Date: / /

لجان الدُّفعات

Soul Academy

تفريغ إحصاء صيدلاني

موضوع المحاضرة : Lecture 12 part 1

رقم المحاضرة : سكند 4

إعداد الصيدلانيه : يارا

Probability Distribution

Discrete	Continuous
↓	↓
Binomial Distribution	Normal Distribution

دائماً (outcome) عندها = 2 .

امثلة:

الحامل ← بنت
← ولد

النتيجة ← ناجح
← راسب

Binomial Probability

Characteristics of a Binomial Experiment (Bernoulli Trial)

1. The experiment consists of n identical trials.
2. There are only two possible outcomes on each trial. We will denote one outcome by S (for success) and the other by F (for failure).
3. The probability of S (success), p remains the same (constant) from trial to trial (for all trails). This probability is denoted by p, and the probability of F is denoted by q. Note that $\boxed{q = 1 - p}$.
4. The trials are independent (outcome of one trial is not affected by the outcome of any other trial) لا تتأثر
5. The binomial random variable x is the number of S's in n trials, is said to follow Binomial Distribution with parameters n and p.
6. X can take on the values $x=0,1,\ldots,n$

Notation: $X \sim \underbrace{Bin}_{\text{Binomial}}(n,p)$

$n \rightarrow$ number of trial.

$p \rightarrow$ Probability of success in one trial.

Example: Binomial Experiment

Testing the effectiveness of a drug

- Suppose that 10 patients with identical infirmities take a drug, for each patient, it s observed whether the drug is effective or not effective. Thus a success is a cure and failure is a non-cure.

رح نرمزله (P)

- The probability of success, p, is the effectiveness of the drug cures a patient, the probability of failure, $\boxed{q=1-p}$, is the probability that the drug does not cure a patient.

$1-p = \boxed{q}$ رمزناله بـ

- Finally, we can assume that the results of administering the drug are independent from one patient to another. Hence the conditions of Binomial experiment are met.

Binomial Probability

- 2-outcome situation are very common in life, for example:

➢ Head/Tail بالعملة النقدية

➢ Effective/Ineffective

➢ Democrat/Republican بالحكم السياسي

➢ Pass/Fail

➢ Left/Right

➢ Approve/Disapprove

➢ Hit a target/Not hit a target

Example of Binomial Random Variable

- Number of customers entering a certain pharmacy out of 15 who make a purchase. عدد الزبائن الي بروحوا عالصيدلية و بيشتروا.
- Number of workers suffering from a certain disease in a random sample of 6 workers. عدد العمال الي بعانوا من مرض معين.
- Number of heads in the experiment of tossing an unbiased coin 3 times. عدد (heads) بس نرمي العملة ٣ مرات
- Number of woman out of 10 developing a breast cancer over a lifetime. عدد النساء المصابات بسرطان الثدي على طول الحياة.
- Number of correct guesses at 30 true-false questions when you randomly guess all answers. عدد التخمينات في ٣٠ سؤال (صح، خطأ).
- Number of purchases at a certain store made with credit card among 10 randomly selected purchases. عدد المبيعات في متجر معين باستخدام بطاقة الائتمان.

Binomial Probability Distribution

بهذا الشرط

- If the discrete random variable X is defined to be as "the number of successes" in the n independent Bernoulli trials, then X is said to have a Binomial distribution denoted by ***X~Bin(n,p)***, where $\boxed{n}$: is the number of trials and $\boxed{p}$: is the probability of success. If X~Bin(n,p), then the probability mass function (pmf) for X denoted by P(X=x) is given as follow:

• اذا عرّفنا (x) على انها [number of successes]
في (n) عندما تكون [independent] ← يعني
كل تجربة مستقلة عن التجربة التانية (ما بتعتمد
عليها). وبكون عنّا [2-outcome] ، بكون ال
[Probability] شبه ثابت في كل تجربة
هون بنقدر نحكي عنها ⟸ Binomial Distribution

Binomial Probability Distribution

بالامتحان نطبق القانون ~~بحساب~~ لحساب Probability

$$p(x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{x!\,(n-x)!} p^x (1-p)^{n-x}$$

$p(x)$ = Probability of x 'Successes'

... 'Success' on a single trial

قيمة الاحتمال وبكون من (1—2) فقط.

... s

... cesses' in n trials

... ilures in n trials

لنفترض مثال بسيط:
(مينا) (coins) [10] مرات، وسألنا:
What is the probability to get (3) heads?

