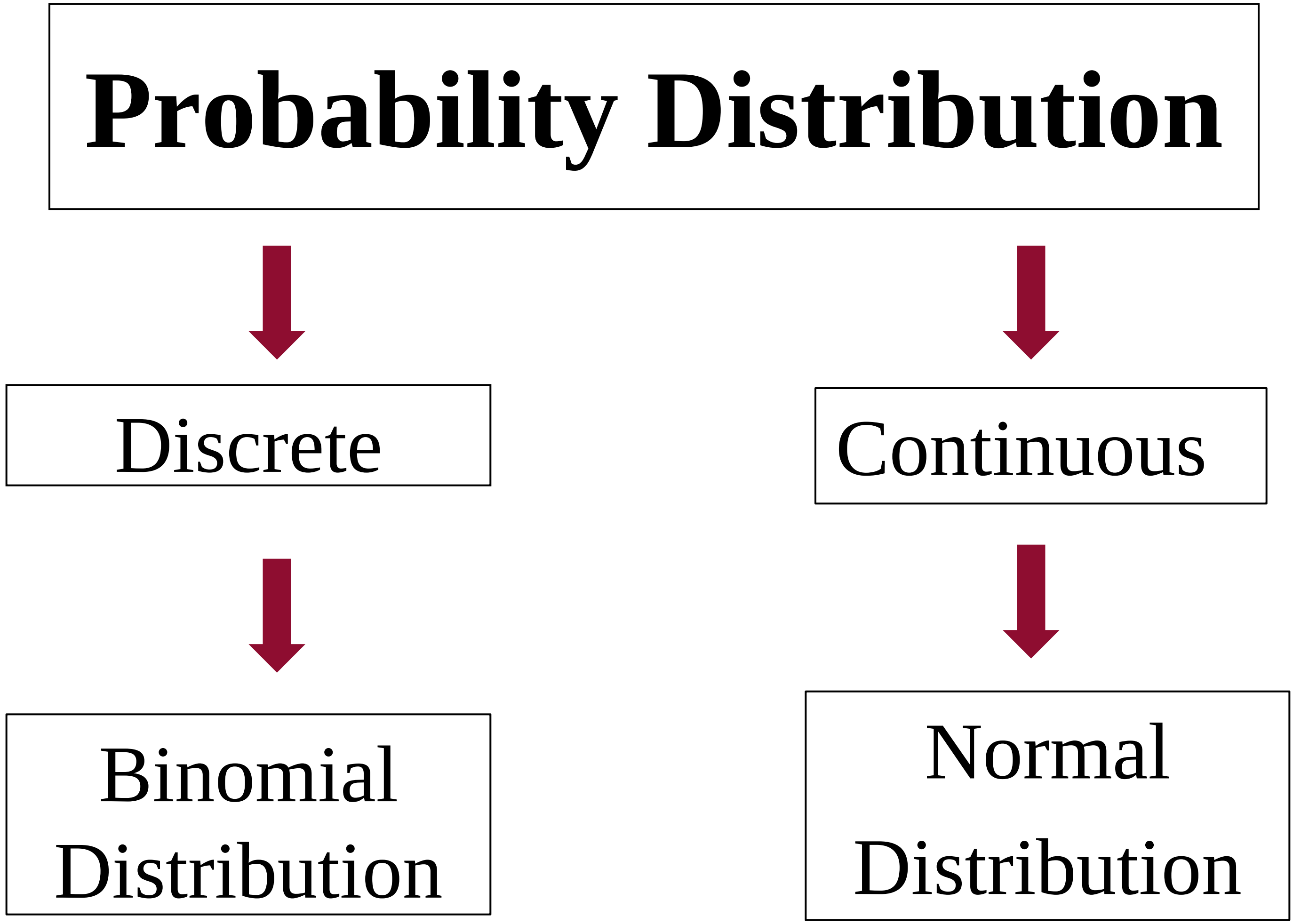

Probability Distribution
Discrete
Continuous
Binomial Distribution
Normal Distribution

Binomial Probability

Characteristics of a Binomial Experiment (Bernoulli Trial)

1. The experiment consists of n identical trials.

2. There are only two possible outcomes on each trial. We will denote one outcome by S (for success) and the other by F (for failure).

3. The probability of S (success), p remains the same (constant) from trial to trial (for all trails). This probability is denoted by p, and the probability of F is denoted by q. Note that $q = 1 - p$.

4. The trials are independent (outcome of one trial is not affected by the outcome of any other trial)

5. The binomial random variable x is the number of S's in n trials, is said to follow Binomial Distribution with parameters n and p.

6. X can take on the values $x=0,1,\ldots,n$

Notation: $X \sim Bin(n,p)$

Example: Binomial Experiment

Testing the effectiveness of a drug

- Suppose that 10 patients with identical infirmities take a drug, for each patient, it s observed whether the drug is effective or not effective. Thus a success is a cure and failure is a non-cure.

- The probability of success, p, is the effectiveness of the drug cures a patient, the probability of failure, q=1-p, is the probability that the drug does not cure a patient.

- Finally, we can assume that the results of administering the drug are independent from one patient to another. Hence the conditions of Binomial experiment are met.

Binomial Probability

- 2-outcome situation are very common in life, for example:
  - Head/Tail
  - Effective/Ineffective
  - Democrat/Republican
  - Pass/Fail
  - Left/Right
  - Approve/Disapprove
  - Hit a target/Not hit a target

Example of Binomial Random Variable

- Number of customers entering a certain pharmacy out of 15 who make a purchase.
- Number of workers suffering from a certain disease in a random sample of 6 workers.
- Number of heads in the experiment of tossing an unbiased coin 3 times.
- Number of woman out of 10 developing a breast cancer over a lifetime.
- Number of correct guesses at 30 true-false questions when you randomly guess all answers.
- Number of purchases at a certain store made with credit card among 10 randomly selected purchases.

Binomial Probability Distribution

- If the discrete random variable X is defined to be as “the number of successes” in the n independent Bernoulli trials, then X is said to have a Binomial distribution denoted by ***X~Bin(n,p),*** where n is the number of trials and p is the probability of success. If X~Bin(n,p), then the probability mass function (pmf) for X denoted by P(X=x) is given as follow:

Binomial Probability Distribution

$$p(x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{x!\,(n-x)!} p^x (1-p)^{n-x}$$

$p(x)$ = Probability of x 'Successes'

p = Probability of a 'Success' on a single trial

q = $1 - p$

n = Number of trials

x = Number of 'Successes' in n trials
($x = 0, 1, 2, ..., n$)

$n - x$ = Number of failures in n trials

Binomial Probability Distribution

If $X \sim Bin(n, p)$, then the approximate shape for the distribution of X is given as follows:

1) If **p < 0.5**, then right-skewed (**positive**) distribution.

2) If **p = 0.5**, then **symmetric** distribution

3) If **p > 0.5**, then left-skewed (**negative**) distribution.

Binomial Probability Distribution Example

Experiment: Toss 1 coin 5 times in a row. Note number of tails. What's the probability of 3 tails?

$$p(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$p(3) = \frac{5!}{3!(5-3)!} .5^3 (1-.5)^{5-3}$$

$$= .3125$$

Binomial Distribution Thinking Challenge

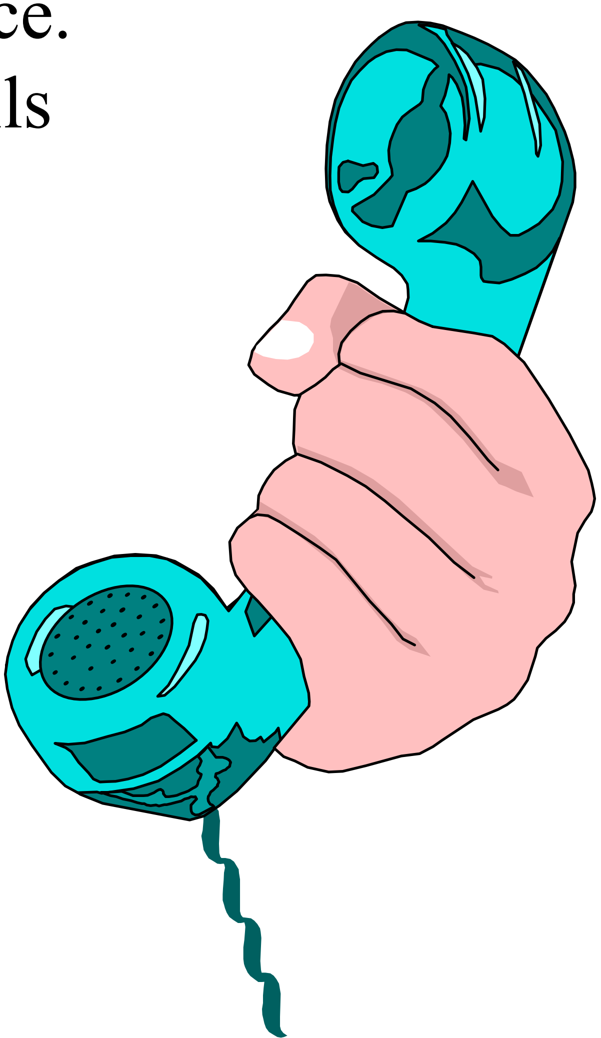
You're a telemarketer selling service. You've sold 20 in your last 100 calls (***p* = .20**). If you call **12** people tonight, what's the probability of

A. No sales?

B. Exactly 2 sales?

C. At most 2 sales?

D. At least 2 sales?



Binomial Distribution Solution*

$\boldsymbol{n = 12, p = .20}$

A. $p(0) =$ **.0687**

B. $p(2) =$ **.2835**

C. $p(\text{at most } 2)$

$$= p(0) + p(1) + p(2)$$
$$= .0687 + .2062 + .2835$$
$$= \mathbf{.5584}$$

D. $p(\text{at least } 2)$

$$= p(2) + p(3)...+ p(12)$$
$$= 1 - [p(0) + p(1)]$$
$$= 1 - .0687 - .2062$$
$$= \mathbf{.7251}$$

Binomial Distribution Characteristics

Mean

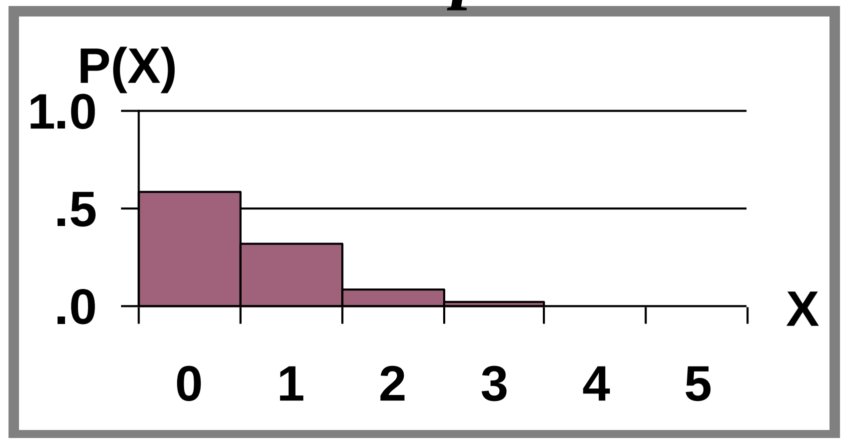
$$\mu = E(x) = np$$

Standard Deviation

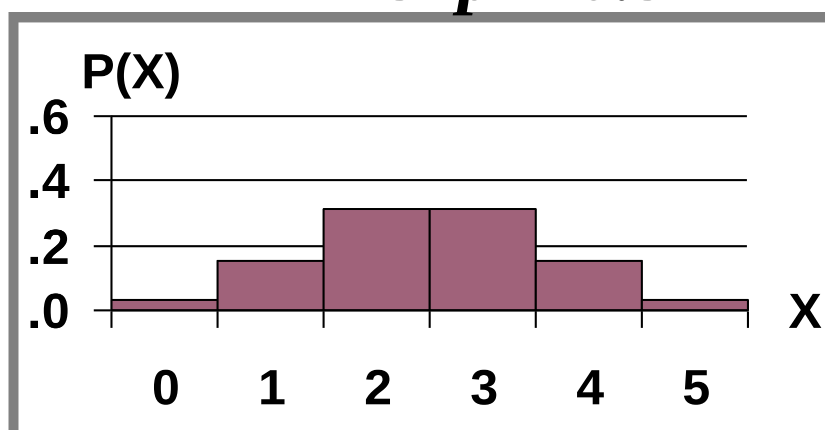
$$\sigma = \sqrt{npq}$$

Recall that q = 1 - p

***n* = 5 *p* = 0.1**



***n* = 5 *p* = 0.5**



Example

- Suppose it is known a new drug is successful in curing a muscular pain in 22% of the cases. If it is tried on a random sample of 5 patients, then answer the following:

a) Find the probability that:

1) No one of patient will be cured?
2) Exactly 3 patients will be cured?
3) At least 2 patients will be cured?
4) At most 4 patients will be cured?
5) From 1 to 4 patients will be cured?

b) Find the mean and standard deviation for the distribution of the number of patients who are cured?

c) What is the approximate shape for the distribution of the number of patients who are cured?

