
مثک : هو اینس الفیزای (قانون السورۃ ، قانون القوة ، ...) ، انا کا نه مثلاً
قانون القوة = الكتلة $\times$ التسارع ، انا کا نه عندی ال mass و کا نه
عندی ال acceleration (التسارعی) بقدر فتنیاً طیه ملیت بالربط رج علاج
ال force ، فنیس یطالع معنا اینس بالربط (exactly) فنیس نه
Deterministic .

Experiment



Experiment



➤ **Deterministic or Non-Random experiment**

➤ In **deterministic** experiment, the outcome can be predicted exactly in advance by using a mathematical mode that allows a perfect prediction of the phenomena's outcome.

* مثال کے ال (Non - Deterministic) ؛ بالحد (2) کہے یا female 51 male 49
بھکے حرکت نتائج بھکے عام لکے لا بھکے توقع انجینس للجنین 100 % (2) فی حال
عدم استعمال السونار (4) وحی احياء بالسونار مکتے بحد 2 خطا في الحق .

- Many examples exist in physics and science.
- Example: $\text{Force} = \text{mass} \times \text{acceleration}$. So if we are given values for mass and acceleration, we exactly know the value of force.

Experiment (Phenomena)

- Non-Deterministic or Random experiment
- In random experiment, no mathematical model exists that allows a perfect prediction of the experiment's outcome.
- In this case, we are unable to predict the outcomes, or they are not known exactly. However, in the long-run, the outcomes exhibit a statistical regularity, so we can describe the probability of the possible outcomes.

➔ **لما نر جی ۱۰ قرو شه او انی عملہ ۱ احتمال یطالع لو رستیا**
! لا احتمال بکونه عنا (عربی اد لاجیز / مہدورہ او کتابہ) ، شی
نسبہ ۱ لا احتمال لک و حدہ ؟ ۵۰٪ لکل وحدہ ، لکن ما بقدر
نجز م ۱۰۰٪ ، ۱۰۰٪ Head بہتر تہ فتح فمطل .



↑

- possible outcomes.
- events ω (event) ω sample space Ω

می پائی، بہت چیز
I will go forever

- n: $\rightarrow$ I will go forever

- fraction

I will go
forever

chance of a successful MS test (2)

لا جیسے علیحدہ
outcome (نہ نتائج)

The Sample Space (S)



• عندیہ؟ یعنی احتمال
(head و tail) اور (head و head)
• ایک جہ کی تجربہ و حدہ
• احتمال کل واحد صفر
• احتمال 50%

Examples of a sample space (S)

1. Tossing a coin. Outcomes, $S = \{\text{Head, Tail}\} = \{H, T\}$
2. Tossing two coins once. Outcomes $S = \{HH, HT, TH, TT\}$. So number of outcomes $= n(S) = 2 * 2 = 4$.
3. Tossing three coins once. Then, outcomes:

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Number of outcomes $= n(S) = 2 * 2 * 2 = 8$ or S^n .

4. Rolling a die, then outcomes $S = \{1, 2, 3, 4, 5, 6\}$,

So number of outcomes $= n(S) = 1 * 6 = 6$.

$2^3 = 8$

$(6)^2 = 36$ The Sample Space (S)



5. Rolling two dice, the outcomes are:

$S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),$
 $(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),$
 $(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),$
 $(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),$
 $(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),$
 $(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$

So number of outcomes $S = n(S) = \underline{6 \times 6} = 36$

6

Examples

Example: Toss three coins. The total number of simple events is:

$$2 \times 2 \times 2 = 8$$

2^3

Example: Toss two dice. The total number of simple events is:

$$6 \times 6 = 36$$

6^2

Example: Two M&Ms are drawn from a dish containing two red and two blue candies. The total number of simple events is:

$$4 \times 3 = 12$$

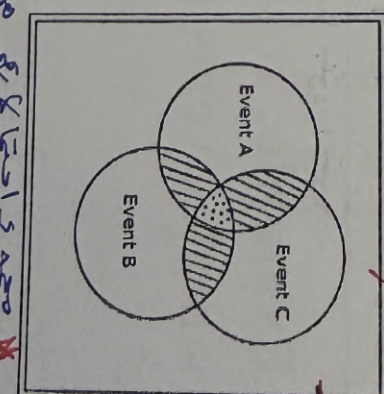
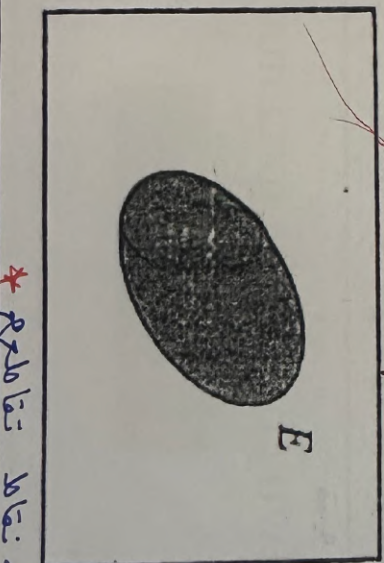
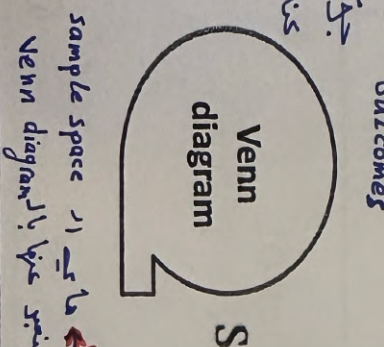
دو عذیر ۲
۱ احقر ۱
۳ عذیر ۳
۴ احقر ۴
۵ عذیر ۵
۶ احقر ۶
۷ عذیر ۷
۸ احقر ۸
۹ عذیر ۹
۱۰ احقر ۱۰
۱۱ عذیر ۱۱
۱۲ احقر ۱۲

The Event (E)

An Event, E

- The event, which is denoted by E, is any subset of the sample space, S, so it is any set of outcomes (not necessarily all outcomes) of the random phenomena.

- The event, E, can be denoted by A, B, C, D,....



