

Date:

/ /



لجان الرِّفعات



تفريغ إحصاء صيدلاني

موضوع المحاضرة : Lecture 8,9,10 part 2

رقم المحاضرة : سكند 2

إعداد الصيدلانيه : جنين



Counting Rule for Permutations

A second useful counting rule enables us to count the number of experimental outcomes when n objects are to be selected from a set of N objects, where the order of selection is important.

► Number of Permutations of N Objects Taken n at a Time

$$P_n^N = n! \binom{N}{n} = \frac{N!}{(N-n)!}$$

where: $N! = N(N-1)(N-2) \dots (2)(1)$
 $n! = n(n-1)(n-2) \dots (2)(1)$
 $0! = 1$

Permutations

- The number of ways you can arrange n distinct objects, taking them r at a time is

$$P_r^n = \frac{n!}{(n-r)!}$$

where $n! = n(n-1)(n-2)\dots(2)(1)$ and $0! \equiv 1$.

Example: How many 3-digit lock combinations can we make from the numbers 1, 2, 3, and 4?

The order of the choice is important!

$$P_3^4 = \frac{4!}{1!} = 4(3)(2) = 24$$

Examples

Example: A lock consists of five parts and can be assembled in any order. A quality control engineer wants to test each order for efficiency of assembly. How many orders are there?

The order of the choice is important!

$$P_5^5 = \frac{5!}{0!} = 5(4)(3)(2)(1) = 120$$

Counting Rule for Combinations

A third useful counting rule enables us to count the number of experimental outcomes when n objects are to be selected from a set of N objects.

Number of Combinations of N Objects Taken n at a Time

$$C_n^N = \binom{N}{n} = \frac{N!}{n!(N-n)!}$$

where: $N! = N(N-1)(N-2) \dots (2)(1)$
 $n! = n(n-1)(n-2) \dots (2)(1)$
 $0! = 1$

Combinations

- The number of distinct combinations of n distinct objects that can be formed, taking them r at a time is

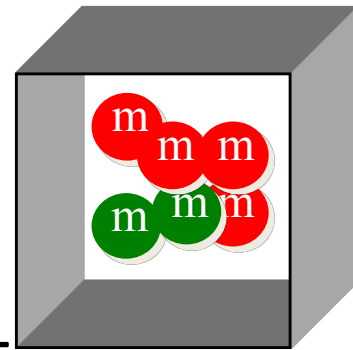
$$C_r^n = \frac{n!}{r!(n-r)!}$$

Example: Three members of a 5-person committee must be chosen to form a subcommittee. How many different subcommittees could be formed?

The order of the choice is not important!

$$C_3^5 = \frac{5!}{3!(5-3)!} = \frac{5(4)(3)(2)1}{3(2)(1)(2)1} = \frac{5(4)}{(2)1} = 10$$

Example



- A box contains six M&Ms[®], four red and two green. A child selects two M&Ms at random. What is the probability that exactly one is red?

The order of the choice is not important!

$$C_2^6 = \frac{6!}{2!4!} = \frac{6(5)}{2(1)} = 15$$

ways to choose 2 M & Ms.

$$C_1^2 = \frac{2!}{1!1!} = 2$$

ways to choose 1 green M & M.

$$C_1^4 = \frac{4!}{1!3!} = 4$$

ways to choose 1 red M & M

$4 \times 2 = 8$ ways to choose 1 red and 1 green M&M.

$P(\text{ exactly one red}) = 8/15$

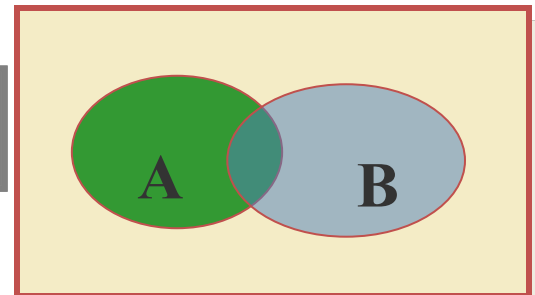
Calculating Probabilities for Unions and Complements

- There are special rules that will allow you to calculate probabilities for composite events.
- **The Additive Rule for Unions:**
- For any two events, **A** and **B**, the probability of their union, $P(A \cup B)$, is

P = Probability احتمال

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

• إذا اطلب مني بالامتحان أطلع $P(A \cup B)$



Addition Law



▶ The addition law provides a way to compute the probability of event A , or B , or both A and B occurring.