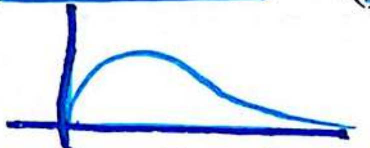
$$P(x) = \binom{10}{3} (1/2)^3 (1-0.5)^{10-3} = \text{----}$$

Binomial Probability Distribution

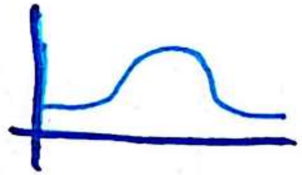
من خلال (p) نحدد الشكل التقريبي

If X~Bin (n, p), then the approximate shape for the distribution of X is given as follows:

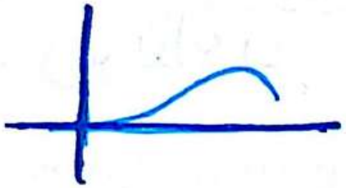
1) If $\boxed{\mathbf{p < 0.5},}$ then right-skewed (**positive**) distribution.



2) If $\boxed{\mathbf{p = 0.5}}$, then **symmetric** distribution



3) If $\boxed{\mathbf{p > 0.5},}$ then left-skewed (**negative**) distribution.



Binomial Probability Distribution Example

=n

Experiment: Toss 1 coin 5 times in a row. Note number of tails. What's the probability of 3 tails?

=x

successes

number of ← يكون

tails.

$$p(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$p(3) = \frac{5!}{3!(5-3)!} .5^3 (1-.5)^{5-3}$$

$$= .3125$$

n = 5

x = 3

P = 0.5

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Binomial Distribution Thinking Challenge

اذا كان بعمل (100) اتصال بيبيع (20) ؛ فالاحتمال

$0.20 = \frac{20}{100} = (P)$

You're a telemarketer selling service. You've sold 20 in your last 100 calls (p = **.20**). If you call **12** people tonight, what's the probability of

$n = 12$

A. No sales? $P(0) \rightarrow$ يعني (x) = صفر

B. Exactly 2 sales?

C. At most 2 sales?

D. At least 2 sales?

Binomial Distribution Solution*

$\boldsymbol{n = 12, p = .20}$

A. $p(0) = \mathbf{.0687}$

B. $p(2) = \mathbf{.2835}$

C. $p(\text{at most } 2)$

$$
\begin{aligned}
&= p(0) + p(1) + p(2)\\
&= .0687 + .2062 + .2835\\
&= \mathbf{.5584}
\end{aligned}
$$

D. $p(\text{at least } 2)$

$$
\begin{aligned}
&= p(2) + p(3) \ldots + p(12)\\
&= 1 - [p(0) + p(1)]\\
&= 1 - .0687 - .2062\\
&= \mathbf{.7251}
\end{aligned}
$$