مثلاً: سب سے پہلے دیکھو کہ احتمال کیا ہے؟

(15)

مثلاً: سب سے پہلے دیکھو کہ احتمال کیا ہے؟

Types of Events



15

مجلس القضاء ع

Types of Events



- ## 1. Null (impossible) event (ϕ)

126.

- ## 2. Entire (sure or certain) event (S) →

4
E

14-1

18
L
-
-
-

4.

$\frac{C_1}{4}$

2.2.5

على ١٥

4. Compound event

↓
مع الاطفال
ممكنه
لا

$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ 0 & 1 \end{pmatrix}$

١١٨٢١

7
 4
 1
 6

آیا من احتمال ببیمک من

- [Compound event

Types of Events



1. The **null event** (the empty event) (ϕ), never occurs, impossible.

$\phi = \{ \} =$ the event that contains no outcomes

2. The **entire** (sure or certain) event, the sample space (S), $S =$ the event that contains all outcomes

So the sure event, S , always occurs. →

* مثال :

مشو ا حصاد آر صی

جیو ا نژد و علاج ا لا رقم

منه (1) خالص (6) .

Types of Events

یاد ایا بلکہ جز جز واحد
sample space سے ۱
زیادہ ہو احتمال آریے جبر الزم
و جملہ عندیہ رقم (۱) نہ جملہ احتمال؟
۱



3. A **simple event** is the outcome that is observed on a single repetition of the experiment.
 - One and only one simple event can occur when the experiment is performed.
 - Simple event is denoted by E with subscript, e.g. $E_1, E_2, \dots$ etc.

The set of all simple events of an experiment is called the sample space, S.

4. **Compound event** is the outcome that contains more than one simple event when the experiment is performed.

compound event
حدود ۳
بجیے
لگتے بجیے ک

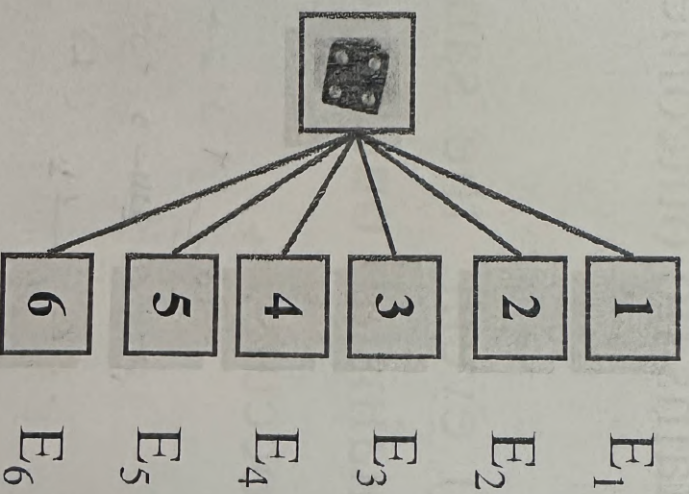
$$0.5 = \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

۱
% 50

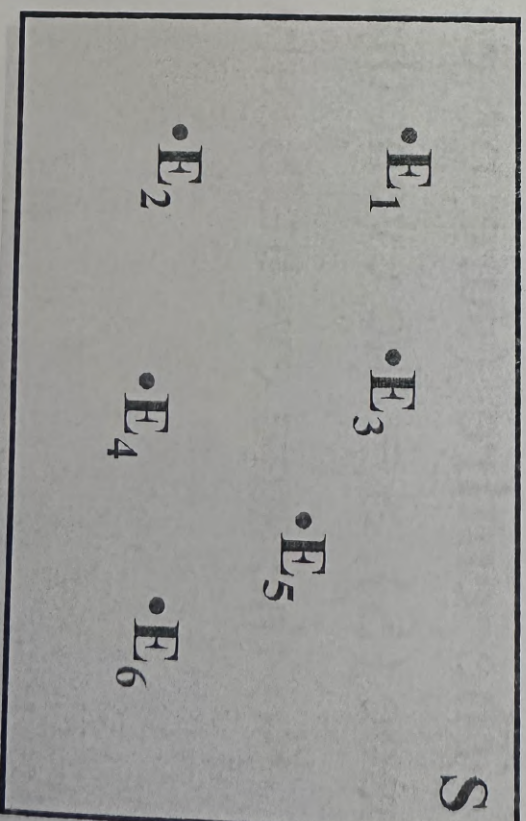
بجیے عندیہ اکثر واحد
بجیے ۲ جیت مشا
نمط ۱
probability

Example

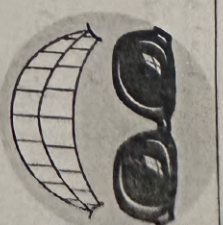
- The die toss:
 - Simple events:
- Sample space:



$$S = \{E_1, E_2, E_3, E_4, E_5, E_6\}$$



Event



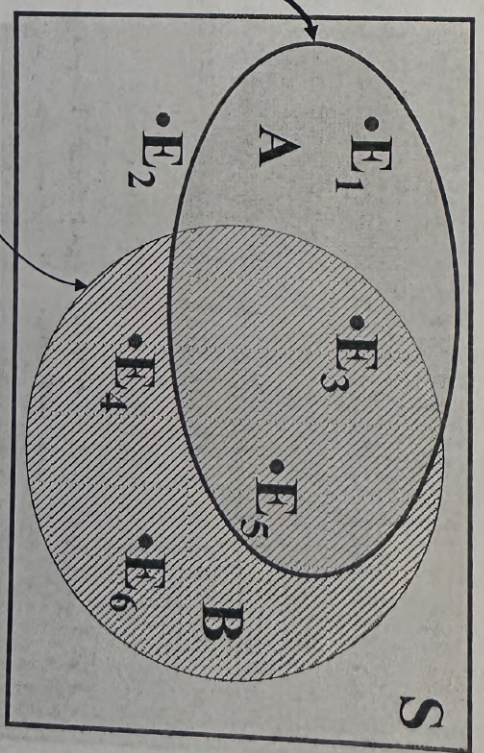
- An event is a collection of one or more simple events.