The law is written as:

▶
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

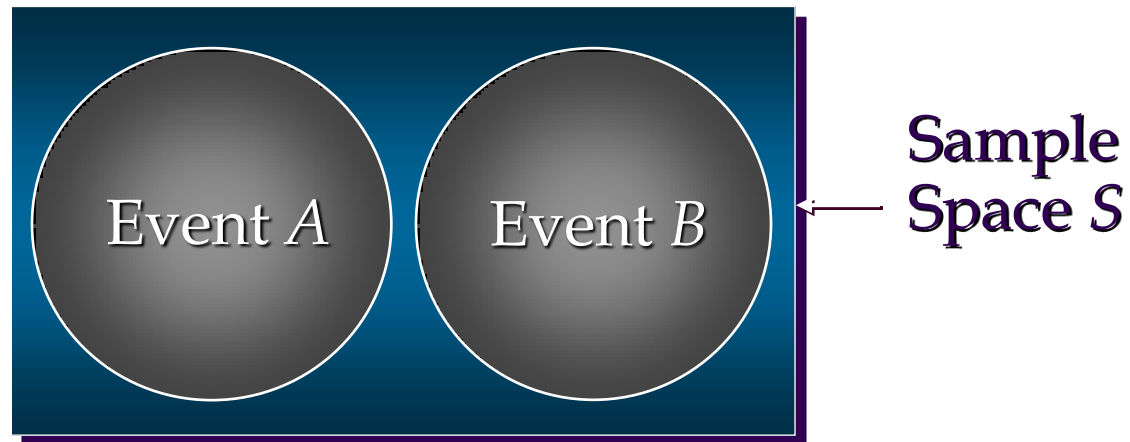
Mutually Exclusive (Disjoint) Events



Two events are said to be mutually exclusive if the events have no sample points (outcomes) in common.

Two events are mutually exclusive if, when one event occurs, the other cannot occur (They can't occur at the same time. The outcome of the random experiment cannot belong to both A and B)

If events A and B are mutually exclusive, $P(A \cap B) = 0$.



Mutually Exclusive Events



- ▶ If events A and B are mutually exclusive, $P(A \cap B) = 0$.

The addition law for mutually exclusive events is:

$$P(A \cup B) = P(A) + P(B)$$

مثلاً :

A : female

B : male

$$P(A \cup B) = P(A) + P(B)$$

there's no need to
include " $- P(A \cap B)$ "

إذا كنت عارف انه :
event A and B
mutually exclusive
فالحق قانون
 $P(A \cap B) = \emptyset$

Mutually Exclusive (Disjoint) Events



- Two events are mutually exclusive if, when one event occurs, the other cannot, and vice versa.

عندما يظهر الرقم فردي وأجبر من 2 بنفس الوقت

- Experiment: Toss a die

–A: observe an odd number

Not Mutually
Exclusive

–B: observe a number greater than 2

–C: observe a 6

–D: observe a 3

Mutually
Exclusive

B and C?
B and D?

لأنه غير ممكن يظهر الرقم 3 و 6
بنفس الوقت
mutually exc

Example: Additive Rule

Example: Suppose that there were 120 students in the classroom, and that they could be classified as follows:

A: brown hair

$$P(A) = 50/120$$

B: female

$$P(B) = 60/120$$

	Brown	Not Brown
Male	20	40
Female	30	30

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 50/120 + 60/120 - 30/120 \\ &= 80/120 = 2/3 \end{aligned}$$

→ Brown hair or female

A Special Case

When two events A and B are **mutually exclusive**, $P(A \cap B) = 0$ and $P(A \cup B) = P(A) + P(B)$.

A: male with brown hair

$$P(A) = 20/120$$

B: female with brown hair

$$P(B) = 30/120$$

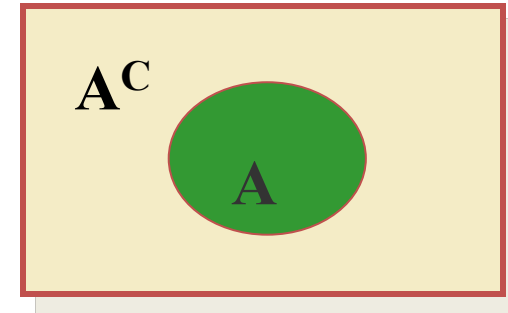
	Brown	Not Brown
Male	20	40
Female	30	30