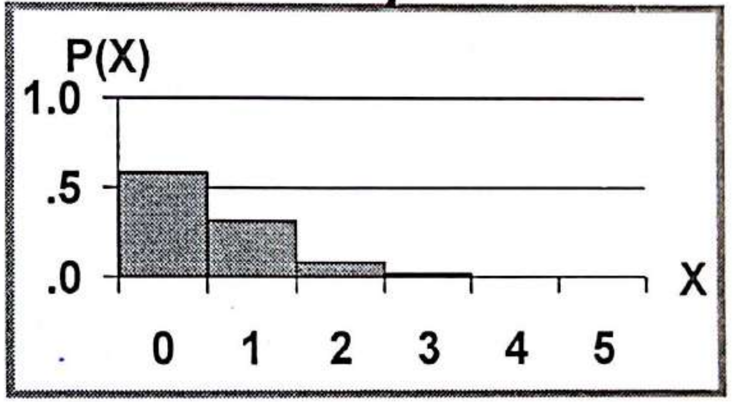
Binomial Distribution Characteristics

Mean

$$\mu = E(x) = np$$

$= 5 * 0.1 =$

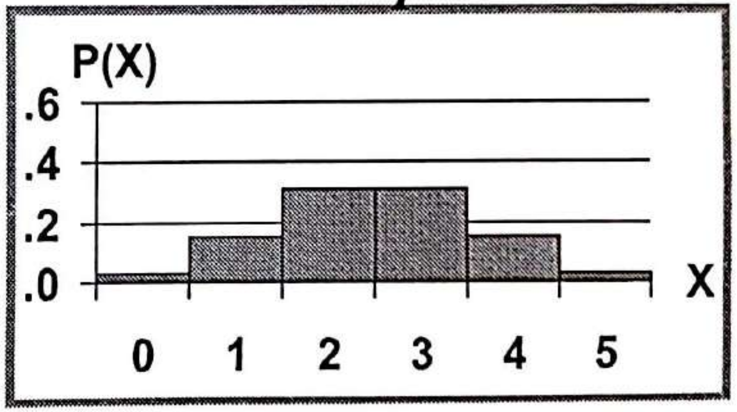
$n = 5 \quad p = 0.1$



Standard Deviation

$$\sigma = \sqrt{npq}$$

$n = 5 \quad p = 0.5$



Recall that q = 1 - p

$P = 22\%$

Example

- Suppose it is known a new drug is successful in curing a muscular pain in $\boxed{22\%}$ of the cases. If it is tried on a random sample of 5 patients, then answer the following:

$n=5$

$6(q = 1 - 0.22)$

a) Find the probability that:

1) No one of patient will be cured? $\rightarrow x=0$
2) Exactly 3 patients will be cured? $\rightarrow x=3$
3) At least 2 patients will be cured? $P(2), P(3), P(4), P(5)$
4) At most 4 patients will be cured? $P(0) + P(1), P(2), P(3), P(4)$ $1 - P(5)$
5) From 1 to 4 patients will be cured? $\rightarrow P(1) + P(2) + P(3) + P(4)$

b) Find the mean and standard deviation for the distribution of the number of patients who are cured? $\mu = n \cdot P = 0.22 * 5$ / $\sqrt{npq}$

c) What is the approximate shape for the distribution of the number of patients who are cured? $P < 0.5 \longrightarrow$ Positive.

Solution

- n=5, p=0.22, X~Bin(5,0.22)

$$P(X=0)=\binom{5}{0}(0.22)^{0}(0.78)^{5-0}=\frac{5!}{0!5!}(1)(0.289)=(1)(1)(0.289)=0.289$$

$$P(X=1)=\binom{5}{1}(0.22)^{1}(0.78)^{5-1}=\frac{5!}{1!4!}(0.22)(0.370)=(5)(0.22)((0.370)=0.407$$

$$P(X=2)=\binom{5}{2}(0.22)^{2}(0.78)^{5-2}=\frac{5!}{2!3!}(0.0484)(0.475)=(10)(0.0484)((0.475)=0.229$$

$$P(X=3)=\binom{5}{3}(0.22)^{3}(0.78)^{5-3}=\frac{5!}{3!2!}(0.010648)(0.6084)=(10)(0.010648)(0.6084)=0.065$$

$$P(X=4)=\binom{5}{4}(0.22)^{4}(0.78)^{5-4}=\frac{5!}{4!1!}(0.00234256)(0.78)=(5)(0.00234256)(0.78)=0.009$$

$$P(X=4)=\binom{5}{5}(0.22)^{5}(0.78)^{5-5}=\frac{5!}{5!0!}(0.0005153632)(1)=(1)(0.0005153632)(1)=0.001$$

هيك بيكون بالامتحان

Continued

x	0	1	2	3	4	5	sum
P(X=x)	0.289	0.407	0.229	0.065	0.009	0.001	1

دائماً

a) The probability:

1) Probability that no one of patients will be cured = $P(X=0)=0.289$

2) Probability that exactly 3 patients will be cured $=P(X=3)=0.065$

3) Probability that at least 2 patients will be cured = $P(X \geq 2)$

$= P(X=2)+P(X=3)+P(X=4)+P(X=5)$

$= 0.229+0.065+0.009+0.001$

$= 0.304$ **OR**

The probability that at least 2 patients will be cured

$= P(X \geq 2) = 1- P(X<2)$

$= 1-(P(X=0)+P(X=1))$

$= 1-(0.289+0.407) = 1- 0.696 = 0.304$

Continued

4) Probability that at most 4 patients will be cured

$= P(X \leq 4)$

$= P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)$

$= 0.289+0.407+0.229+0.065+0.009 = 0.999$

OR

Probability that at most 4 patients will be cured

$= P(X \leq 4) = 1 - (P > 4) = 1 - (P(X=5) = 1 - 0.001 = 0.999$

5) Probability that from 1 to 4 patients will be cured

$= P(1 \leq X \leq 4)$

$= P(X=1)+P(X=2)+P(X=3)+P(X=4)$

$= 0.407+0.229+0.065+0.009$

$= 0.71$

Continued

b) The mean and the standard deviation for the number of patients who are cured can be calculated as follows:

- The mean $\mu = E(x) = np$

$$= (5)(0.22) = 1.1 \text{ patients}$$

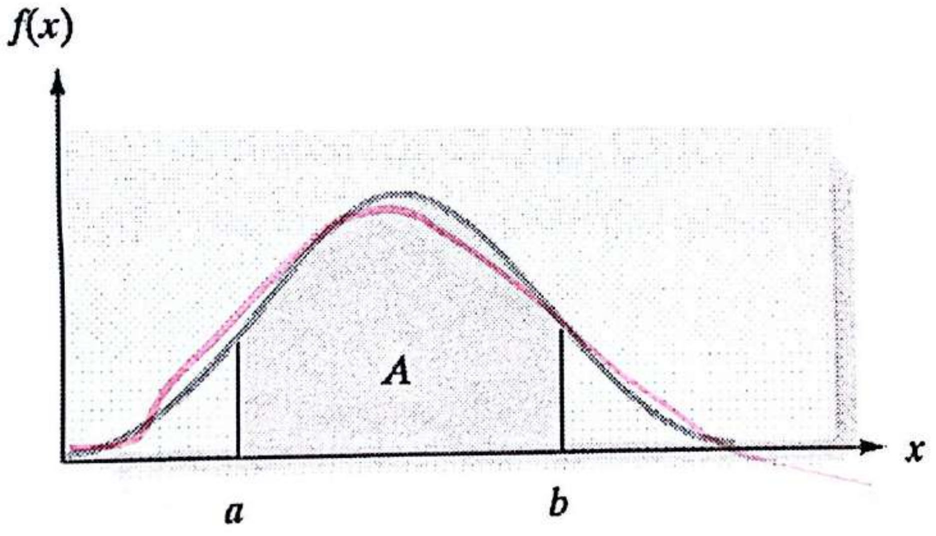
- The standard deviation $\sigma = \sqrt{npq}$

$$\sigma = \sqrt{np(1-q)} = \sqrt{(5)(0.22)(0.78)} = 0.926 \text{ patient.}$$

c) The approximate shape of the distribution of the number of patients who are cured is right-skewed (positive) because $p = 0.22$ which is less than 0.5.

Continuous Probability Density Function

The graphical form of the probability distribution for a continuous random variable x is a smooth curve

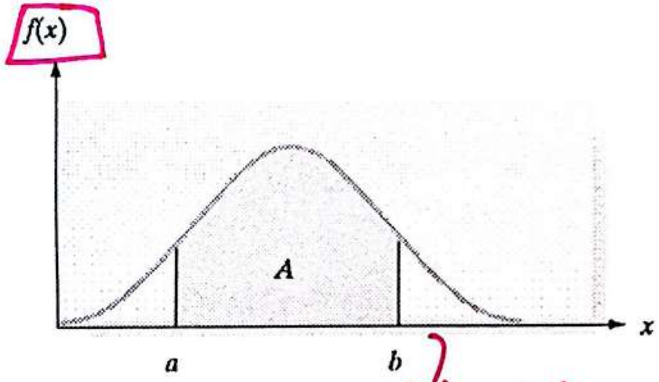


Probability Distributions for Continuous Random Variables

Continuous Probability Density Function

This curve, a function of x, is denoted by the symbol $f(x)$ and is variously called a **probability density function (pdf)**, a **frequency function**, or a **probability distribution**.

The areas under a probability distribution correspond to probabilities for $\boxed{x}$. The area A beneath the curve between two points $\boxed{a}$ and $\boxed{b}$ is the probability that $\boxed{x}$ assumes a value between a and b.



probability $= \underline{\underline{1}}$

Probability Distributions for Continuous Random Variables

- ~~The probability distribution of a continuous random~~ variable is called $\boxed{\text{a continuous probability distribution.}}$
- The most important of ~~continuous probability~~ distribution in statistics (in life) is the normal distribution (some times referred to as Gaussian distribution).