Solution

- n=5, p=0.22, X~Bin(5,0.22)

$$P(X=0)=\binom{5}{0}(0.22)^0(0.78)^{5-0}=\frac{5!}{0!5!}(1)(0.289)=(1)(1)(0.289)=0.289$$

$$P(X=1)=\binom{5}{1}(0.22)^1(0.78)^{5-1}=\frac{5!}{1!4!}(0.22)(0.370)=(5)(0.22)((0.370)=0.407$$

$$P(X=2)=\binom{5}{2}(0.22)^2(0.78)^{5-2}=\frac{5!}{2!3!}(0.0484)(0.475)=(10)(0.0484)(0.475)=0.229$$

$$P(X=3)=\binom{5}{3}(0.22)^3(0.78)^{5-3}=\frac{5!}{3!2!}(0.010648)(0.6084)=(10)(0.010648)(0.6084)=0.065$$

$$P(X=4)=\binom{5}{4}(0.22)^4(0.78)^{5-4}=\frac{5!}{4!1!}(0.00234256)(0.78)=(5)(0.00234256)(0.78)=0.009$$

$$P(X=4)=\binom{5}{5}(0.22)^5(0.78)^{5-5}=\frac{5!}{5!0!}(0.0005153632)(1)=(1)(0.0005153632)(1)=0.001$$

Continued

x	0	1	2	3	4	5	sum
P(X=x)	0.289	0.407	0.229	0.065	0.009	0.001	1

a) The probability:

1) Probability that no one of patients will be cured = $P(X=0)=0.289$.

2) Probability that exactly 3 patients will be cured $=P(X=3)=0.065$.

3) Probability that at least 2 patients will be cured = $P(X \geq 2)$

$= P(X=2)+P(X=3)+P(X=4)+P(X=5)$

$= 0.229+0.065+0.009+0.001$

$= 0.304$ **OR**

The probability that at least 2 patients will be cured

$= P(X \geq 2) = 1- P(X<2)$

$= 1-(P(X=0)+P(X=1))$

$= 1-(0.289+0.407) = 1- 0.696 = 0.304$

Continued

4) Probability that at most 4 patients will be cured

$= P(X \leq 4)$

$= P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)$

$= 0.289+0.407+0.229+0.065+0.009 = 0.999$

OR

Probability that at most 4 patients will be cured

$= P(X \leq 4) = 1 - (P > 4) = 1 - (P(X=5) = 1 - 0.001= \ 0.999$

5) Probability that from 1 to 4 patients will be cured

$= P(1 \leq X \leq 4)$

$= P(X=1)+P(X=2)+P(X=3)+P(X=4)$

$= 0.407+0.229+0.065+0.009$

$= 0.71$

Continued

b) The mean and the standard deviation for the number of patients who are cured can be calculated as follows:

- The mean $\mu = E(x) = np$

$$= (5)(0.22) = 1.1 \text{ patients}$$

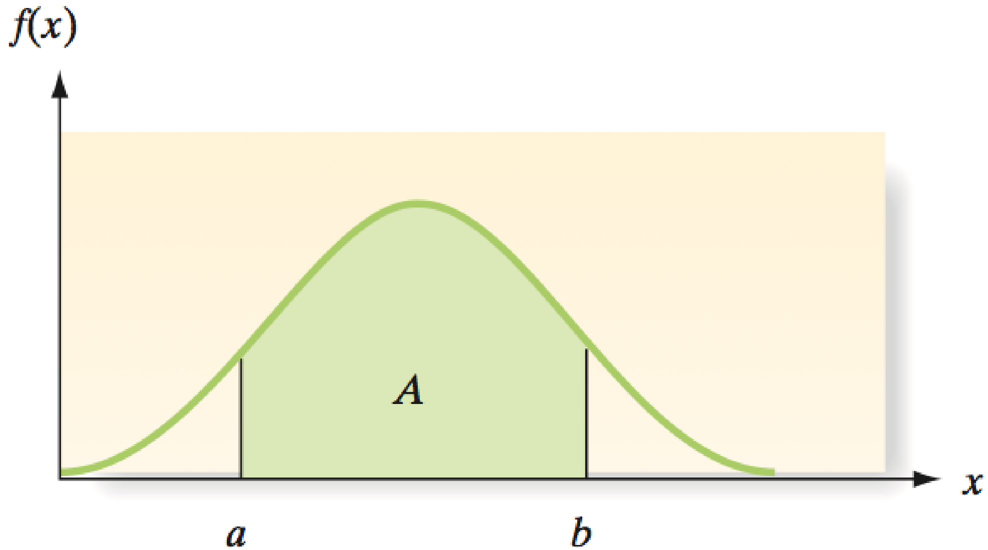
- The standard deviation $\sigma = \sqrt{npq}$

$\sigma = \sqrt{np(1-q)} = \sqrt{(5)(0.22)(0.78)} = 0.926$ patient.

c) The approximate shape of the distribution of the number of patients who are cured is right-skewed (positive) because p = 0.22 which is less than 0.5.

Continuous Probability Density Function

The graphical form of the probability distribution for a continuous random variable *x* is a smooth curve

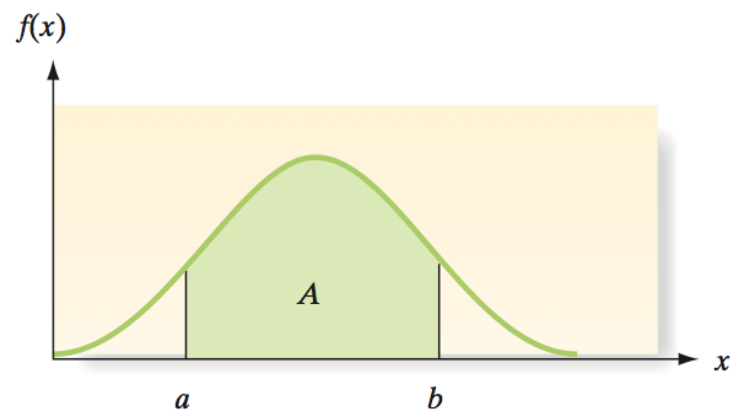


Probability Distributions for Continuous Random Variables

Continuous Probability Density Function

This curve, a function of x, is denoted by the symbol $f(x)$ and is variously called a **probability density function (pdf),** a **frequency function**, or a **probability distribution**.

The areas under a probability distribution correspond to probabilities for x. The area A beneath the curve between two points a and b is the probability that x assumes a value between a and b.



Probability Distributions for Continuous Random Variables

- The probability distribution of a continuous random variable is called a continuous probability distribution.
- The most important of continuous probability distribution in statistics (in life) is the normal distribution (some times referred to as Gaussian distribution).

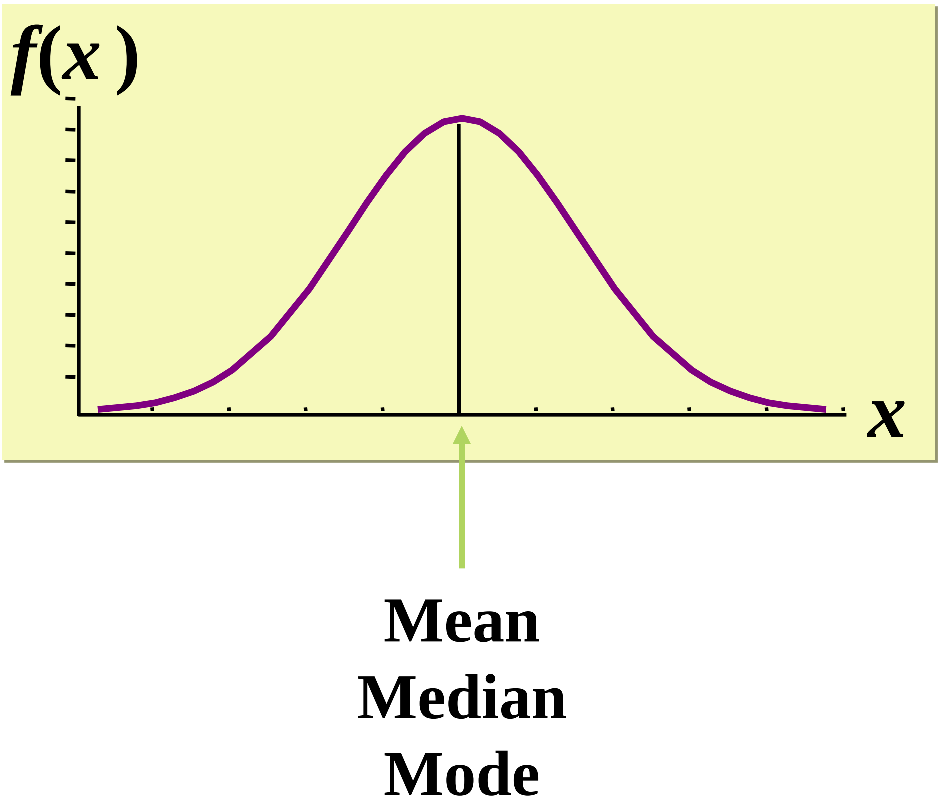
The Normal Distribution

Importance of Normal Distribution

1. Describes many random processes or continuous phenomena
2. Can be used to approximate discrete probability distributions
   - Example: binomial
3. Basis for classical statistical inference

Normal Distribution

1. 'Bell-shaped' & symmetrical
2. Mean, median, mode are equal
3. Continuous probability distribution



Probability Density Function (pdf)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\left(\frac{1}{2}\right)\left(\frac{x-\mu}{\sigma}\right)^2}$$

where

μ = Mean of the normal random variable x

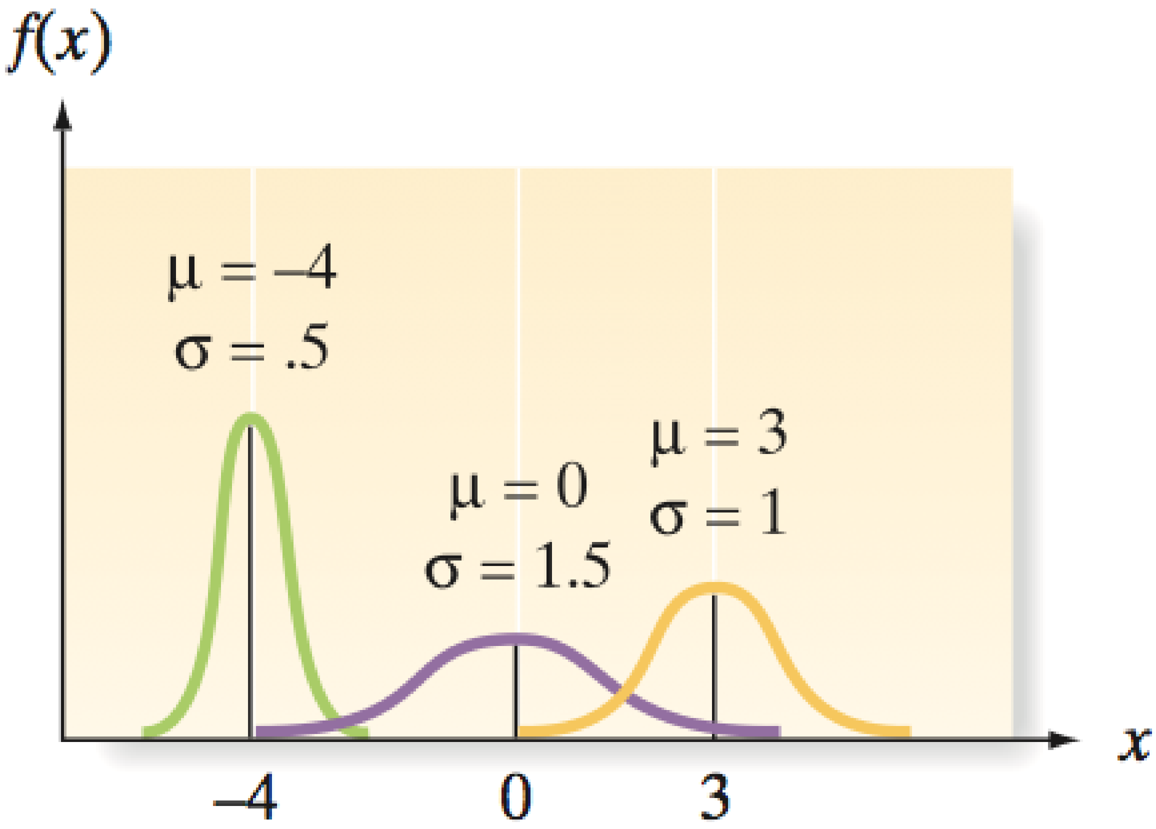
σ = Standard deviation

π = 3.1415 . . .

e = 2.71828 . . .

