Statistics

لجان الدُّفعات

DOPAMINE ACADEMY

Subject: Lecture 12+13

Lec 12

Subject: probability Distribution

↓	↓
Discrete	Continuous
↓	↓
Binomial Distribution	Normal Distribution

Characteristis of a Binomial Experiment :-

1. the experiment consists of n identical trials
2. there are only two possible outcomes on each trial
3. the probability of S, remains the same from trial to trial (P) → P + q = 1 (q: probability of F)
4. the trials are independent
5. $x \sim Bin(n, p)$ n: number of trial   p: probability of success in one trial.

Approximate shape for the distribution of X is given as follows :-

1) If $P < 0.5$ then right-skewed "positive" distribution.
2) If $P = 0.5$ then symmetric distribution
3) If $P > 0.5$ then left-skewed "negative" distribution.

Mean $(\mu) = np$ standard deviation $(\sigma) = \sqrt{npq}$

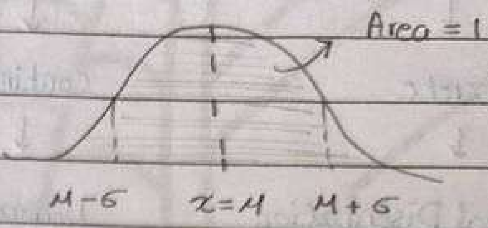
Ex:-

x	0	1	2	3	4	5
$P(X=x)$	0.289	0.407	0.229	0.065	0.009	0.001

1. Probability that no one of patients will be cured? $x=0$ $P(x=0) = 0.289$
2. Probability that exactly 3 patients will be cured? $x=3$ $P(x=3) = 0.065$
3. probability that at least 2 patients will be cured? $P(x \geq 2) = 0.304$
4. Propability that at most 4 patients will be cured? $P(x \leq 4) = 0.999$

Subject :

Normal distribution :-



1: Bell shaped "Symmetrical"

2. Mean = Meadian = Mode

3. Continuous probability distribution.

- We have "3" standard deviation 68 – 95 – 99.7
  σ 2σ 3σ

- The standard normal distribution has a mean $\mu=0$ and $\sigma=1$ $Z\sim N(0,1)$

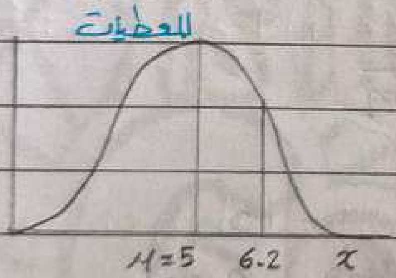
Normal distribution $\longrightarrow$ standard Normal distribution

$x\sim N(\mu,\sigma^2)$ $\quad Z=\dfrac{x-\mu}{\sigma} \rightarrow Z\sim N(\mu,\sigma^2)$

Z: describes the number of standard deviations between x and μ.

ex: non standard normal $\mu=5$, $\sigma=10$, $P(5<x<6.2)$?

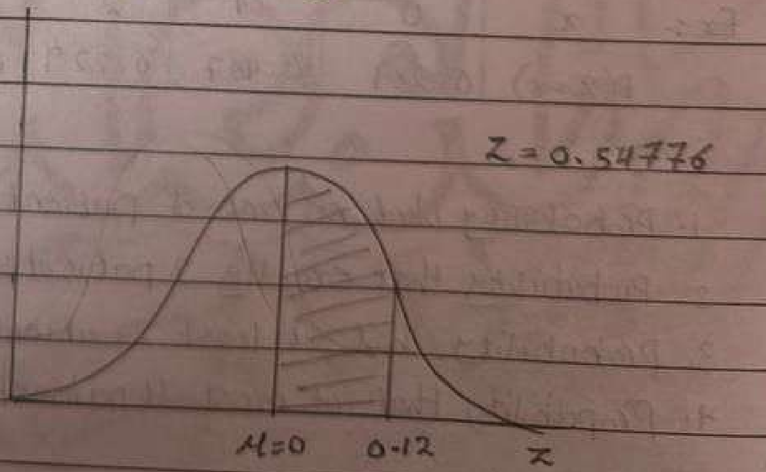
1. نرسم الرسمة ونحط المعطيات



2. نحول $x \rightarrow z$

$$Z=\frac{x-\mu}{\sigma}=\frac{5-5}{10}=0$$

$$Z=\frac{x-\mu}{\sigma}=\frac{6.2-5}{10}=0.12$$



$0.54776-0.5=0.04776$

ex: $P(0<z<1.96)$

$\rightarrow 0.97500 - 0.5 = 0.47500$

ex: $\overset{0.89617}{P(z>1.26)}$

$\rightarrow 1 - 0.89617 = 0.10383$

ex: $\overset{0.01659}{P(z>-2.13)}$

$\rightarrow 1 - 0.01659 = 0.98341$

ex: $\overset{\cancel{0.10383} \quad \cancel{0.89617}}{P(-1.26 \leq z \leq 1.26)}$

$\rightarrow 0.89617 - 0.10383 = 0.79234$

ex: $P(\overset{0.0027}{-2.78} \leq z \leq \overset{0.0228}{-2.00})$

$\rightarrow 0.0228 - 0.0027 = 0.0201$

ex: $\overset{\cancel{0.4522}}{P(3.8 \leq x \leq 5)}$ $\mu = 5$, $\sigma = 10$

$P(-0.12 \leq z \leq 0)$

$\rightarrow 0.5 - 0.4522 = 0.0478$

Lec 13

Approximating a Binomial Distribution with a Normal Distribution :-

Theorem: Normal distribution "approximation" to a Binomial distribution.

- If $np>5$ and $n(1-p)>5$

Correction for continuity :-

1. A 0.5 Unit adjustment to discrete variable.
2. Used when approximating a discrete distribution with a continuous distribution.
3. Improves accuracy.

Interval $\mu + 3\sigma$ If interval lies in the range $0 \rightarrow n$ the normal distribution will provide a reasonable approximation to the probabilities of most Binomial events.

Subject :

What is the normal approximation of $P(x=4)$ given $n=10, p=0.5$?

$P(3.5 < x < 4.5)$

Interval $= \mu \pm 3\sigma = 5 \pm 4.74 = (0.26, 9.74)$ $0 \rightarrow 10$ ✓

$z_1 = \frac{3.5-5}{\sqrt{2.5}} = -0.95$ $z_2 = \frac{4.5-5}{\sqrt{2.5}} = -0.32$

$P(\underset{0.17106}{-0.95} < z < \underset{0.37448}{-0.32}) = 0.37448 - 0.17106 = 0.20342$