مهمة بحياتنا لانه كل الاشياء الي بتنتمي:

- Biologic
- medicine
- Plant cell

→ normal distribution عادةً اتكون

Gaussian distribution أو نسميها

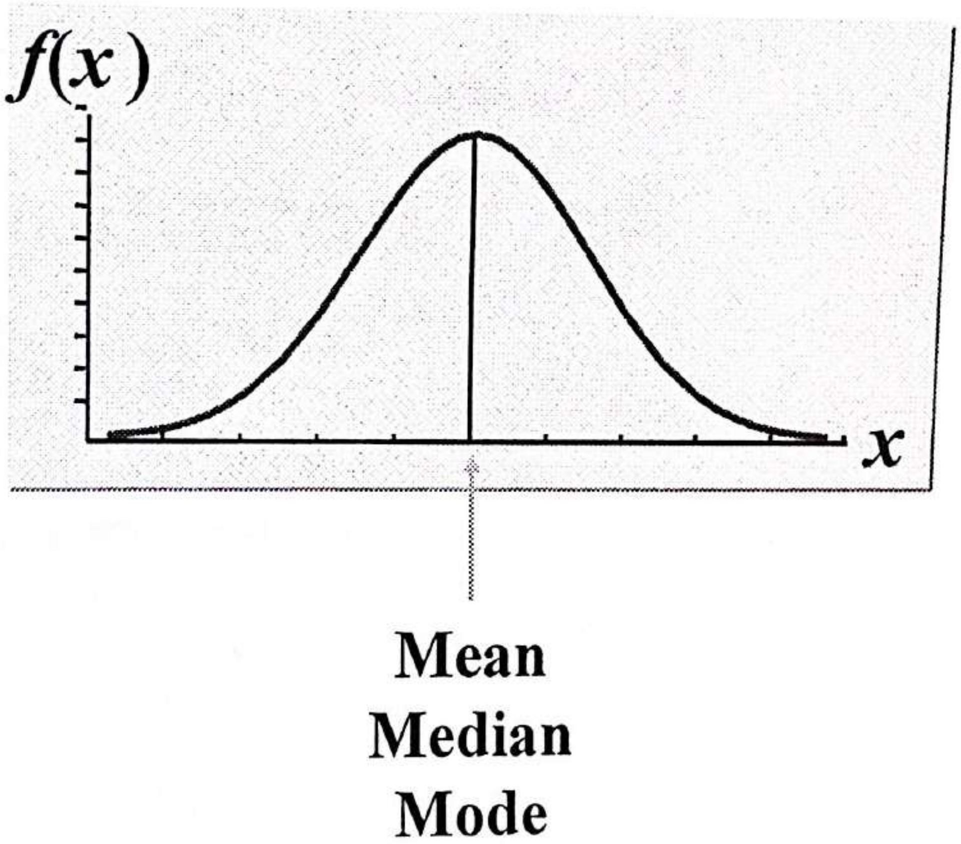
The Normal Distribution

Importance of Normal Distribution

1. Describes many random processes or continuous phenomena
2. Can be used to approximate discrete probability distributions
   - Example: binomial
3. Basis for classical statistical inference

Normal Distribution

1. 'Bell-shaped' & symmetrical
2. Mean, median, mode are equal
3. Continuous probability distribution



Probability Density Function (pdf)

هذه المعادلة لن نستخدمها

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{1}{2}\right)\left(\frac{x-\mu}{\sigma}\right)^2}$$

where

μ = Mean of the normal random variable x

σ = Standard deviation

π = 3.1415 . . .