$P(x < a)$ is obtained from a table of normal probabilities

Effect of Varying Parameters (μ & σ)

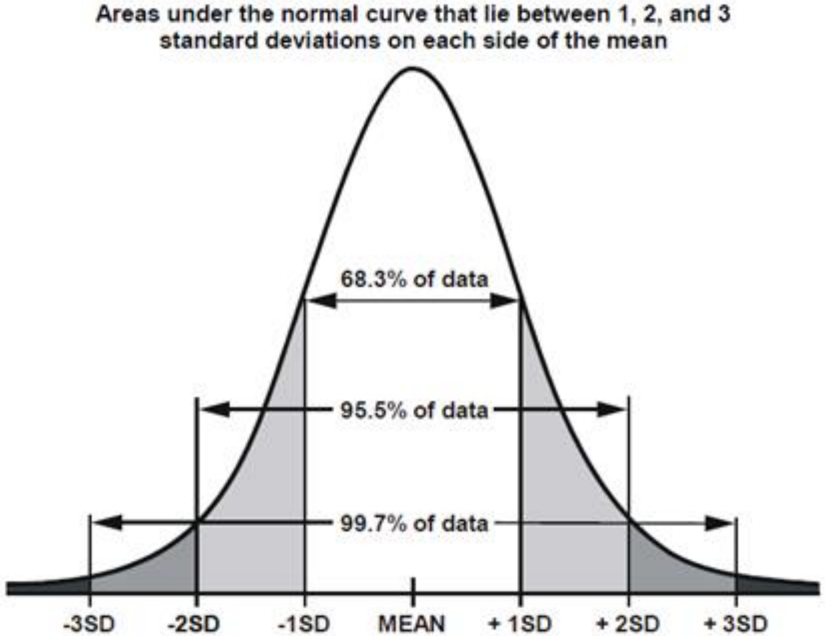


Properties of the Normal Density Curve

1. It is symmetric about its mean μ
2. The highest point occur at $x=\mu$
3. It has inflection points at $\mu - \sigma$ and $\mu + \sigma$
4. The area under the curve is one.
5. The area under the curve to the right of μ equals the area under the curve to the left of μ equals ½.
6. The mean median, and mode are equal.

The Beauty of the Normal Curve

- No matter what μ and σ are, the area between $\mu - \sigma$ and $\mu + \sigma$ is about 68%; the area between $\mu - 2\sigma$ and $\mu + 2\sigma$ is about 99.7%. Almost all values fall within 3 standard deviations (68-95-99.7 Rule)



The Beauty of the Normal Curve

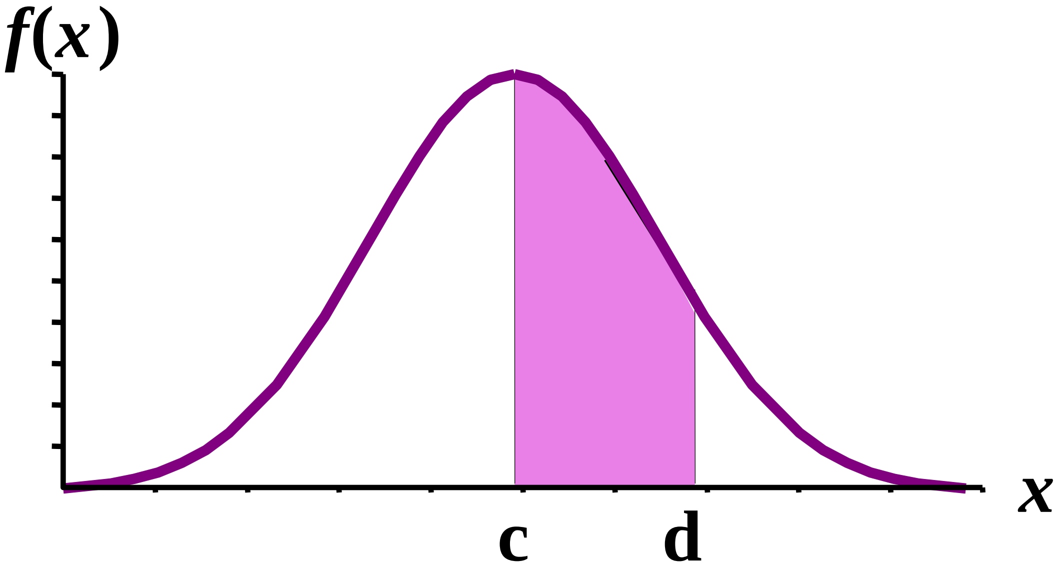
- Note that, when X is normally distributed with the mean μ and standard deviation σ, then we refer to that as follows: $\boldsymbol{X \sim N(\mu,\sigma^2)}$.
- Example: 68-95-99 Rule

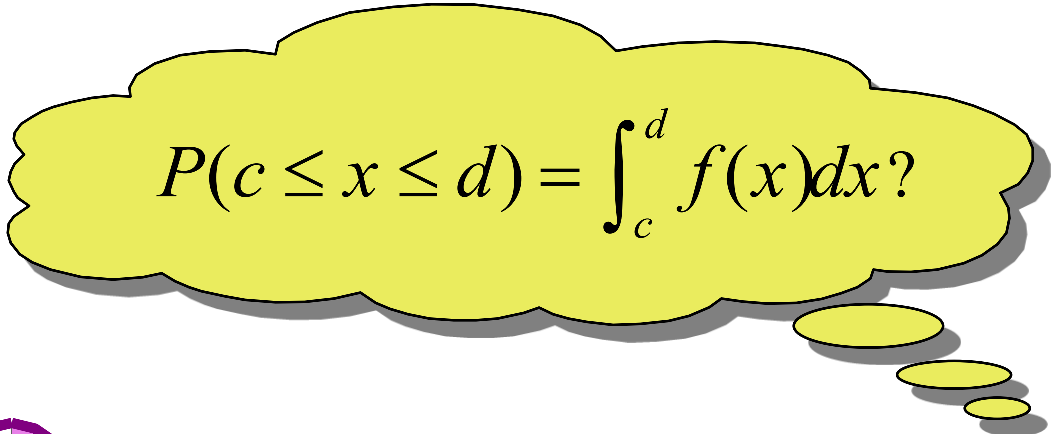
The Hashemite University students intelligence scores (X) are normally distributed with μ = 100 and σ =15; that is $\boldsymbol{X \sim N(100,255)}$.

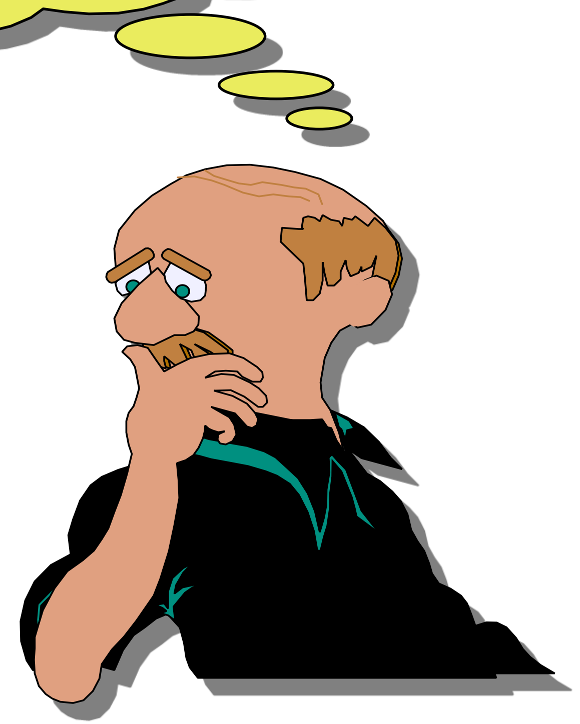
- **68% of scores within μ ± σ =100 ± 15=85 to 115**
- **95% of scores within μ ± 2σ =100 ± (2)(15) = 70 to 130**
- **99.7% of scores within μ ± 3σ =100 ± (3)(15) = 55 to 145**

Normal Distribution Probability

Probability is area under curve!

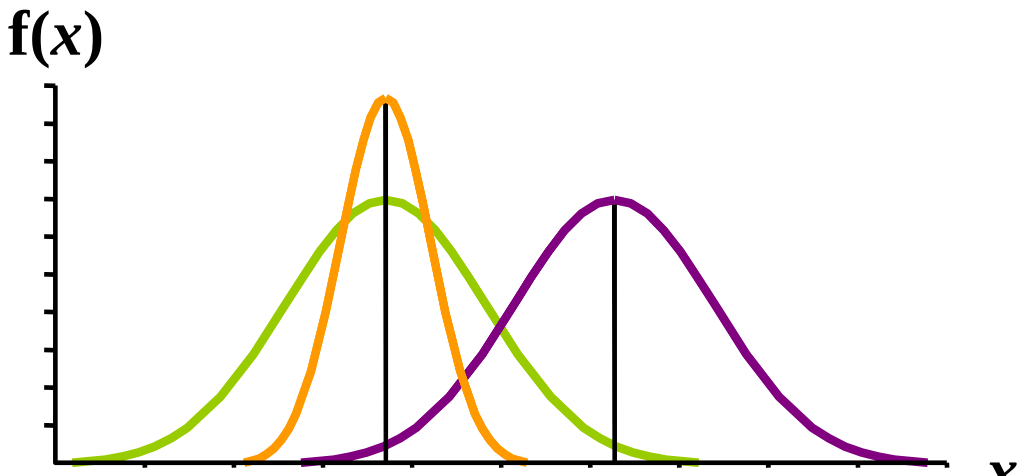






Normal Distribution Probability

Normal distributions differ by mean & standard deviation. Since the shapes are different, the areas under the curves between any two points are also different.



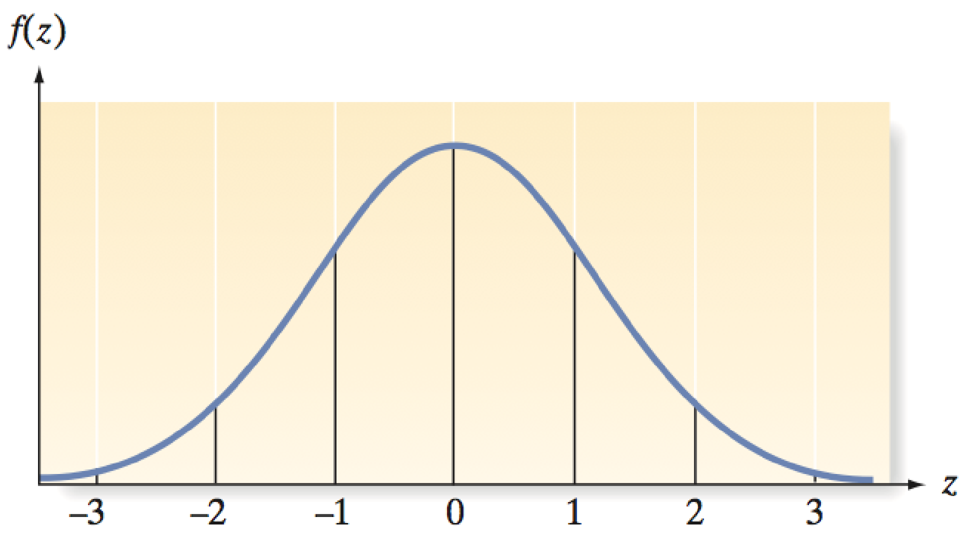
Non-standard Normal Distribution

- To make life easier, all the normal distributions can be converted to a standard normal distribution.
- The standard normal distribution has a mean $\mu = 0$ and $\sigma = 1$.

Standard Normal Distribution

The **standard normal distribution** is a normal distribution with $\mu = 0$ and $\sigma = 1$. A random variable with a standard normal distribution, denoted by the symbol z, is called a standard normal random variable.

Z~N(0, 1).