• The die toss:

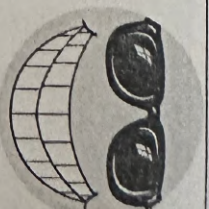
- A: an odd number
- B: a number > 2

$$A = \{E_1, E_3, E_5\}$$

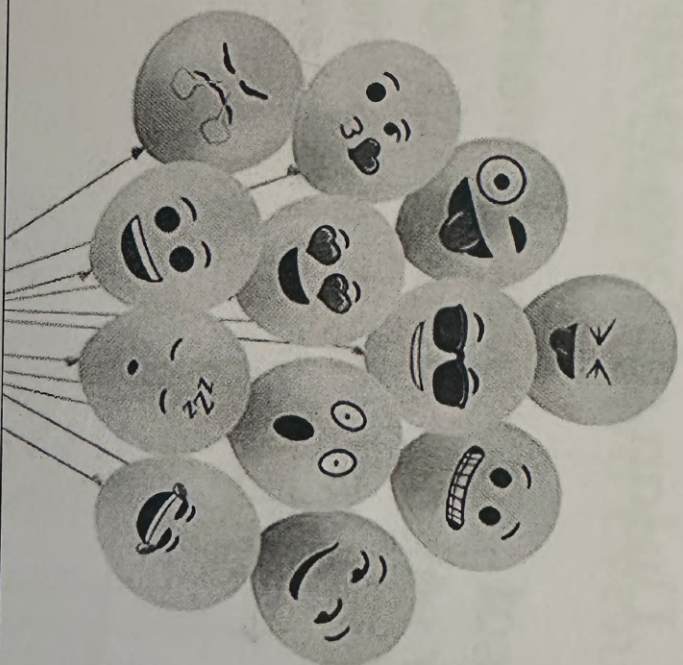
$$B = \{E_3, E_4, E_5, E_6\}$$



Examples of Event



- Rolling a die, then outcomes, $S = \{1, 2, 3, 4, 5, 6\}$
- **A** = The event that the number comes up is greater than 5 = $\{6\}$ which is simple event.
 sample space S ہونے کا
 ممکنہ احوال اُس سے جس پر توجہ دیا جائے
 (مثلاً احوال واحد)
 (simple event)
 ایک سے 6
- **B** = The event that the number comes up is an even number = $\{2, 4, 6\}$, which is a compound event.
 عدد زوجہ ہے
- **C** = Then event that the number comes is less than 7 = $\{1, 2, 3, 4, 5, 6\}$, which is a sure event.
 احوال چلیج دیا
 7 سے 6
- **D** = The event that the number comes up is greater than 6 = $\{\}$ = ϕ , which is an impossible event.
 احوال چلیج
 دیا 7 سے 6



Event Relations

Some Basic Relationships of Probability

There are some basic probability relationships that can be used to compute the probability of an event without knowledge of all the sample point probabilities.

▶ Union of Two Events

(۱ اتحاد)

▶ Intersection of Two Events

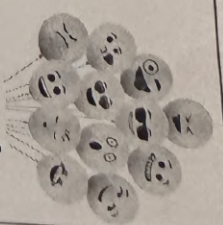
(۱ تقاطع)

▶ Complement of an Event

(۱ المكملة)

▶ Mutually Exclusive Events

Union of Two Events



- The union of two events, A and B, is the event that either A or B or both occur when the experiment is performed. We write

$$A \cup B$$

یعنی کہ اگر دو events
ایک وقوع پر A و B
ایک وقوع پر،
both A اور B

☆ سو صرف اتحاد؟ سو صرف اتحاد پر A و B؟
کل الفتح الوجود

☆ A و B

☆ ہاں بنجیے عنہ

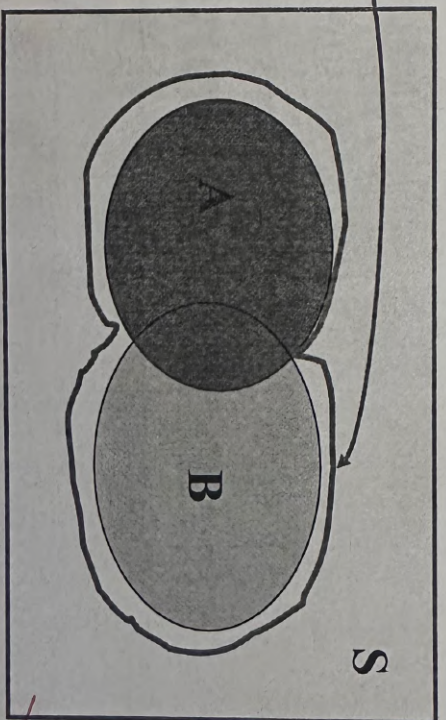
• Union

$$A \cup B$$

☆ د اٹھا کلمہ

(OR) بنجیے ہاں

• Union کا



☆ ہاں

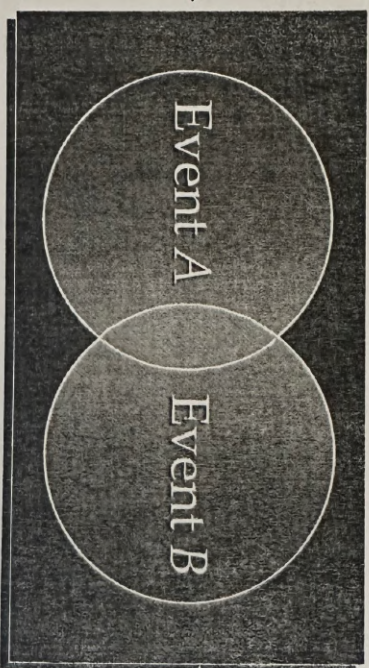
• Venn diagram

Union of Two Events



The union of events A and B is the event containing all sample points that are in A or B or both.

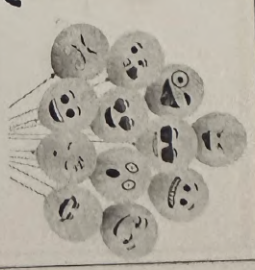
The union of events A and B is denoted by $A \cup B$.