A and B are mutually exclusive, so that

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) \\ &= 20/120 + 30/120 \\ &= 50/120 \end{aligned}$$

إذا ما يجب $P(A \cap B)$
في القانون

Calculating Probabilities for Complements



- We know that for any event **A**:

$$P(A \cap A^C) = 0$$

- Since either **A** or A^C must occur,

$$P(A \cup A^C) = 1$$

ليس 1 ؟

- so that $P(A \cup A^C) = P(A) + P(A^C) = 1$

اعادة صياغة القانون

$$P(A^C) = 1 - P(A)$$

if $P(\text{female}) = 0.3$, $P(\text{male}) = ?$
 $\rightarrow P(\text{male}) = 1 - 0.3 = \boxed{0.7}$

لأنه مجموع
الحدثين مع صنفه
1 =

Example



Select a student at random from the classroom. Define:

A: male

$$P(A) = 60/120$$

B: female

	Brown	Not Brown
Male	20	40
Female	30	30

A and B are complementary, so that

$$P(B) = 1 - P(A) = 1 - 60/120 = \frac{60}{120}$$

Calculating Probabilities for Intersections

- we can find $P(A \cap B)$ directly from the table. Sometimes this is impractical or impossible. The rule for calculating $P(A \cap B)$ depends on the idea of **independent and dependent events**.

$P(A)$ بجدون B أو
عدم حدوثه لن يتغير

$P(A)$ متاثر
بحدوث B

العلاقة بين A, B ←

Two events, **A** and **B**, are said to be **independent** if and only if the probability that event **A** occurs does not change, depending on whether or not event **B** has occurred.

Conditional Probabilities

- The probability that A occurs, given that event B has occurred is called the **conditional probability** of A given B and is defined as

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \text{ if } P(B) \neq 0$$

“given”

→ given

حساب P(A) بعد حدوث B

Conditional Probability



▶ The probability of an event given that another event has occurred is called a conditional probability.

▶ The conditional probability of A given B is denoted by $P(A | B)$.

A conditional probability is computed as follows :

▶
$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

Multiplication Law

لأنها عملية ضرب



► The multiplication law provides a way to compute the probability of the intersection of two events.

The law is written as:

►
$$P(A \cap B) = P(B)P(A | B)$$

Example 1



- Toss a fair coin twice. Define
 - A: head on second toss
 - B: head on first toss

$$P(A|\text{not } B) = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A|B) = \frac{1}{2}$$

$$P(A|\text{not } B) = \frac{1}{2}$$

$$= \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

$P(A \cap B)$

HH	1/4
HT	1/4
TH	1/4
TT	1/4

not B

P(A) does not change, whether B happens or not...

A and B are independent!

$$P(A|B) = P(A) \leftarrow$$

Independent Events



► If the probability of event A is not changed by the existence of event B , we would say that events A and B are independent.

► Two events A and B are independent if:

$$P(A | B) = P(A)$$

or

$$P(B | A) = P(B)$$

$P(A)$ does change,
depending on
whether B happens
or not...



A and B are
dependent!

Defining Independence

- We can redefine independence in terms of conditional probabilities:

Two events A and B are **independent** if and only if

$$P(A|B) = P(A) \quad \text{or} \quad P(B|A) = P(B)$$

Otherwise, they are **dependent**.

- Once you've decided whether or not two events are independent, you can use the following rule to calculate their intersection.

Multiplication Law for Independent Events



► The multiplication law also can be used as a test to see if two events are independent.

The law is written as:

►
$$P(A \cap B) = P(A)P(B)$$

القوانين ليست حقة

The Multiplicative Rule for Intersections

- For any two events, **A** and **B**, the probability that both **A** and **B** occur is

$$\begin{aligned} P(A \cap B) &= P(A) P(B \text{ given that } A \text{ occurred}) \\ &= P(A)P(B|A) \end{aligned}$$

- If the events **A** and **B** are independent, then the probability that both **A** and **B** occur is

$$P(A \cap B) = P(A) P(B)$$

Example 1 not important

In a certain population, 10% of the people can be classified as being high risk for a heart attack. Three people are randomly selected from this population.

What is the probability that exactly one of the three are high risk?