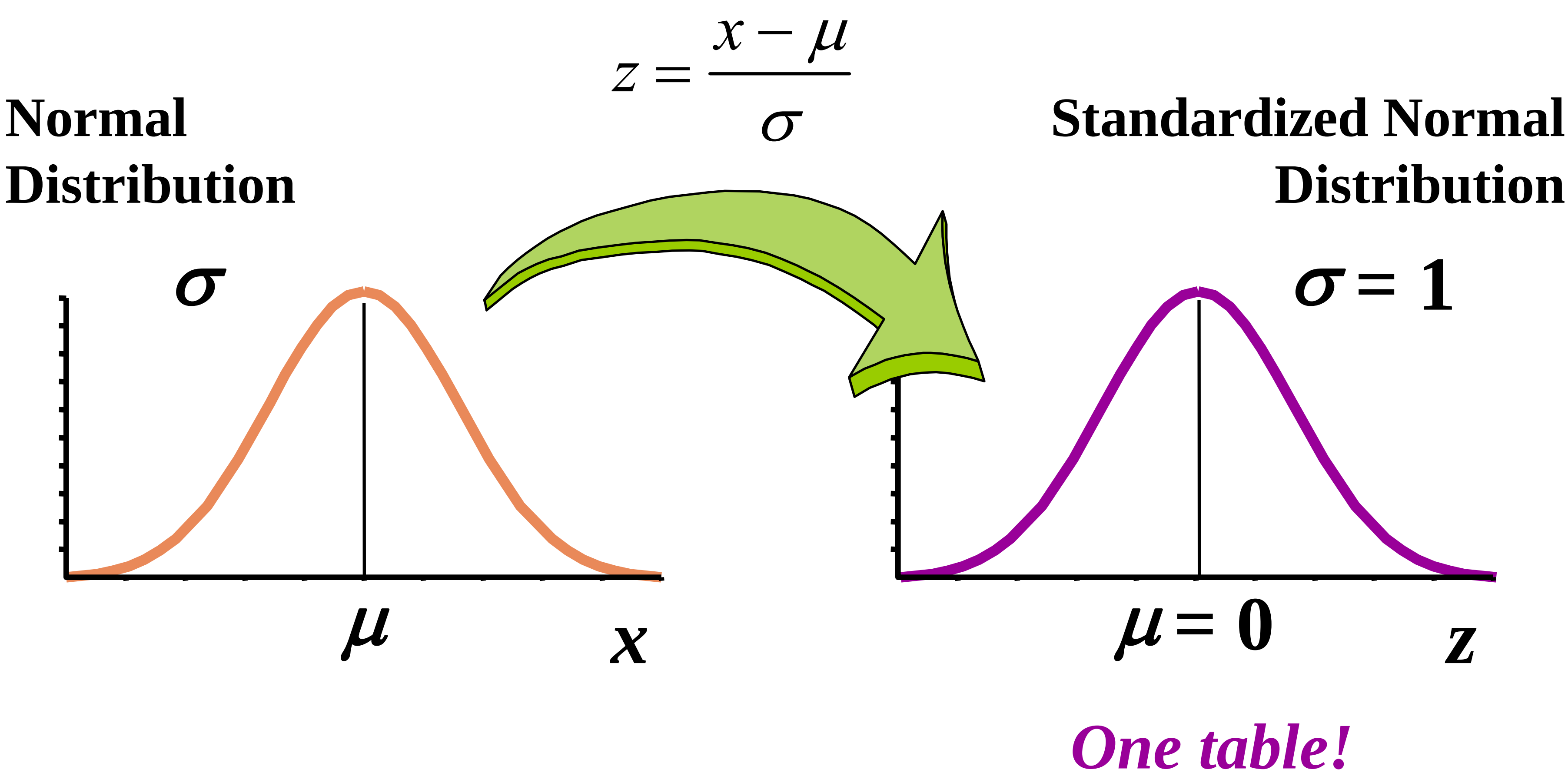
Property of Normal Distribution

If x is a normal random variable with mean μ and standard deviation σ, then the random variable z, defined by the formula

$$z = \frac{x - \mu}{\sigma}$$

has a standard normal distribution. The value z describes the number of standard deviations between x and μ.

Standardize the Normal Distribution



Finding a Probability Corresponding to a Normal Random Variable

1. Sketch normal distribution, indicate mean, and shade the area corresponding to the probability you want.
2. Convert the boundaries of the shaded area from x values to standard normal random variable z

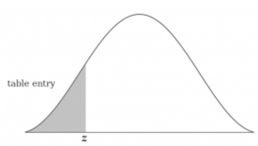
$$z = \frac{x - \mu}{\sigma}$$

   Show the z values under corresponding x values.

3. Use Z score table to find the areas corresponding to the z values.

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414
-0.1	.46017	.45620	.45224	.44828	.44433	.44034	.43640	.43251	.42858	.42465
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-1	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08692	.08534	.08379	.08226
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-2	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-3	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-4	.00003	.00003	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002

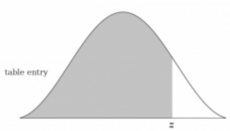
Negative Z score table



Use the negative Z score table below to find values on the left of the mean as can be seen in the graph alongside. Corresponding values which are less than the mean are marked with a negative score in the z-table and respresent the area under the bell curve to the left of z.

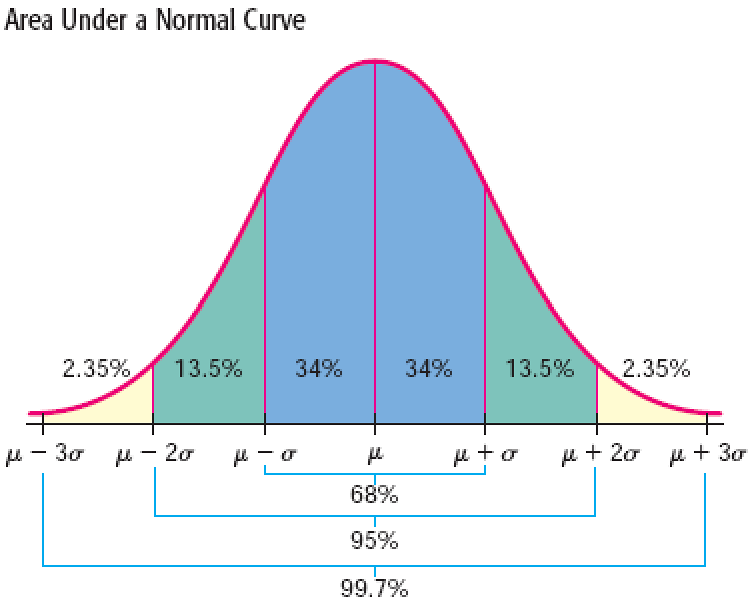
Positive Z score table

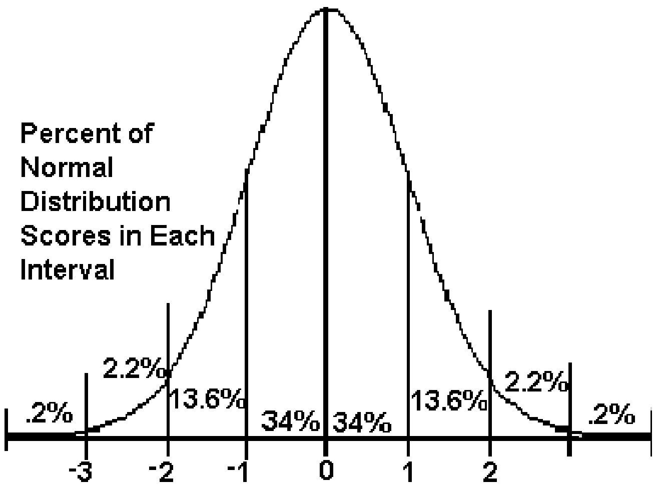
Use the positive Z score table below to find values on the right of the mean as can be seen in the graph alongside. Corresponding values which are greater than the mean are marked with a positive score in the z-table and respresent the area under the bell curve to the left of z.

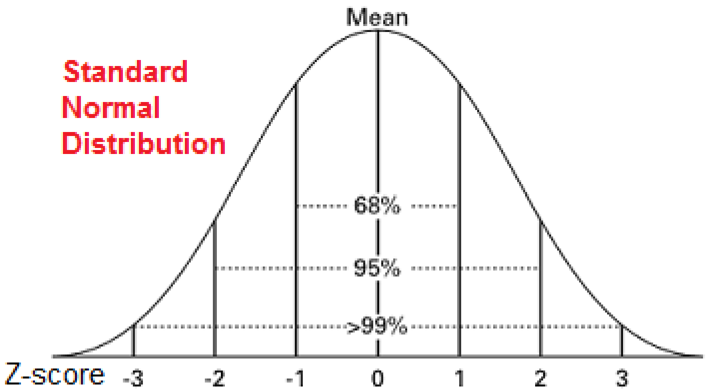


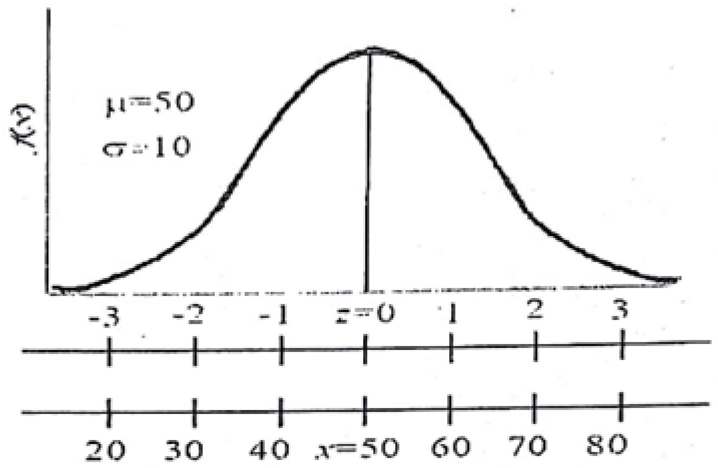
z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
+0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
+0.1	.53983	.54380	.54776	.55172	.55567	.55966	.56360	.56749	.57142	.57535
+0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
+0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
+0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
+0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
+0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
+0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
+0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
+0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
+1	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
+1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
+1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
+1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91308	.91466	.91621	.91774
+1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
+1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
+1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
+1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
+1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
+1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
+2	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
+2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
+2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
+2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
+2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
+2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
+2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
+2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
+2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
+2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
+3	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
+3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
+3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
+3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
+3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
+3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
+3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
+3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
+3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
+3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997
+4	.99997	.99997	.99997	.99997	.99997	.99997	.99998	.99998	.99998	.99998