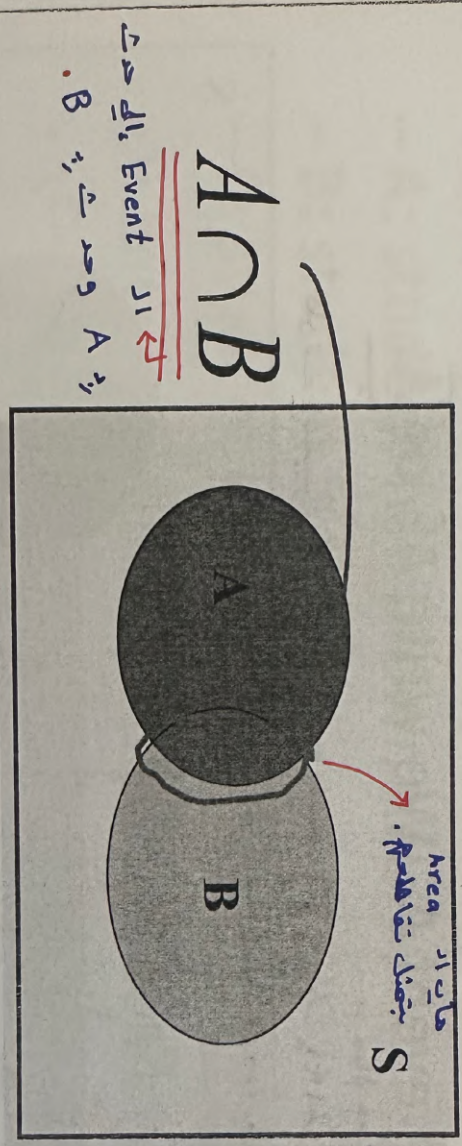
Sample
Space S

* التقاطع : القيمة المشتركة فقط

Intersection of Two Events



- The intersection of two events, A and B, the event that both A and B occur when the experiment is performed. We write $A \cap B$.



مثال آخر : لو عرفت
A جبر الزد إنه يطلع
B و عرفت B إنه
يطلع جبر الزد عنده
نمو تقاطع A و B ؟
Φ - علاه مستحيل
إنه ٣ و ٦ بانه نفسه
يطلع .

- If two events A and B are mutually exclusive, then $P(A \cap B) = 0$.

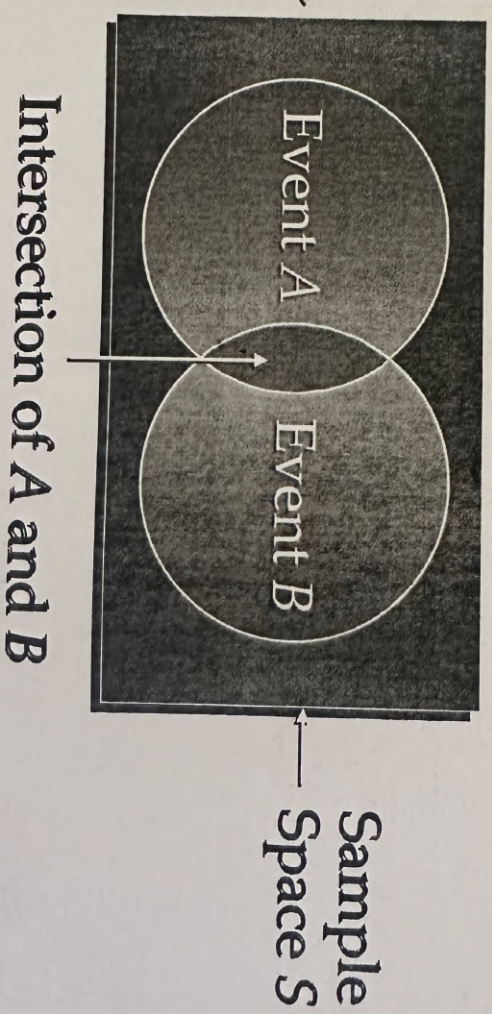
يعني إنه حدين
مستحيل يتروا
بعضه ملك بكونه
تقاطع
26

student in
male
female
event (B) student
event (A) student
Φ 0
تقاطع
26

Intersection of Two Events

The intersection of events A and B is the set of all sample points that are in both A and B .

The intersection of events A and B is denoted by $A \cap B$.



Complement of an Event

outcomes جيتي كد ار

complement of event (A) جيتي كد ار

يعني لو حكيته انه ار (A) event (A) باله مو موجوده بار

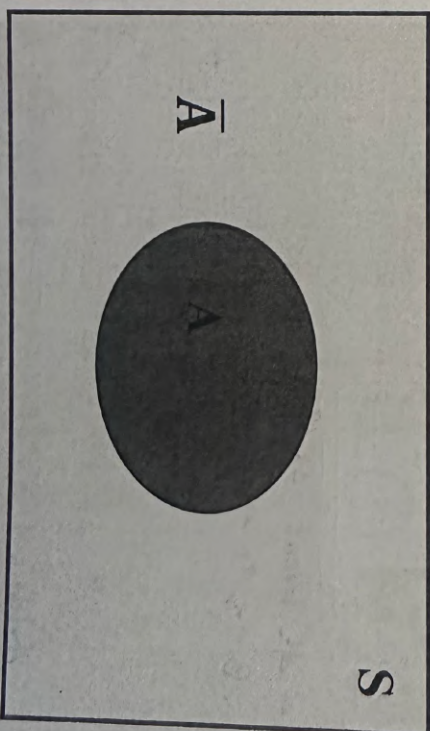
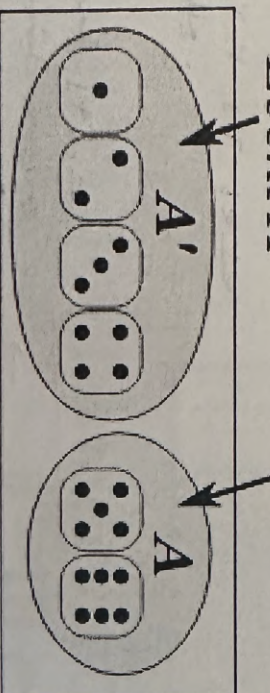


- The complement of an event A consists of all outcomes of the experiment that do not result in event A . We write A^c or $\bar{A}$.