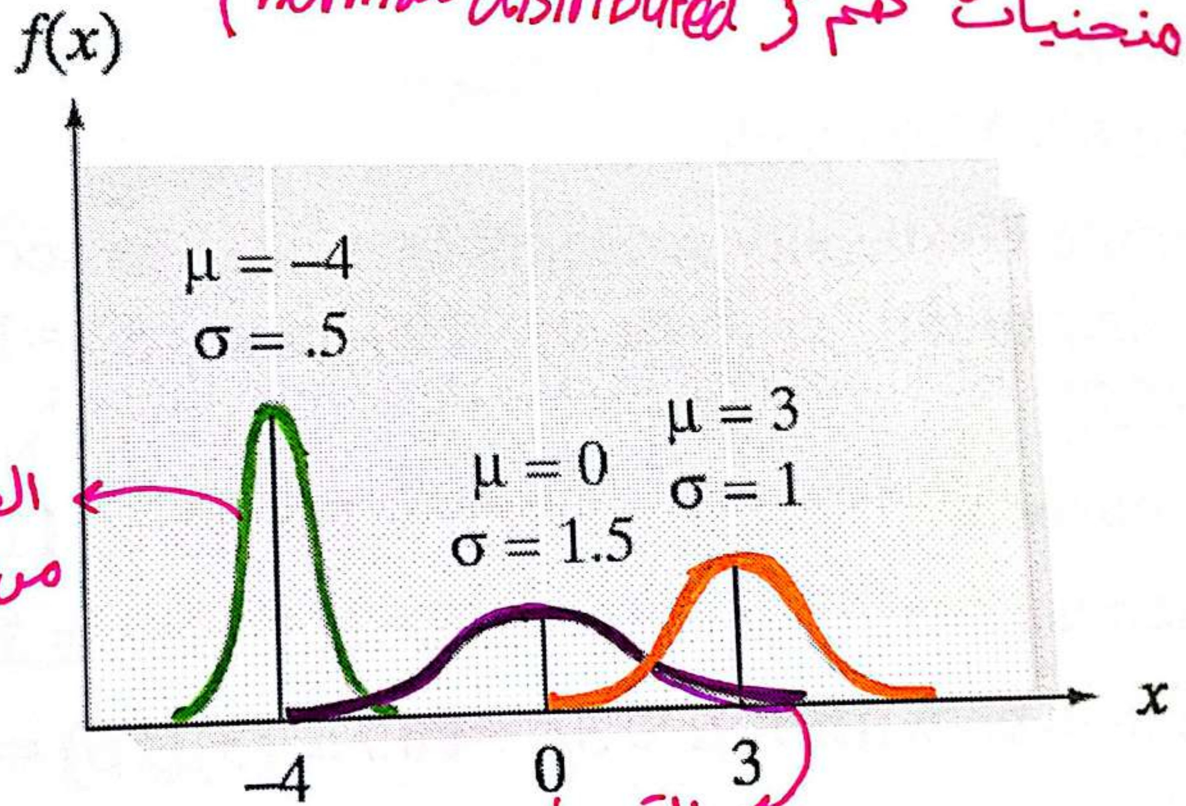
e = 2.71828 . . .

$P(x < a)$ is obtained from a table of normal probabilities

X

Effect of Varying Parameters (μ & σ)

* الثلاثة منحنيات هم ك (normal distributed



القيم قريبة من بعض

القيم فيه متباعدة

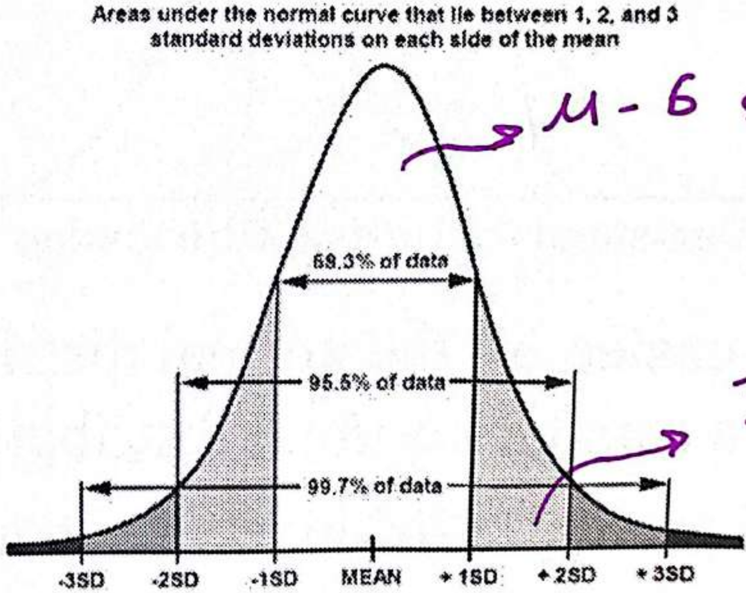
Properties of the Normal Density Curve

خصائصه:

1. It is symmetric about its mean μ
2. The highest point occur at x=μ
3. It has inflection points at μ - σ and μ + σ
4. The area under the curve is one.
5. The area under the curve to the right of μ equals the area under the curve to the left of μ equals ½.
6. The mean median, and mode are equal.