Standard Normal Distribution

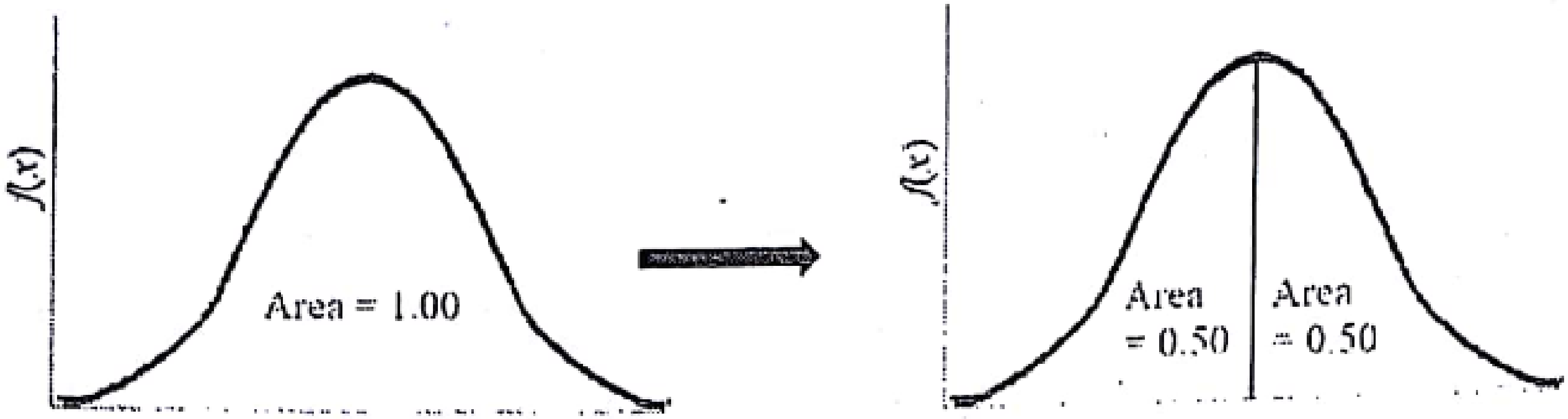






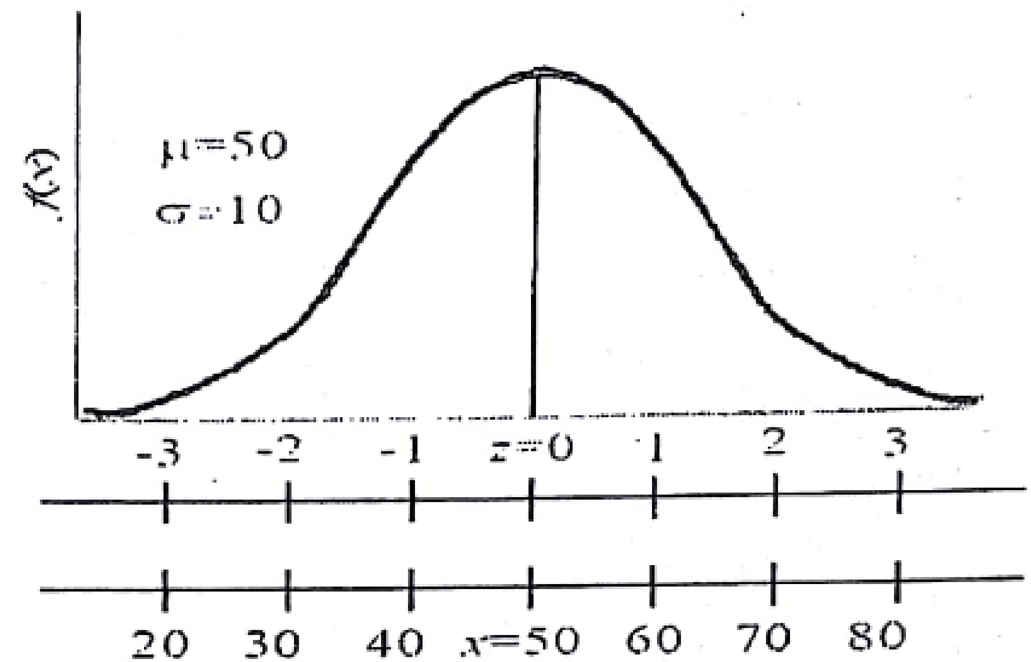


STANDARD NORMAL DISTRIBUTION



$$z = \frac{X - \mu}{\sigma}$$

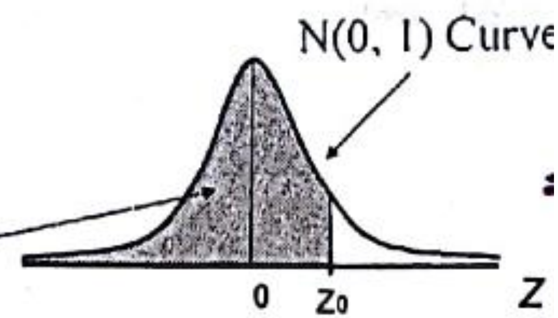
or, $X = \mu + z\sigma$



Finding Areas Under the Standard Normal Curve

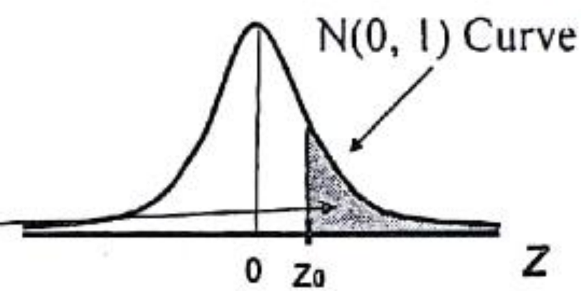
(a) To find the area to the *left* of z_0, find the area that corresponds to z_0 in the Standard Normal Table by using the following rule:

$$P(Z \leq z_0) = N(z_0)$$



(b) To find the area to the *right* of z_0, use the Standard Normal Table to find the area that corresponds to *left* of z_0. Then subtract the area from 1 by using the following formula:

$$P(Z > z_0) = 1 - P(Z \leq z_0) = 1 - N(z_0)$$



Or

$$P(Z > z_0) = P(Z \leq -z_0) = N(-z_0)$$

Non-standard Normal $\mu = 5$, $\sigma = 10$: $P(5 < x < 6.2)$

0= 0.5 0.12 = 0.5478

P(0 < z < 0.12)= **0.5478 – 0.5 =0.0478**

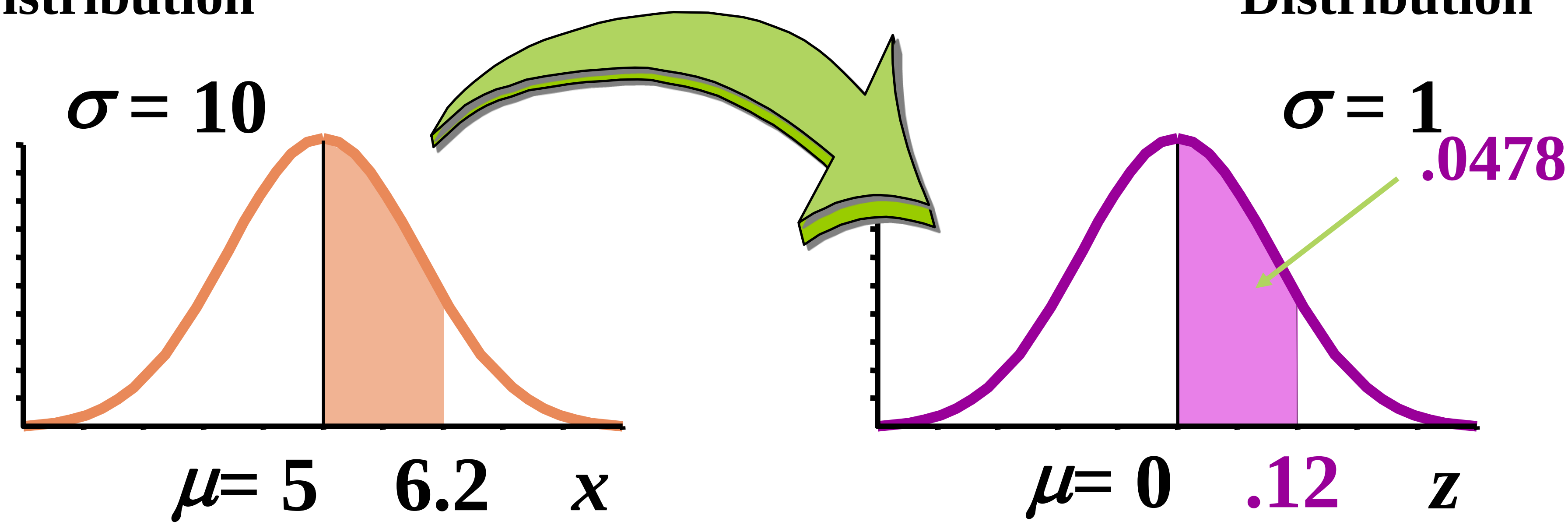
$0 < z < 0.12$

$$z = \frac{x-\mu}{\sigma} = \frac{6.2-5}{10} = .12$$

Z=(5-5)/10=0

Normal Distribution

Standardized Normal Distribution



Shaded area exaggerated

The Standard Normal Table: $P(0 < z < 1.96)$

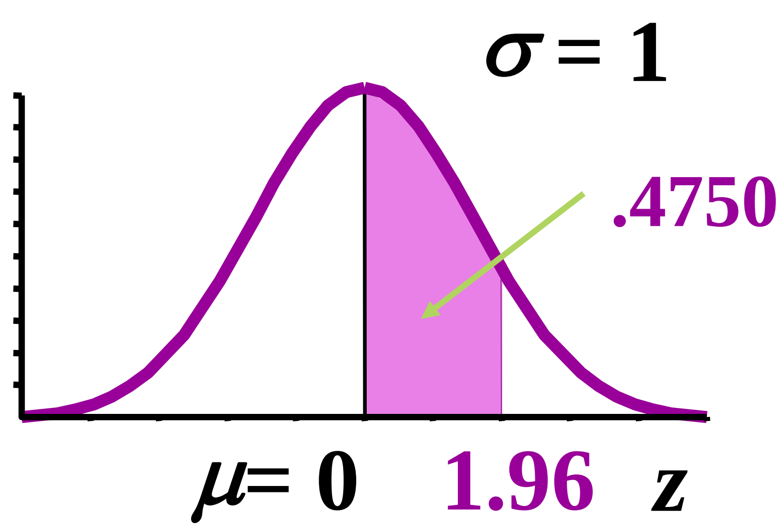
Standardized Normal Probability Table

Z	.04	.05	**.06**
1.8	.96512	.96784	.96856
1.9	.97381	.97441	.97500
2.0	.97932	.9798	.98030
2.1	.98382	.98422	.984616

Probabilities

0= 0.5 1.96 = 0.9750

P(0 < z < 0.12)= 0.9750 – 0.5 = 0.4750

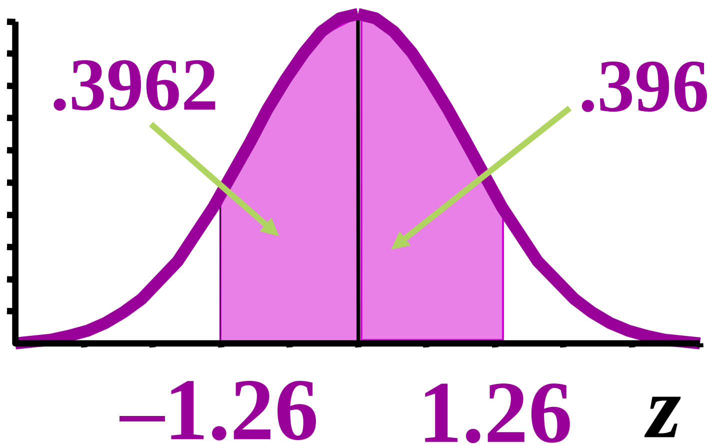


Shaded area exaggerated

The Standard Normal Table: $P(-1.26 \le z \le 1.26)$

Standardized Normal Distribution

$\sigma = 1$



$\mu = 0$

Shaded area exaggerated

-1.26 = 0.1038 1.26 = 0.8962

P(-1.26 ≤ z ≤ 1.26)= **0.8962 – 0.1038 = 0.7924**

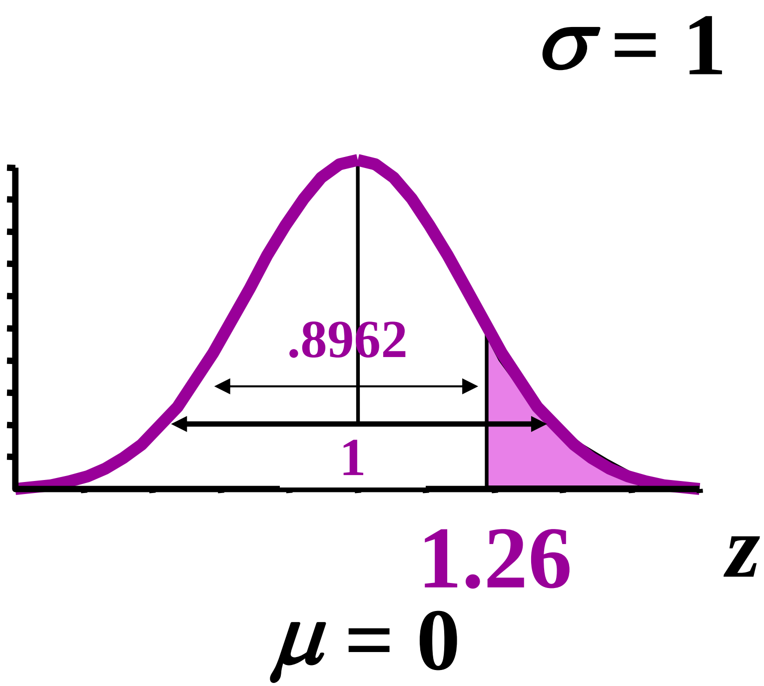
$$P(-1.26 \le z \le 1.26)$$

$$= 0.8962 - 0.1038$$

$$= 0.7924$$

The Standard Normal Table: $P(z > 1.26)$

Standardized Normal Distribution



$P(z > 1.26) = 1 - P(z \leq 1.26$

$= 1 – .8962$

$= .1038$

Or

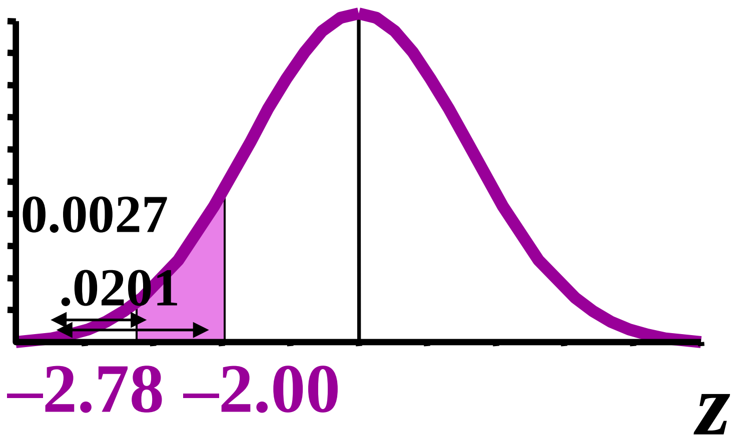
$P(z > 1.26) = P(z \leq -1.26)$

$= .1038$

The Standard Normal Table: $P(-2.78 \le z \le -2.00)$

Standardized Normal Distribution

$\sigma = 1$



$\mu = 0$

Shaded area exaggerated

$P(-2.78 \le z \le -2.00)$

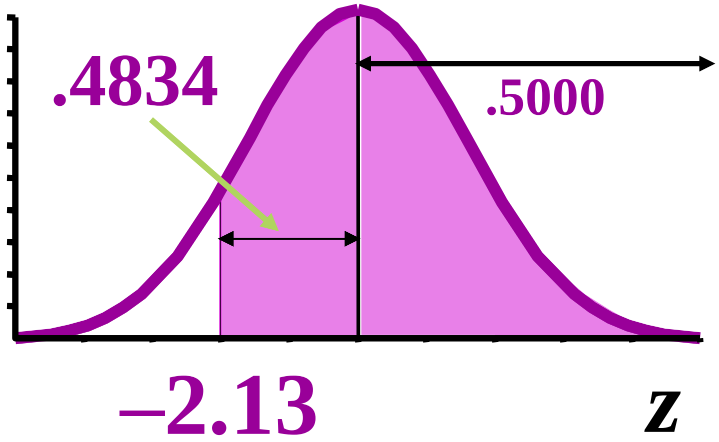
$= .0228 - .0027$

$= .0201$

The Standard Normal Table: $P(z > -2.13)$

Standardized Normal Distribution

$\sigma = 1$



$\mu = 0$

Shaded area exaggerated

$P(z > -2.13) = P(z \leq 2.13)$

$= 0.9834$

OR

$P(z > -2.13) = 1 - P(z \leq -2.13)$

$= 1 - 0.0166$

$= .9834$

Non-standard Normal μ = 5, σ = 10: $P(3.8 \le x \le 5)$

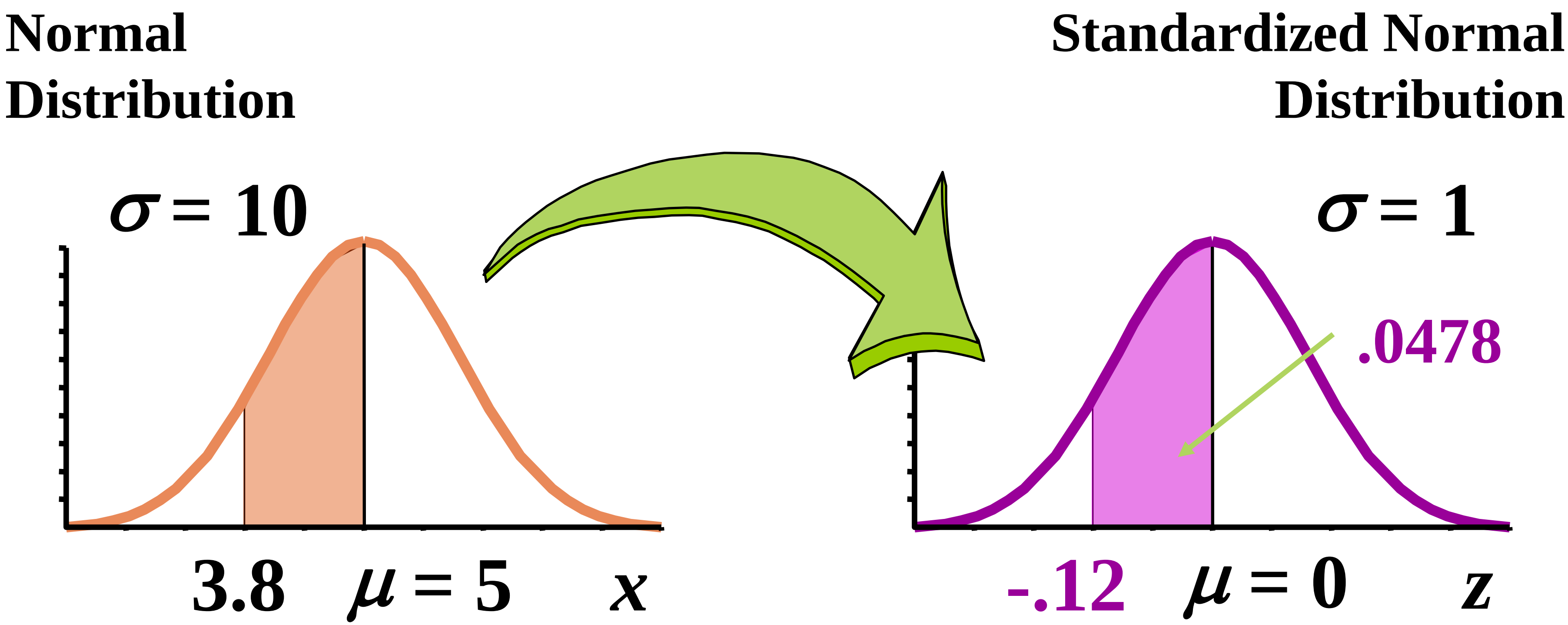
$$z = \frac{x-\mu}{\sigma} = \frac{3.8-5}{10} = -.12$$

Z score for -.12= 0.4522

P(-.12 ≤ **z ≤ 0)**

=**0.5-**0.4522

=0.0478



Shaded area exaggerated

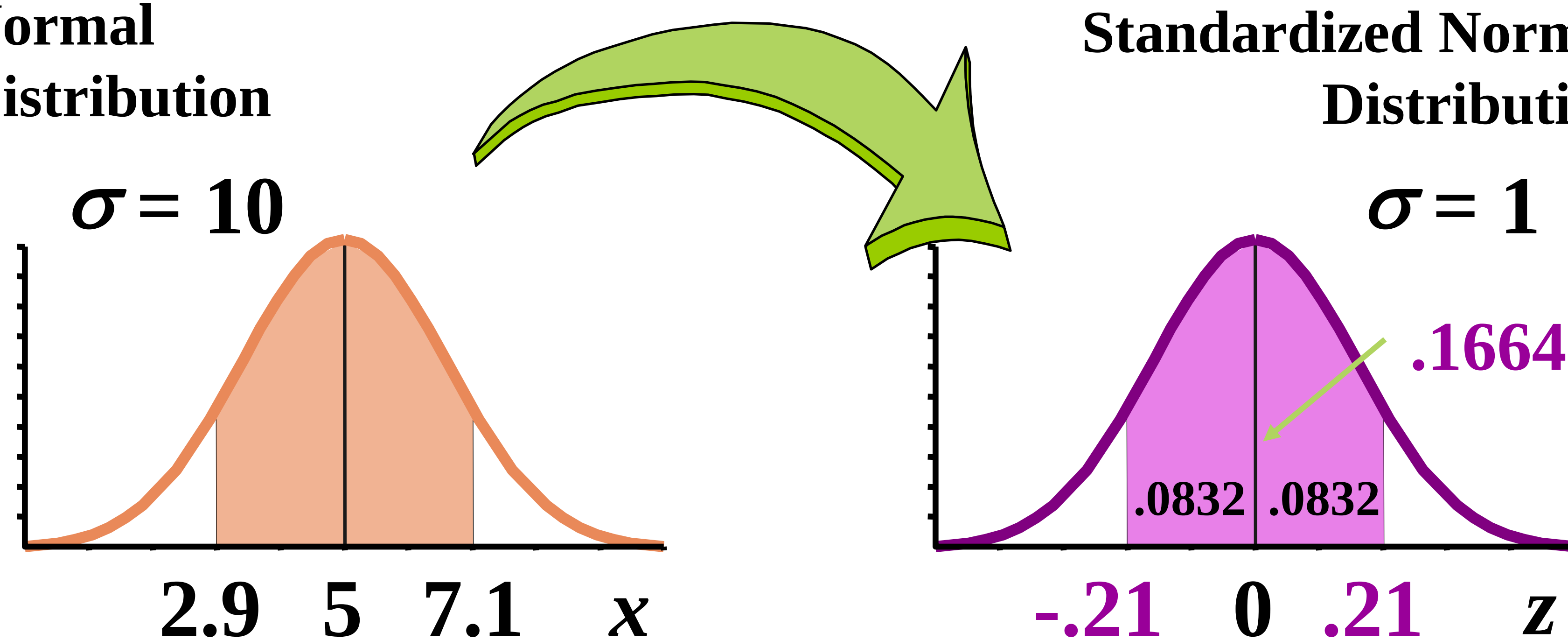
Non-standard Normal $\mu = 5$, $\sigma = 10$: $P(2.9 \le x \le 7.1)$

$$z = \frac{x-\mu}{\sigma} = \frac{2.9-5}{10} = -.21 \qquad z = \frac{x-\mu}{\sigma} = \frac{7.1-5}{10} = .21$$

Normal Distribution

Standardized Normal Distribution

$\sigma = 10$ $\qquad$ $\sigma = 1$



Shaded area exaggerated

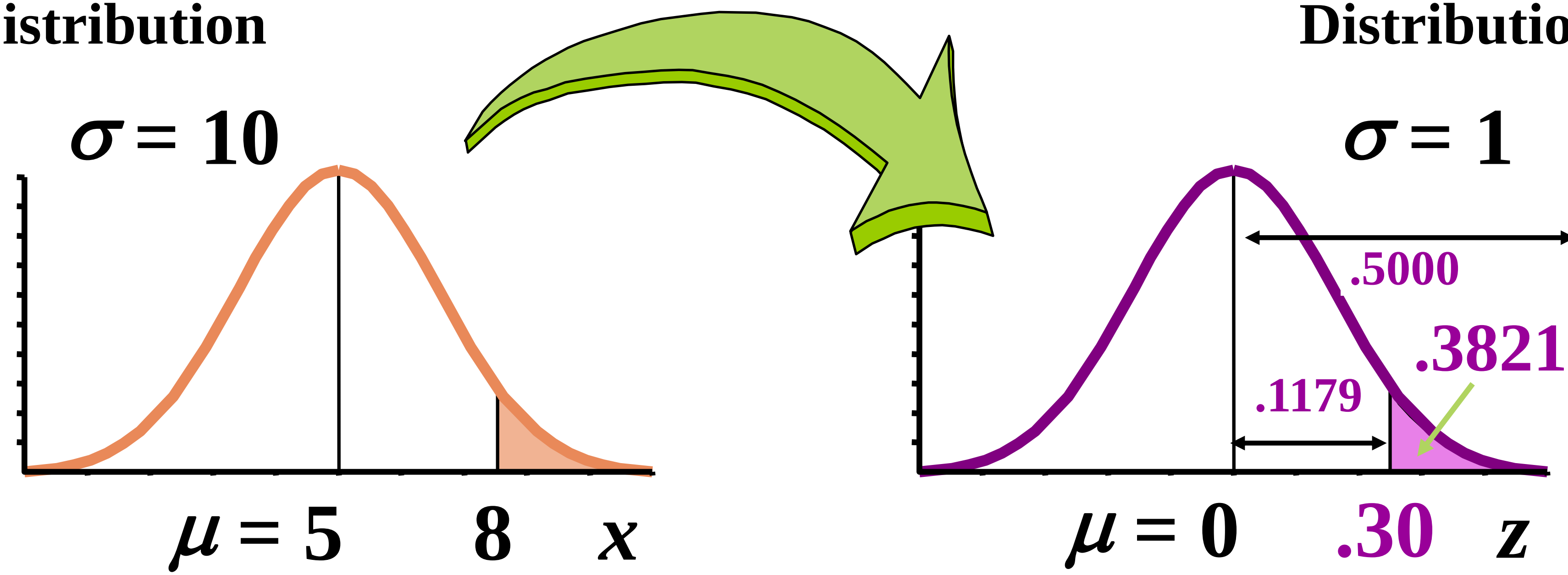
P(-.21≤ z ≤ 0.21)
=0.5832-0.4522
=0.1664

Non-standard Normal $\mu = 5$, $\sigma = 10$: $P(x \geq 8)$

$$z = \frac{x - \mu}{\sigma} = \frac{8 - 5}{10} = .30$$

Normal Distribution

Standardized Normal Distribution



Shaded area exaggerated

P(z ≥ 0.3) = 1- P(z≤ 0.3)