مثال عن جيتي النور : اذا عرفت (A) انا الاوراق الخودية ٤ مصناحه ار
complement (A) دة صفحت الاركان الخودية

طرق التعبير عن ال complement

Complement of Event A



complement صحتها سوي دة مساوية ؟

جميع الية

تساوية

ار

سوي ال صفر

سو سو جو ٿا

ڪل possible outcomes

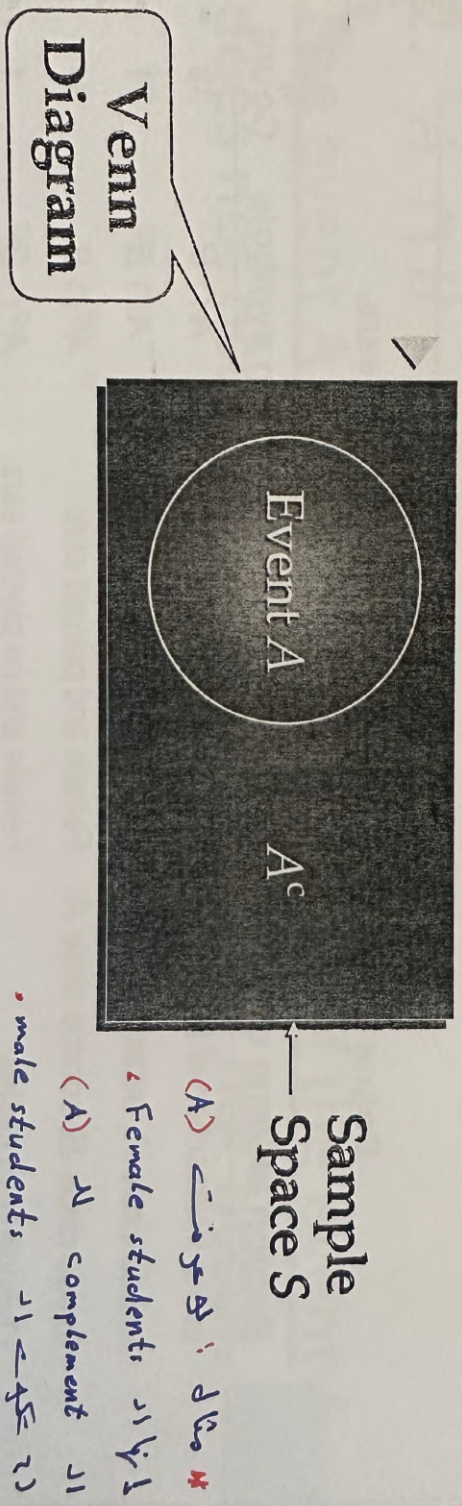
ڪل ٿا (A) ۾

Complement of an Event



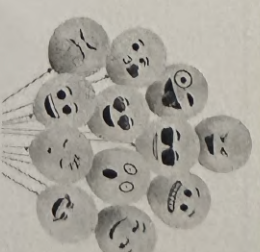
The complement of event A is defined to be the event consisting of all sample points that are not in A .

The complement of A is denoted by A^c .



مثال : ٿا جي ٿا (A)
 Female students ۾
 complement ۾ (A)
 male students ۾

Example



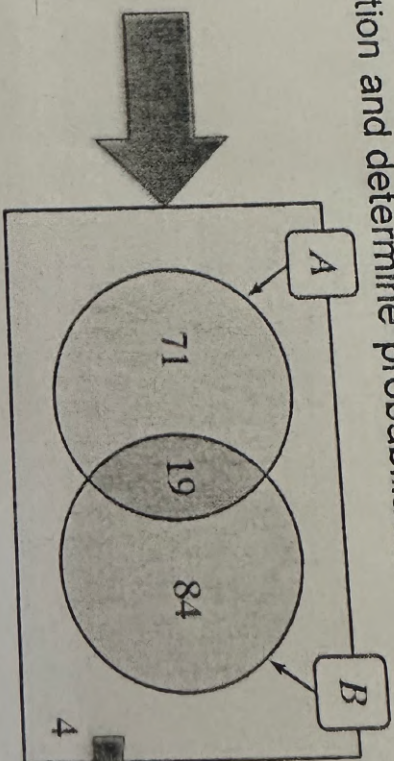
- Select a student from the classroom and record his/her hair color and gender.
 - A: student has brown hair
 - B: student is female
 - C: student is male
- What is the relationship between events **B** and **C**?
 - A^C : Student does not have brown hair
 - $B \cap C$: Student is both male and female = $\emptyset$
 - $B \cup C$: Student is either male and female = all students = S

Mutually exclusive; $B = C^C$

Venn Diagrams and Probability

Recall the example on gender and pierced ears. We can use a Venn diagram to display the information and determine probabilities.

Gender	Pierced Ears?		Total
	Yes	No	
Male	19	71	90
Female	84	4	88
Total	103	75	178



Probability Rules

Define events A : is male and B : has pierced ears.

تعريف

Region in Venn diagram	In words	In symbols	Count
In the intersection of two circles	Male and pierced ears	$A \cap B$	19
Inside circle A , outside circle B	Male and no pierced ears	$A \cap B^c$	71
Inside circle B , outside circle A	Female and pierced ears	$A^c \cap B$	84
Outside both circles	Female and no pierced ears	$A^c \cap B^c$	4

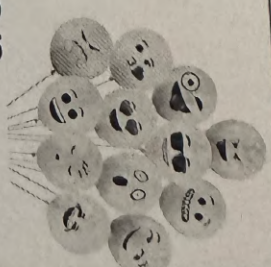
• دالة على التفاضل

(And)

د 1

30

Event Relations



- Key words to recognise which event relation you should perform:

1. Union: if you see the word or or at least one of the two event occurs.
2. Intersection: if you see the word and or both events occur.
3. Complement: if you see the word not.