Define H: high risk

N: not high risk

$$\begin{aligned} P(\text{exactly one high risk}) &= P(HNN) + P(NHN) + P(NNH) \\ &= P(H)P(N)P(N) + P(N)P(H)P(N) + P(N)P(N)P(H) \\ &= (.1)(.9)(.9) + (.9)(.1)(.9) + (.9)(.9)(.1) = 3(.1)(.9)^2 = .243 \end{aligned}$$

male = 51%

Example 2



Suppose we have additional information in the previous example. We know that only 49% of the population are female. Also, of the female patients, 8% are high risk. A single person is selected at random. What is the probability that it is a high risk female?

Define H: high risk

F: female

From the example, $P(F) = .49$ and $P(H|F) = .08$.

Use the Multiplicative Rule:

↳ $P(\text{high risk given they are female})$

↳ high risk and female → intersection

$$P(\text{high risk female}) = P(H \cap F)$$

$$= P(F)P(H|F) = .49(.08) = .0392$$

Rules Summary

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A \cup B) = P(A) + P(B)$
- $P(A^c) = 1 - P(A)$
- $P(A/B) = \frac{P(A \cap B)}{P(B)}$
- $P(A \cap B) = P(B) \cdot P(A/B)$
- $P(A/B) = P(A)$
- $P(B/A) = P(B)$
- $P(A \cap B) = P(B) \cdot P(A)$

تعطى بالامتحان

- Additional rules:

- $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

- $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

- $P(A \cup \bar{B}) = P(\bar{B}) + P(A \cap B)$

- $P(\bar{A} \cup B) = P(\bar{A}) + P(A \cap B)$

- De Morgan's Laws

- $P(\overline{A \cup B}) = P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$

- $P(\overline{A \cap B}) = P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B)$

Examples

For the random experiment of rolling two fair dice, suppose that we define the two events A and B as follows:

A: First die show 1 → اول حجر نند بهای رقم 1

B: Second die show 1 → ثانی حجر نند بهای رقم 1

Then find the value of $P(A \cup B)$?

Solution

The outcomes of the two events are as follows:

$$A = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)\} \Rightarrow P(A) = 6 / 36$$

$$B = \{(1,1), (2,1), (3,1), (4,1), (5,1), (6,1)\} \Rightarrow P(B) = 6 / 36$$

$$A \cap B = \{(1,1)\} \Rightarrow P(A \cap B) = 1 / 36$$

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 6/36 + 6/36 - 1/36 \\ &= 11/36 \\ &= 0.306 \end{aligned}$$

Example-Disjoint Events

- For random experiment of rolling two fair dice, suppose that we define the two events A and B as follows:

A: Sum of the two numbers comes up to 3 مجموع الرقمين على حدين النرد = 3

B: Sum of the two numbers comes up to 6 مجموع الرقمين على حدين النرد = 6

Then find $P(A \cup B)$?

$$A = \{ (1,2), (2,1) \} \quad P(A) = 2/36$$

$$B = \{ (1,5), (2,4), (3,3), (4,2), (5,1) \} \quad P(B) = 5/36$$

$$P(A \cap B) = \phi = \text{zero}$$

$$\text{So } P(A \cup B) = (2/36) + (5/36) = (7/36) = 0.194$$

Examples

* شركة هيدلانية بالامارات عندها خطين تلفونات

- Suppose that in the Gulf pharmaceutical industries company in UAE, there are two telephone lines 1 and II. Let A be the event that line I is busy and let B be the event that line II is busy.

الخطين يكونوا Busy

$$P(A)=0.55 \quad P(B)=0.65 \quad \text{and} \quad P(A \cap B)=0.35$$

Answer the following: معلومات لخم اعطائها بالسؤال

a) Find the probability that both lines are free?

تقاطع
intersection

عكس
Busy
يعني متمم
complement

Examples

Solution

$$\begin{aligned}P(\text{Both lines are free}) &= P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) \\&= 1 - P(A \cup B) \\&= 1 - (P(A) + P(B) - P(A \cap B)) \\&= 1 - (0.55 + 0.65 - 0.35) \\&= 1 - 0.85 \\&= 0.15\end{aligned}$$

Continued

قاطع
↖

$\bar{B}$

b) Find the probability that line I is busy and line II is free?

Solution

$$P(\text{Line I is busy and line II is free}) = P(A \cap \bar{B})$$

$$= P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

$$= 0.55 - 0.35$$

$$= 0.20$$

Continued

c) Find the probability that line I is free or line II is busy?