=1- 0.6179

= 0.3821

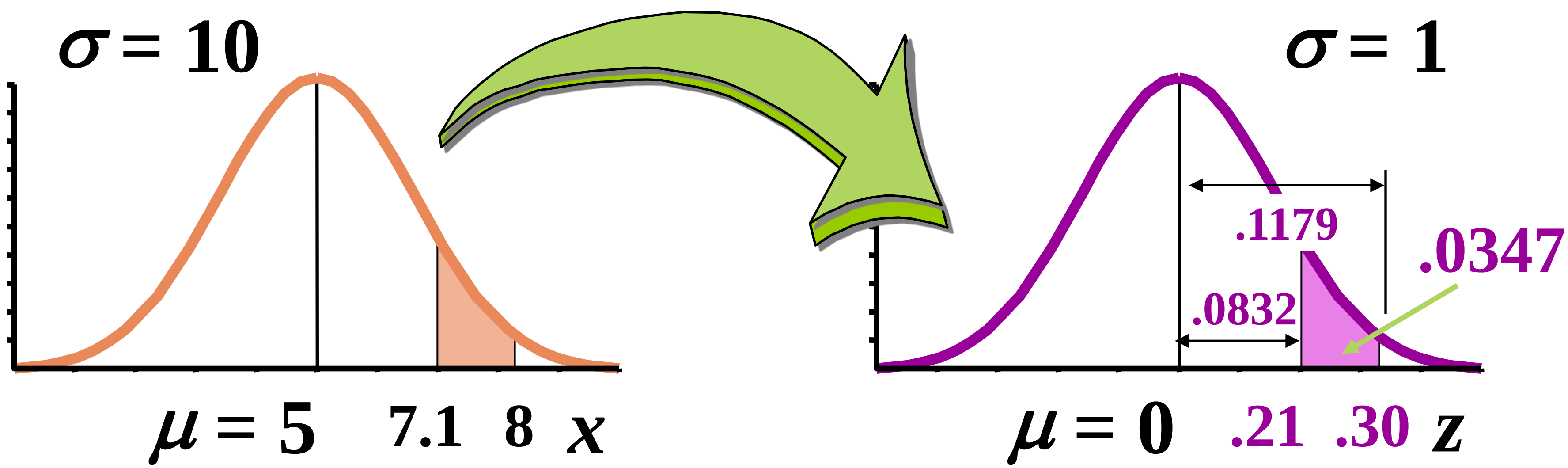
Non-standard Normal μ = 5, σ = 10: P(7.1 ≤ X ≤ 8)

$$z = \frac{x-\mu}{\sigma} = \frac{7.1-5}{10} = .21$$

$$z = \frac{x-\mu}{\sigma} = \frac{8-5}{10} = .30$$

Normal Distribution

Standardized Normal Distribution



Shaded area exaggerated

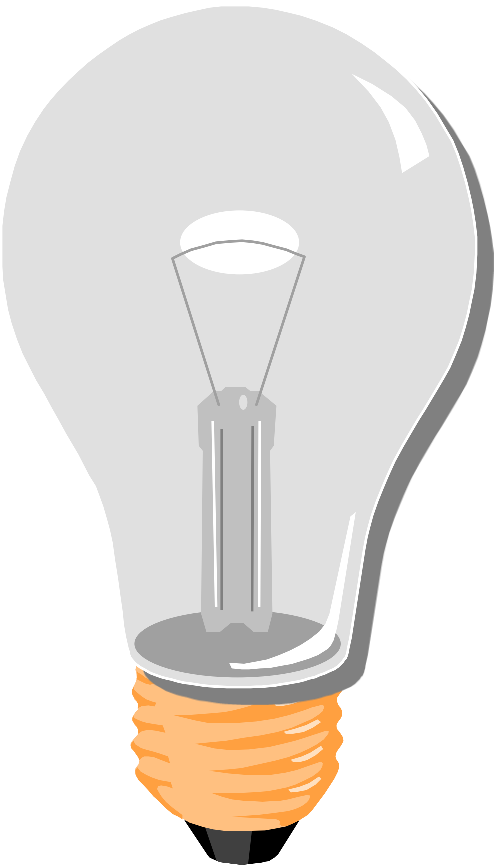
P(0.21≤ z ≤ 0.3)
=0.6179 - 0.5832
=0.0347

Normal Distribution Thinking Challenge

You work in Quality Control for GE. Light bulb life has a **normal distribution** with μ = **2000** hours and σ = **200** hours. What's the probability that a bulb will last