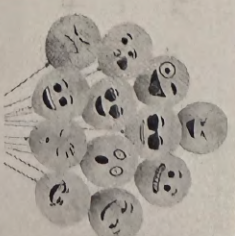
not happen
does not happen

De Moivre's Laws

اسم العالم

المبدأ محل حدود المتوابع

المقاييس من المحقق
بمساعدة من مبرمجها



$$1. \overline{A \cup B} = \bar{A} \cap \bar{B}$$

The event A or B does not occur if the event A does not occur and the event B does not occur.

$$2. \overline{A \cap B} = \bar{A} \cup \bar{B}$$

complement

The event A and B does not occur if the event A does not occur or the event B does not occur.

A cluster of approximately 12 balloons, each featuring a different black-and-white smiley face expression. The expressions include various combinations of eyes (smiling, winking, closed, open) and mouths (curved up, straight, open in a wide smile or laugh). The balloons are tied together at the bottom by several thin strings.

1. $A \cup \Phi = A$

- sample space Ω

3. $A \cap \Phi = \Phi$

4. $AUS = S$

5. $A \cap \bar{A} = \phi$

6. $AU\bar{A} = S$

outcomes

is the

↑

$$I \rightarrow 0$$

probabilities

11
 2
 3
 4
 5
 6
 7
 8
 9
 10
 11
 12
 13
 14
 15
 16
 17
 18
 19
 20
 21
 22
 23
 24
 25
 26
 27
 28
 29
 30
 31
 32
 33
 34
 35
 36
 37
 38
 39
 40
 41
 42
 43
 44
 45
 46
 47
 48
 49
 50
 51
 52
 53
 54
 55
 56
 57
 58
 59
 60
 61
 62
 63
 64
 65
 66
 67
 68
 69
 70
 71
 72
 73
 74
 75
 76
 77
 78
 79
 80
 81
 82
 83
 84
 85
 86
 87
 88
 89
 90
 91
 92
 93
 94
 95
 96
 97
 98
 99
 100
 101
 102
 103
 104
 105
 106
 107
 108
 109
 110
 111
 112
 113
 114
 115
 116
 117
 118
 119
 120
 121
 122
 123
 124
 125
 126
 127
 128
 129
 130
 131
 132
 133
 134
 135
 136
 137
 138
 139
 140
 141
 142
 143
 144
 145
 146
 147
 148
 149
 150
 151
 152
 153
 154
 155
 156
 157
 158
 159
 160
 161
 162
 163
 164
 165
 166
 167
 168
 169
 170
 171
 172
 173
 174
 175
 176
 177
 178
 179
 180
 181
 182
 183
 184
 185
 186
 187
 188
 189
 190
 191
 192
 193
 194
 195
 196
 197
 198
 199
 200
 201
 202
 203
 204
 205
 206
 207
 208
 209
 210
 211
 212
 213
 214
 215
 216
 217
 218
 219
 220
 221
 222
 223
 224
 225
 226
 227
 228
 229
 230
 231
 232
 233
 234
 235
 236
 237
 238
 239
 240
 241
 242
 243
 244
 245
 246
 247
 248
 249
 250
 251
 252
 253
 254
 255
 256
 257
 258
 259
 260
 261
 262
 263
 264
 265
 266
 267
 268
 269
 270
 271
 272
 273
 274
 275
 276
 277
 278
 279
 280
 281
 282
 283
 284
 285
 286
 287
 288
 289
 290
 291
 292
 293
 294
 295
 296
 297
 298
 299
 300
 301
 302
 303
 304
 305
 306
 307
 308
 309
 310
 311
 312
 313
 314
 315
 316
 317
 318
 319
 320
 321
 322
 323
 324
 325
 326
 327
 328
 329
 330
 331
 332
 333
 334
 335
 336
 337
 338
 339
 340
 341
 342
 343
 344
 345
 346
 347
 348
 349
 350
 351
 352
 353
 354
 355
 356
 357
 358
 359
 360
 361
 362
 363
 364
 365
 366
 367
 368
 369
 370
 371
 372
 373
 374
 375
 376
 377
 378
 379
 380
 381
 382
 383
 384
 385
 386
 387
 388
 389
 390
 391
 392
 393
 394
 395
 396
 397
 398
 399
 400
 401
 402
 403
 404
 405
 406
 407
 408
 409
 410
 411
 412
 413
 414
 415
 416
 417
 418
 419
 420
 421
 422
 423
 424
 425
 426
 427
 428
 429
 430
 431
 432
 433
 434
 435
 436
 437
 438
 439
 440
 441
 442
 443
 444
 445
 446
 447
 448
 449
 450
 451
 452
 453
 454
 455
 456
 457
 458
 459
 460
 461
 462
 463
 464
 465
 466
 467
 468
 469
 470
 471
 472
 473
 474
 475
 476
 477
 478
 479
 480
 481
 482
 483
 484
 485
 486
 487
 488
 489
 490
 491
 492
 493
 494
 495
 496
 497
 498
 499
 500
 501
 502
 503
 504
 505
 506
 507
 508
 509
 510
 511
 512
 513
 514
 515
 516
 517
 518
 519
 520
 521
 522
 523
 524
 525

← $\mathcal{S}$ sample space

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

(sample space) event (حادثہ) probabilities (احتمالات) (1) .

35

probabilities $\{p_1, 1-p_1\}$

احداث و انحصارات كيفية Probabilities

An event is a collection of sample points.

The probability of any event is equal to the sum of the probabilities of the sample points in the event.

If we can identify all the sample points of an experiment and assign a probability to each, we can compute the probability of an event.

Assigning Probabilities to Events

Probability of an event $P(E)$: "Chance" that an event will occur

- Must lie between 0 and 1
- "0" implies that the event will not occur
- "1" implies that the event will occur

Types of Probability:

• Objective

- ✓ Relative Frequency Approach
- ✓ Equally-likely Approach

• Subjective

سو كل وحدة ليا
نفست اذ حصاد

relative frequency
كل وحدة
مجموعه
كل وحدة
مجموعه
مجموعه

$$\frac{1}{n} + \frac{1}{n} + \frac{1}{n} + \dots$$

كل وحدة
مجموعه
مجموعه
مجموعه

Relative Frequency Approach: Relative frequency of an event occurring in an infinitely large number of trials

Time Period	Number of Male Live Births	Total Number of Live Births	Relative Frequency of Live Male Birth
1965	1927,054	3,760,358	0.51247
1965-1969	9219,202	17,989,360	0.51248
1965-1974	17,857,860	34,832,050	0.51268

$$\frac{1927.054}{3760.358}$$

جو کہ جو کہ 1
ایک event مختلف

Equally-likely Approach: If an experiment must result in n equally likely outcomes, then each possible outcome must have probability $1/n$ of occurring.