Solution

$$\begin{aligned} P(\text{line I is free or line II is busy}) &= P(\bar{A} \cup B) \\ &= P(\bar{A}) + P(A \cap B) \\ &= 1 - P(A) + P(A \cap B) \\ &= 1 - (0.55 + 0.35) \\ &= 0.8 \end{aligned}$$

Additional Rules

- $P(A/\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})} = \frac{P(A) - P(A \cap B)}{1 - P(B)}$

$$P(\bar{A}/B) = \frac{P(\bar{A} \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)} = 1 - P(A/B)$$

$$P(\bar{A}/\bar{B}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = \frac{P(\overline{A \cup B})}{1 - P(B)} = \frac{1 - P(A \cup B)}{1 - P(B)} = \frac{1 - [P(A) + P(B) - P(A \cap B)]}{1 - P(B)}$$

Example

For the random experiment of rolling two fair dice, suppose that we define the two events A and B as follows:

A: First die shows 1

B: Second die show 2

Then find the value of $P(A/B)$

$P(\text{first die show 1 given that second die show 2})$

Continued

Solution

النتيجة الأولى يعطى 7 والنتيجة
الثانية يعطى 2

$$\bullet P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\bullet A = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)\}$$

$$B = \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2)\}$$

$$A \cap B = \{(1,2)\} = P(A \cap B) = 1/36$$

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/36}{6/36}$$

$$= \frac{1}{6}$$

$$= 0.167$$

Example

In the Queen Alia International Airport, suppose that the probability that a regularly scheduled flight departs on time is $P(A) = 0.83$; the probability that a regularly scheduled flight arrives on time is $P(B) = 0.92$; and the probability that it departs and arrives on time is $P(A \cap B) = 0.78$. Find the probability that a plane

$P(B \setminus A)$

- a) Arrives on time given that it departed on time, and
- b) Departed on time given that it has arrived on time?

Solution

$$\text{a) } P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{0.78}{0.83} = 0.94$$

$$\text{b) } P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.78}{0.92} = 0.85$$

Example

Suppose that we have a drug box containing 20 tablets, of which 5 are defective, if 2 tablets are selected at random and removed from the box in succession without replacing the first, what is the probability that both tablets are defective?

Solution علبة فيها 20 حبة دواء ، 5 حبات خرابة

We shall let:

مطلوب سحب حبتين

A: the event that the first tablet is defective

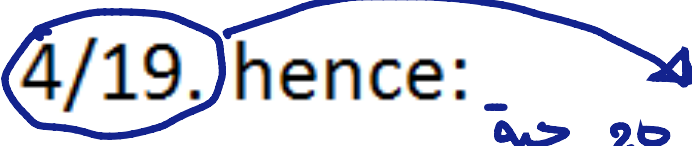
B: the event that the second tablet is defective

$A \cap B$: the event that A occurs, and then B occurs after A has occurred (both events occur)

* سحب الحبة الاولى

بعد سحب الحبة الثانية

Continued

- The probability of first removing a defective tablet is $5/20 = 1/4$, then the probability of removing a second defective tablet from the remaining 4 is $4/19$. hence:


سحبت واحدة من 20 حبة
فقط 19 والحبة طلعت defective
من الخمسة فقط 4
- $P(A \cap B) = P(B) \cdot P(A/B)$
 $= (1/4) \cdot (4/19)$
 $= 1/19 = 0.053$

Additional Rules

Definition of Independence

Two events A and B are said to be (statistically) independent if any one of the following equivalent conditions holds:

- 1) $P(A \cap B) = P(A)P(B)$
- 2) $P(A/B) = P(A)$
- 3) $P(B/A) = P(B)$

Otherwise, they are said to be dependent events

Then,

- 1) A and not B are independent, that is: $P(A \cap \bar{B}) = P(A)P(\bar{B})$
- 2) Not A and B are independent, that is: $P(\bar{A} \cap B) = P(\bar{A})P(B)$
- 3) Not A and not B are independent, that is: $P(\bar{A} \cap \bar{B}) = P(\bar{A})P(\bar{B})$

Example

Two dice are rolled. Suppose that we define the two events A and B as follows:

A: First die show 1.

B: The sum of the two numbers comes up on the two dice is 7.

Are the two events A and B independent?