The Beauty of the Normal Curve

- No matter what μ and σ are, the area between μ - σ and μ + σ is about $\boxed{68\%}$; the area between μ - 2σ and μ + 2σ is about $\boxed{99.7\%.}$ Almost all values fall within 3 standard deviations (68-95-99.7 Rule)



μ - σ & μ + σ

3 standard deviation

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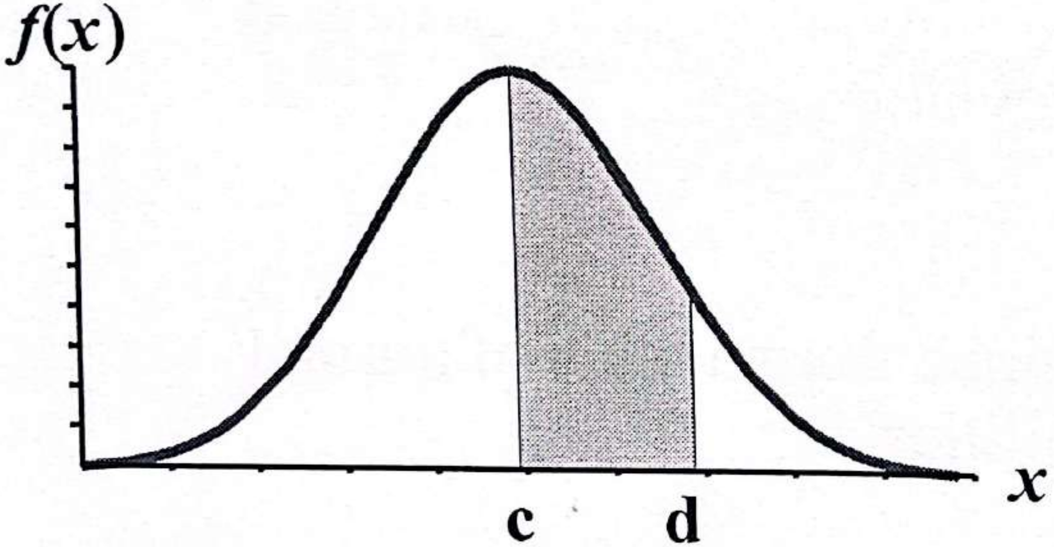
Normal Distribution Probability

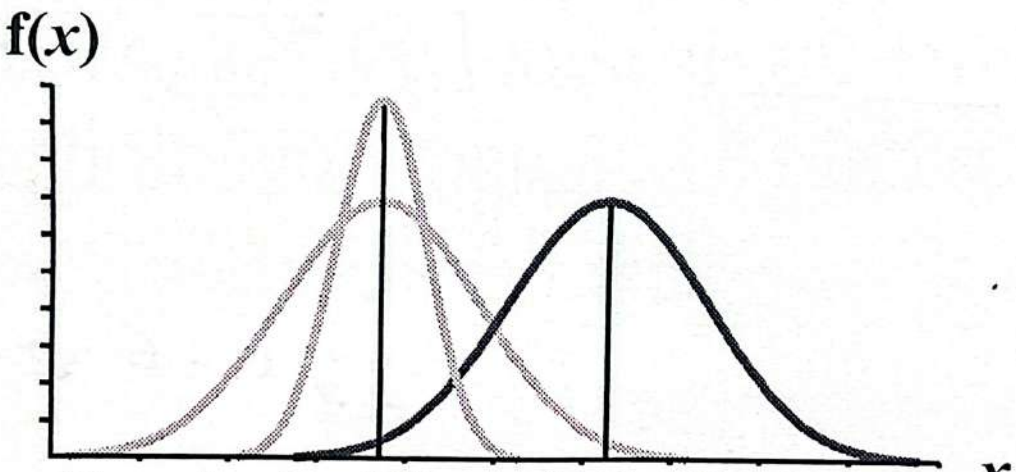
Probability is area under curve!

$$P(c \le x \le d) = \int_c^d f(x)dx\ ?$$



Normal Distribution Probability

Normal distributions differ by mean & standard deviation. Since the shapes are different, the areas under the curves between any two points are also different.