Examples:

1. Roll a fair die
2. Select a SRS of size 2 from a population

زیر ہا حکمتی یا فرقیہ جو فریڈ ، آڈ فریڈ
1 coin ، ہو کہ کو کہ معادلہ
event ، ایک احتمال مستوی
event

مستند و یہ لکھ ، و سر ت ا حسب
بملاچ جو کہ 1 مساویہ (1)
لکے ہو کہ معادلہ کے مختلف
فلا تو ملو 1 ایسا ، ہو کہ جملہ

Subjective Probability: A number between 0 and 1 that reflects a person's degree of belief that an event will occur

Example: Predictions for rain

ذہب بچو 1 نذر

کنا سکی انفریڈ

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2} = \frac{1}{2} = 50\%$$

Assigning Probabilities

Example: Predictions for rain

دیت پیمبر الورد

کنا سبکی - الفرزیه
 $\frac{1}{2} + \frac{1}{3} + \frac{3}{6} = \frac{3}{6} = \frac{1}{2} = 50\%$

Assigning Probabilities

Classical Method

Assigning probabilities based on the assumption of equally likely outcomes → دیت جبر الورد

Relative Frequency Method

Assigning probabilities based on experimentation or historical data → نو خد ادر تم و یعتقه یا total

Subjective Method

تجربین / تکلیف
Assigning probabilities based on judgment

ذکر با ط خبار پیمبریا خبر استراپیجی بالخطفه ، او خبر الیوه ، او خبر بالسیاسه ، او ایہ امین ، پیمبر بنبیہ لانه صلا فی منو خبره معجله نو توقع پیمبر ایہ حقیقہ ، حو مو جانب درکہ دقل و بحسب ، حو مو بعل judgment من خبره بعل judgment ، و حو مو اخبار بیجی دورم ، من کتر ما پیمبر عند تم

(38)

100% probabilities لا Assigning بعلو خبره

Classical Method

If an experiment has n possible outcomes, this method

would assign a probability of $1/n$ to each outcome.

► Example

► Experiment: Rolling a die

► Sample Space: $S = \{1, 2, 3, 4, 5, 6\}$

Probabilities: Each sample point has a $1/6$ chance of occurring



* کیف بنجھلہ ؟

لفورنہ ہو ے کو یہ جبر اندر

عدد ۱۲ احتمال ے ۴

ف' ۴ = ۶ ۴ و کد حد ے

• صفر جبر بنجھلہ ۱/۶

• $\frac{1}{6}$ بھائی بھائی

(39)

Relative Frequency Method

Each probability assignment is given by dividing the frequency (number of days) by the total frequency (total number of days).

Number of Polishers Rented	Number of Days	Probability
0	4	.10
1	6	.15
2	18	.45
3	10	.25
4	<u>2</u>	<u>.05</u>
	40	1.00

4/40

• 11 منجھو کمر چسارے

sample space

کد ۱۱

بھائی ۱ کد ۱۱

کد ۱۱

(40)

made the following probability assignments.

<u>Exper. Outcome</u>	<u>Net Gain or Loss</u>	<u>Probability</u>
(10, 8)	\$18,000 Gain	.20
(10, -2)	\$8,000 Gain	.08
(5, 8)	\$13,000 Gain	.16
(5, -2)	\$3,000 Gain	.26
(0, 8)	\$8,000 Gain	.10
(0, -2)	\$2,000 Loss	.12
(-20, 8)	\$12,000 Loss	.02
(-20, -2)	\$22,000 Loss	.06

- probabilities $\rightarrow$ احتمالات

The Probability of an Event

• $P(A)$ must be between 0 and 1.

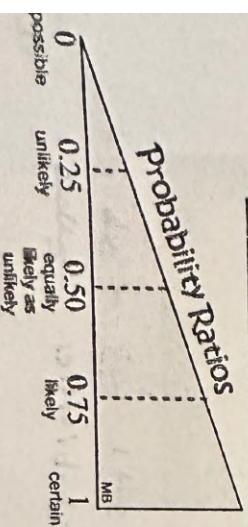
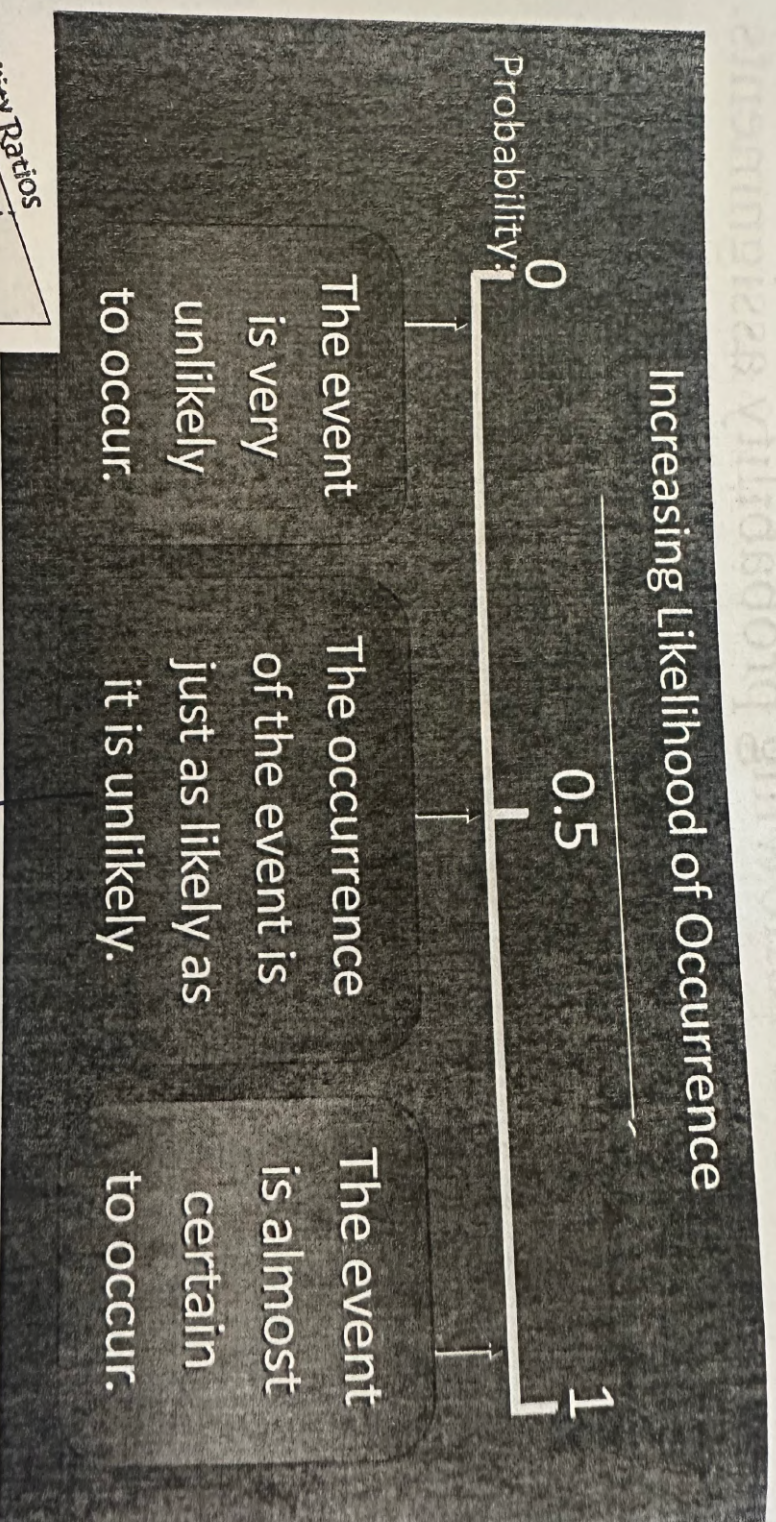
– If event A can never occur, $P(A) = 0$. If event A