Solution

$$A = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)\}$$

$$B = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$$

$$A \cap B = \{(1,6)\}$$

$$(1) \quad P(A) = \left(\frac{6}{36}\right) \text{ and } P(B) = \left(\frac{6}{36}\right)$$

$$\text{then } P(A)P(B) = \frac{1}{36}$$

$$(2) \quad P(A \cap B) = \frac{1}{36}$$

$$\text{Then we get that } P(A \cap B) = \frac{1}{36} = P(A)P(B)$$

independent ::

اذا كان $P(A)$ ،

سواء حدث B

أم لم يحدث ، لا

يتغير

Example

Two sets of cards with a letter on each card as follows are placed into separate bags :

Bag Number I



Bag Number II



} independent

Sara randomly picked one card from each bag. Find the probability that:

a) She picked the letter J and R?

b) Both letters are L?

c) Both letters are vowels?

تقاطع

إذا كان عنا كيسين ، الكيس الأول فيه
خمسة أحرف والكيس الثاني فيه ستة أحرف

Solution

(a) Probability that she picked J and R

$$= (1/5)(1/6) = 1/30 = 0.033$$

(b) Probability that both letters are L

$$= (1/5)(1/6) = 1/30 = 0.033$$

(c) Probability that both letters are vowels

$$= (3/5)(2/6) = 6/30 = 0.2$$

Contingency Table

A contingency table provides a different way of calculating probabilities. The table helps in determining conditional probabilities quite easily. The table displays sample values in relation to two different variables that may be dependent or contingent on one another. The two variables are divided into several categories with their frequencies.

Example

- A random sample of size 200 adults are classified below according to the sex and the level of education attained:

Education	Male (M)	Female (F)	Total
Elementary (E)	38	45	83
Secondary (S)	28	50	78
Higher (H)	22	17	39
Total	88	112	200

Continued

Suppose that a person is selected at random from this group, then find the probability that:

(a) the person is male?

Solution

$$P(\text{the person is male}) = P(M) = 88 / 200 = 0.44$$

(b) the person has a secondary education?

Solution

$$\begin{aligned} P(\text{the person has a secondary education}) \\ = P(S) = 78 / 200 = 0.39 \end{aligned}$$

(c) the person is not a male?

Solution

$$\begin{aligned} P(\text{the person is not a male}) &= 1 - P(\text{the person is male}) \\ &= 1 - P(M) = 1 - 0.44 = 0.56 \end{aligned}$$

-OR

$$\begin{aligned} P(\text{the person is not a male}) &= P(\text{the person is female}) \\ &= P(F) = 112 / 200 = \end{aligned}$$

Continued

d) The person is a male or has higher education?

Solution:

P(the person is a male or has higher education)

$$P(M \cup H) = P(M) + P(H) - P(M \cap H) = (88/200) + (39/200) - (22/200) = 105/200 = 0.525$$

e) The person is female and not has an elementary education

Solution

P(the person is a female and not has an elementary education)

$$P(F \cap \bar{E}) = P(F) - P(F \cap E) = (112/200) - (45/200) = 67/200 = 0.335$$

f) The person is a male or has a secondary education?

P(the person is not a female or has a secondary education)

$$P(\bar{F} \cup S) = P(\bar{F}) + P(F \cap S) = (88/200) + (50/200) = (138/200) = 0.69$$

Continued

g) The person is neither a male nor has a higher education?

solution#

P(the person is neither a male nor has a higher education)

$$=P(\bar{M} \cap \bar{H})$$

$$=P(\overline{M \cup H})$$

$$=1-P(M \cup H)$$

$$=1-[P(M)+P(H)-P(M \cap H)]$$

$$=1-[88/200+39/200-22/200]=1-(105/200)=0.475$$

h) The person is a male given that the person has a secondary education?

Solution

P(the person is a male given that the person has a secondary education)

$$P(M/S) = P(M \cap S)/P(S) = (28/200)/(78/200) = 28/78 = 0.359$$

Continued

- i) The person does not have a higher education degree given that the person is a female?

Solution

P(the person does not have a higher education degree given that the person is a female)

$$\begin{aligned} P(\overline{H}/F) &= \frac{P(\overline{H} \cap F)}{P(F)} = \frac{P(F) - P(H \cap F)}{P(F)} = 1 - P(H/F) = \\ &= 1 - \frac{P(H \cap F)}{P(F)} = 1 - ((17/200)/(112/200)) = 1 - (17/112) = 0.848 \end{aligned}$$