Non-standard Normal Distribution

- To make life easier, all the normal distributions can be converted to a standard normal distribution. بنقدر نحولها
- The standard normal distribution has a $\boxed{\text{mean } \mu = 0}$ and σ $\boxed{\sigma = 1.}$ خصائص

دائماً ثابتة

Property of Normal Distribution

If x is a normal random variable with mean μ and standard deviation σ, then the random variable z, defined by the formula

(Z) ← (X) تحول

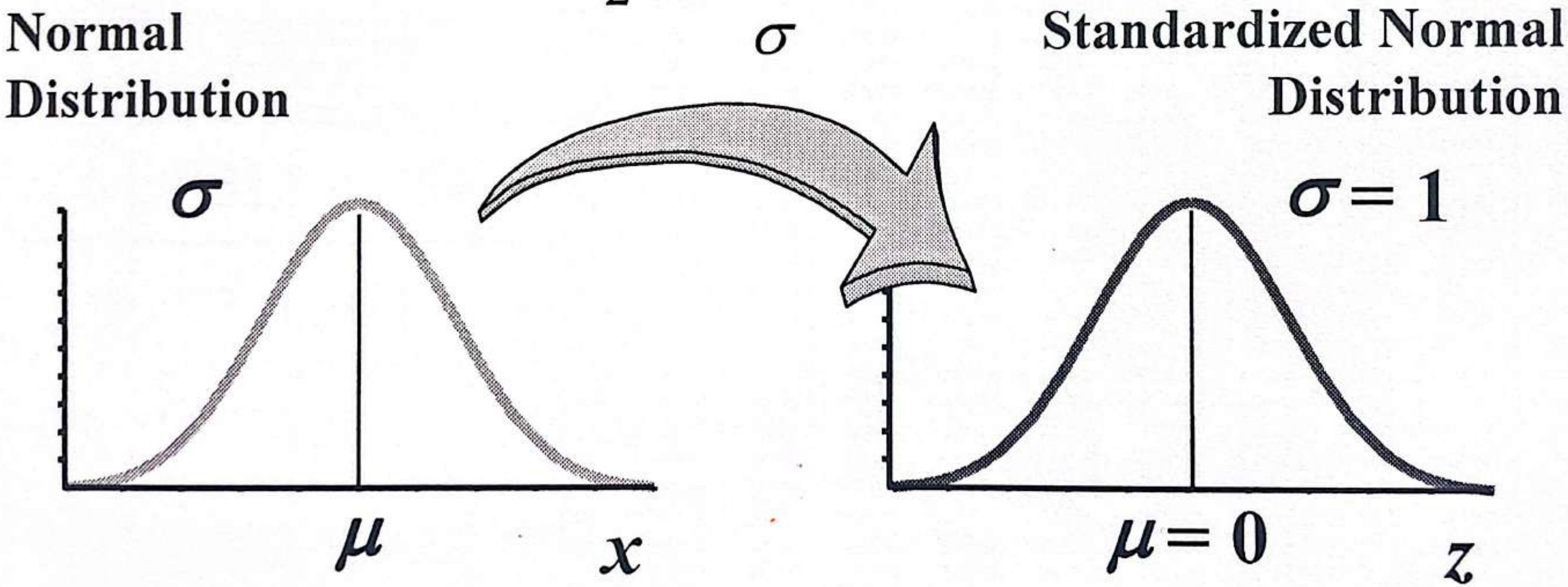
$$\boxed{z = \frac{x - \mu}{\sigma}}$$

تمثل

has a standard normal distribution. The value z describes the number of standard deviations between x and μ.

Standardize the Normal Distribution

$$z = \frac{x - \mu}{\sigma}$$



One table!

Finding a Probability Corresponding to a Normal Random Variable

1. Sketch normal distribution, indicate mean, and shade the area corresponding to the probability you want.
2. Convert the boundaries of the shaded area from x values to standard normal random variable z

$$\boxed{z = \frac{x - \mu}{\sigma}}$$

Show the z values under corresponding x values.

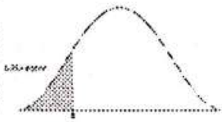
3. Use Z score table to find the areas corresponding to the z values.

بنستخدم الجدول

✗ ليست حفظ
وموجودة في
الامتحان، فقط
كيفية الاستخدام.

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414
-0.1	.46017	.45620	.45224	.44828	.44433	.44034	.43640	.43251	.42858	.42465
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-1	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08692	.08534	.08379	.08226
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-2	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-3	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-4	.00003	.00003	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002

Negative Z score table



Use the negative Z score table below to find values on the left of the mean as can be seen in the graph alongside. Corresponding values which are less than the mean are marked with a negative score in the z-table and represent the area under the bell curve to the left of z.