always occurs when the experiment is performed, $P(A) = 1$.

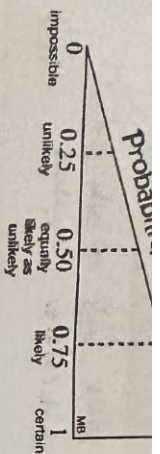
- The sum of the probabilities for all simple events in S equals 1.

• The probability of an event A is found by adding the probabilities of all the simple events contained in A .

Probability as a Numerical Measure of the Likelihood of Occurrence



Equal chance.

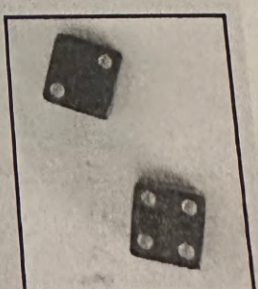


Equal chance.

(43)

* التكرير
يكون
احد
احتمال

The Probability of an Event



- The probability of an event A measures "how often" we think A will occur. We write $P(A)$.
- Suppose that an experiment is performed n times. The relative frequency for an event A is

$$\text{Number of times } A \text{ occurs} = \frac{f}{n}$$

Let A be the event $A = \{O_1, O_2, \dots, O_k\}$, where $O_1, O_2, \dots, O_k$ are k different outcomes. Then

$$P(A) = P(O_1) + P(O_2) + \dots + P(O_k)$$

(44)

Finding Probabilities

- Probabilities can be found using
 - Estimates from empirical studies
 - Common sense estimates based on equally likely events.

• Examples:

- Toss a fair coin.

$$P(\text{Head}) = 1/2$$

- 10% of the U.S. population has red hair.

Select a person at random. $P(\text{Red hair}) = .10$

* What is the probability of observing two tails?

$$\frac{1}{4}$$

Example



- Toss two coins. What is the probability of observing at least one head?

بسته‌های انتخاب

لیکن همه ما به نیت خود

یا انتخاب به نیت خود با علوف
استغفیریه

1st Coin	2nd Coin	E_i	$P(E_i)$
H	H	HH	1/4
H	T	HT	1/4
T	H	TH	1/4
T	T	TT	1/4

$P(\text{at least 1 head})$

$$= P(E_1) + P(E_2) + P(E_3)$$

$$= 1/4 + 1/4 + 1/4 = 3/4$$

$$n = 4$$

خانه در ۱۱ اسامی

۴

Example

- A bowl contains three M&Ms[®], one red, one blue and one green. A child selects two M&Ms at random. What is the probability that at least one is red?

1st M&M	2nd M&M	E_i	$P(E_i)$
Red ← m	m → Blue	RB	1/6
	m → Green	RG	1/6
Blue ← m	m → Red	BR	1/6
	m → Green	BG	1/6
Green ← m	m → Blue	GB	1/6
	m → Red	GR	1/6

$$\begin{aligned}
 &P(\text{at least 1 red}) \\
 &= P(RB) + P(BR) + P(RG) \\
 &\quad + P(GR) \\
 &= 4/6 = 2/3
 \end{aligned}$$

Counting Rules

- If the simple events in an experiment are **equally likely**, you can calculate

$$P(A) = \frac{n_A}{N} = \frac{\text{number of simple events in } A}{\text{total number of simple events}}$$

- You can use **counting rules** to find n_A and N .

A Counting Rule for Multiple-Step Experiments

If an experiment consists of a sequence of k steps in which there are n_1 possible results for the first step, n_2 possible results for the second step, and so on, then the total number of experimental outcomes is given by $(n_1)(n_2) \dots (n_k)$.

A helpful graphical representation of a multiple-step experiment is a tree diagram.

Which Sides Are Greater Than 3?



How Many Possibilities Are There?



Number of Favorable Outcomes = 3

Total Number of Possibilities = 6

Number of Favorable Outcomes / Total Number of Outcomes = $3/6 = 50\%$



PROBABILITY = $\frac{\text{EVENT}}{\text{OUTCOMES}}$