A. between **2000** and **2400** hours?

B. **less** than **1470** hours?

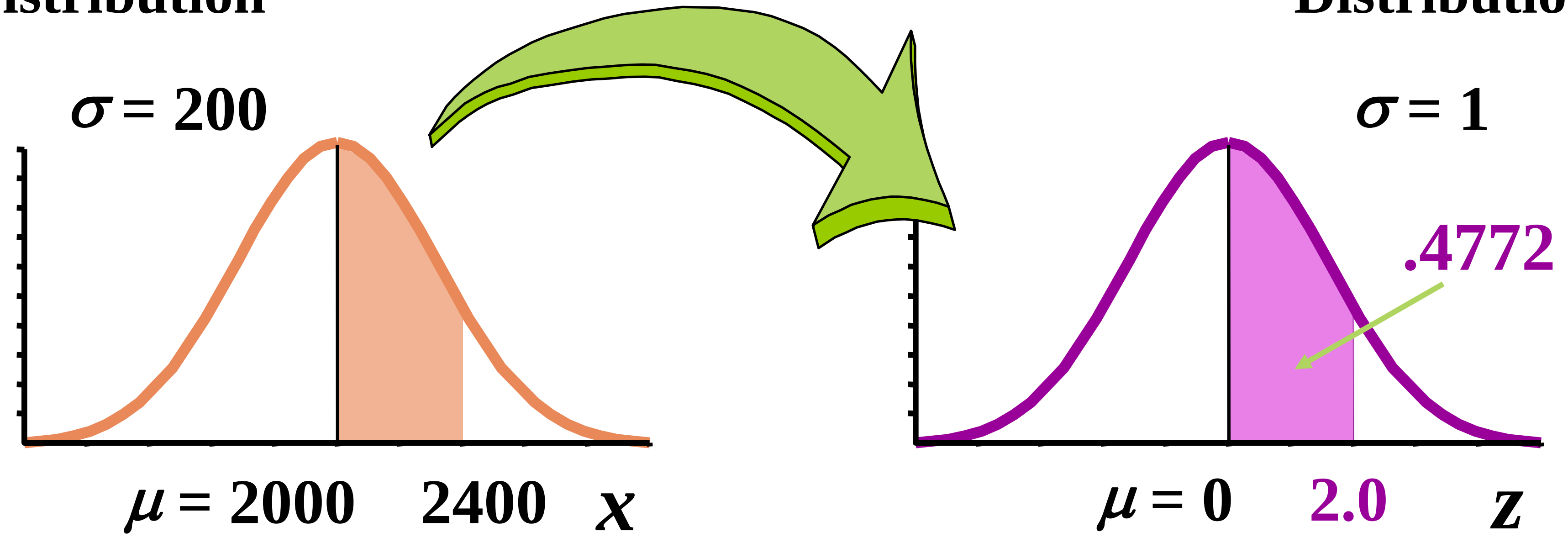


Solution* $P(2000 \le x \le 2400)$

$$z = \frac{x - \mu}{\sigma} = \frac{2400 - 2000}{200} = 2.0$$

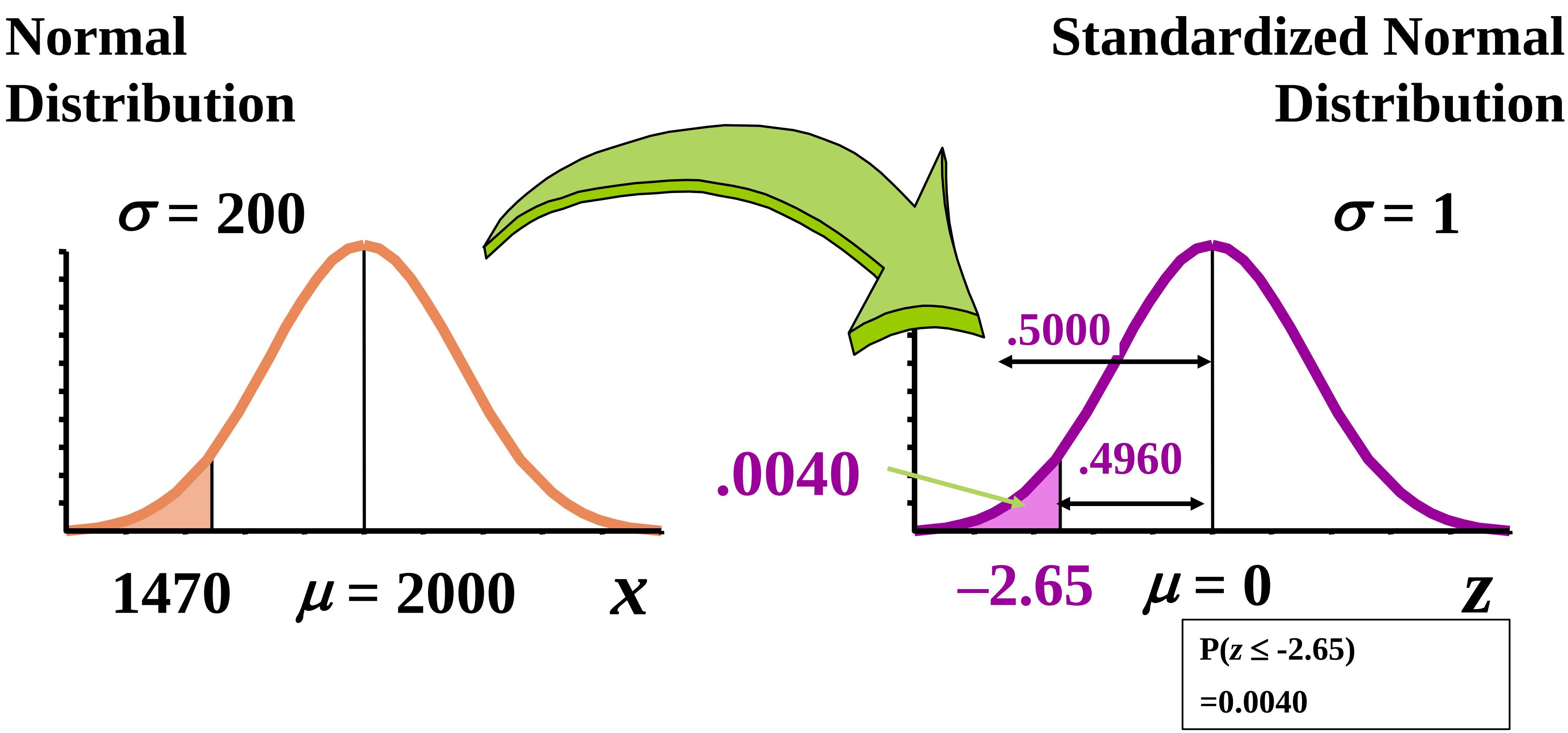
Normal Distribution

Standardized Normal Distribution



Solution* $P(x \le 1470)$

$$z = \frac{x - \mu}{\sigma} = \frac{1470 - 2000}{200} = -2.65$$



Example

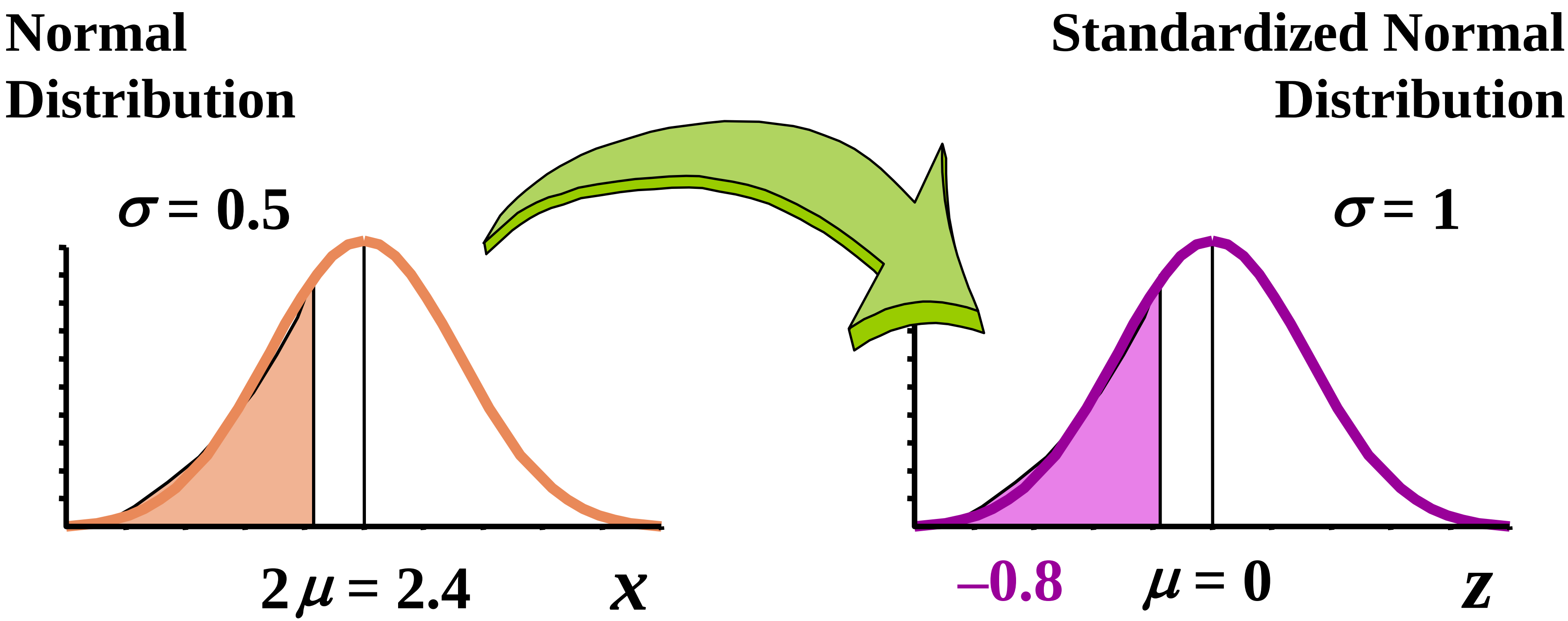
A survey in Jordan indicates that pharmacies use their computers in an average of 2.4 years before upgrading to a new machine. The standard deviation is 0.5 year. A pharmacy is selected at random. Find the probability that the pharmacy will use it for less than or equal 2 years before upgrading. Assume that the variable X is normally distributed?

Continued

Solution

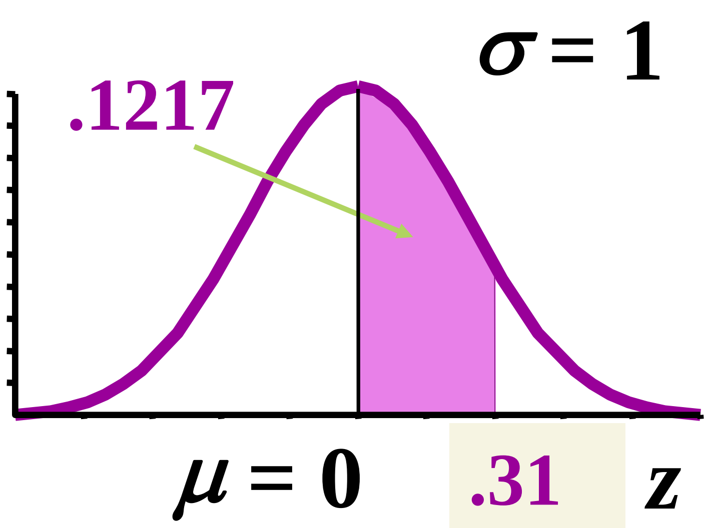
We need to find P(X ≤ 2) as follows: (x-μ)/σ = (2-2.4)/0.5 = -0.8

So P(z ≤ -0.8) = 0.2119



Finding z-Values for Known Probabilities

What is Z, given $P(z) = .1217$?



Shaded area exaggerated

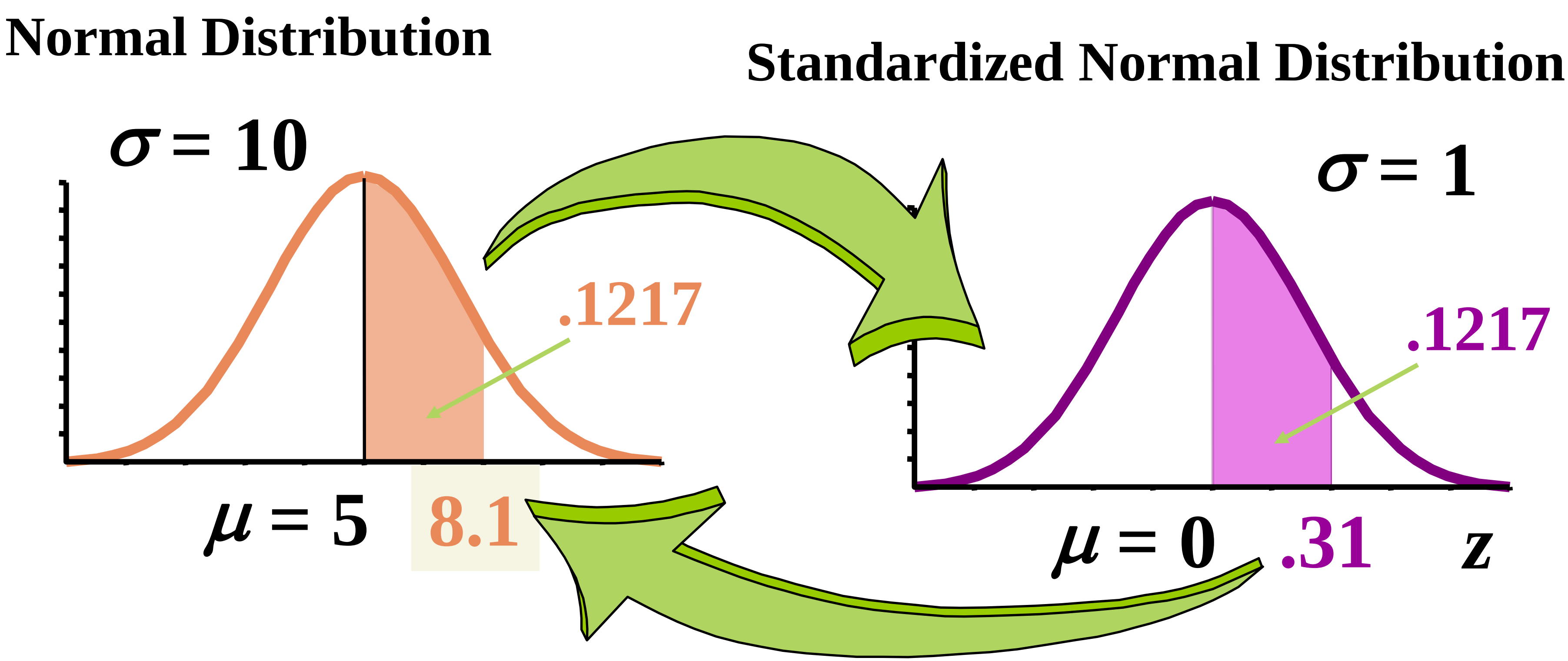
z	0	0.01	0.02
+0	.50000	.50399	.50798
+0.1	.53983	.54380	.54776
+0.2	.57926	.58317	.58706
+0.3	.61791	.62172	.62552

The probability between the z and the mean =0.1217

0.1217 + 0.500 = 0.6217

So it is corresponding to 0.31

Finding x Values for Known Probabilities



$$x = \mu + z \cdot \sigma = 5 + (.31)(10) =$$

Shaded areas exaggerated

Example

The daily sales volume in JD for a given pharmacy in Jordan is normally distributed with a mean of 67 JD per day and a standard deviation of 4 JD per day. Find the daily sales volume x corresponding to z-score 1.96, -2.33, and 0?

- $z = 1.96$, $x = 67+1.96(4) = 74.84$ JD per day
- $z = -2.33$, $x = 67+(-2.33)(4) = 57.68$ JD per day.
- $z = 0$, $x = 67+0(4) = 67$ JD per day.
- Notice that 74.84 JD is above the mean, 57.68 JD is below the mean, and 67 JD is equal to the mean.

Other Discrete Distributions: Poisson

Poisson Distribution

1. Number of events that occur in an interval
   - events **per unit**
     - Time, Length, Area, Space
2. Examples
   - Number of customers arriving in 20 minutes
   - Number of strikes per year in the U.S.
   - Number of defects per lot (group) of DVD’s

Characteristics of a Poisson Random Variable

1. Consists of counting number of times an event occurs during a given unit of time or in a given area or volume (any unit of measurement).
2. The probability that an event occurs in a given unit of time, area, or volume is the same for all units.
3. The number of events that occur in one unit of time, area, or volume is independent of the number that occur in any other mutually exclusive unit.
4. The mean number of events in each unit is denoted by λ.

Poisson Probability Distribution Function

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!} \qquad (x = 0, 1, 2, 3, \ldots)$$

$$\mu = \lambda$$

$$\sigma^2 = \lambda$$

$p(x)$ = Probability of x given λ

λ = Mean (expected) number of events in unit

e = 2.71828 . . . (base of natural logarithm)

x = Number of events **per unit**

Poisson Probability Distribution Function

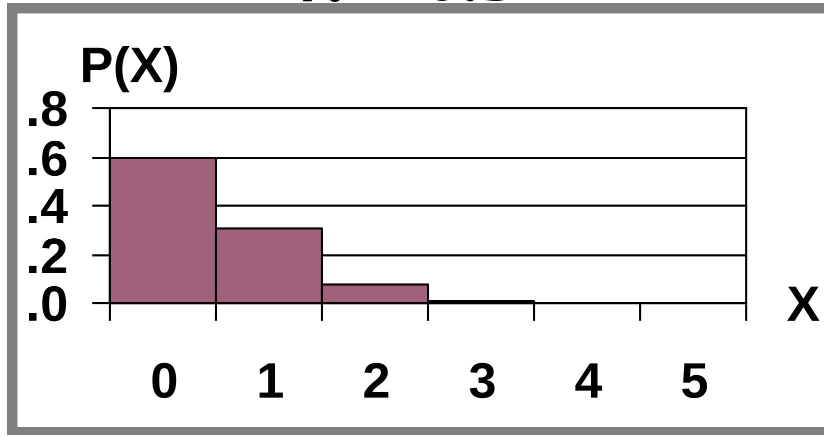
Mean

$$\mu = E(x) = \lambda$$

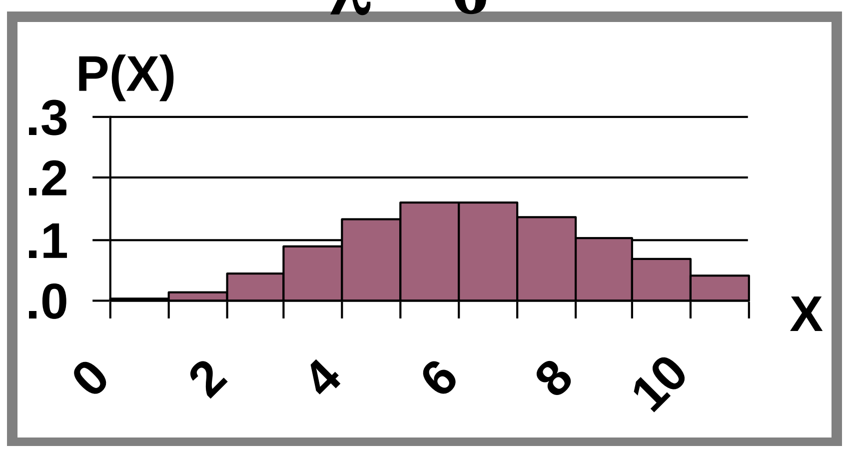
Standard Deviation

$$\sigma = \sqrt{\lambda}$$

λ = 0.5



λ = 6



Poisson Distribution Example

Customers arrive at a rate of 72 per hour. What is the probability of 4 customers arriving in 3 minutes?

Poisson Distribution Solution

72 Per Hr. = 1.2 Per Min. = 3.6 Per 3 Min. Interval

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$p(4) = \frac{(3.6)^4 e^{-3.6}}{4!} = .1912$$

Thinking Challenge

You work in Quality Assurance for an investment firm. A clerk enters **75** words per minute with **6** errors per hour. What is the probability of **0 errors** in a **255-word** bond transaction?

Poisson Distribution Solution: Finding λ*

- 75 words/min = (75 words/min)(60 min/hr)

  = 4500 words/hr

- 6 errors/hr = 6 errors/4500 words

  = **.00133** errors/word

- In a **255-**word transaction (interval):

  λ = **(.00133** errors/word)**(255** words)

  = .34 errors/255-word transaction

Poisson Distribution Solution: Finding p(0)*

$$p(x) = \frac{\lambda^{x} e^{-\lambda}}{x!}$$

$$p(0) = \frac{(.34)^{0} e^{-.34}}{0!} = .7118$$